Lecture 24: Reductions

CSE 373: Data Structures and Algorithms

Slido Event #8930126
https://app.sli.do/event/x9Uf8FRMoxbJA52gEN6E1U

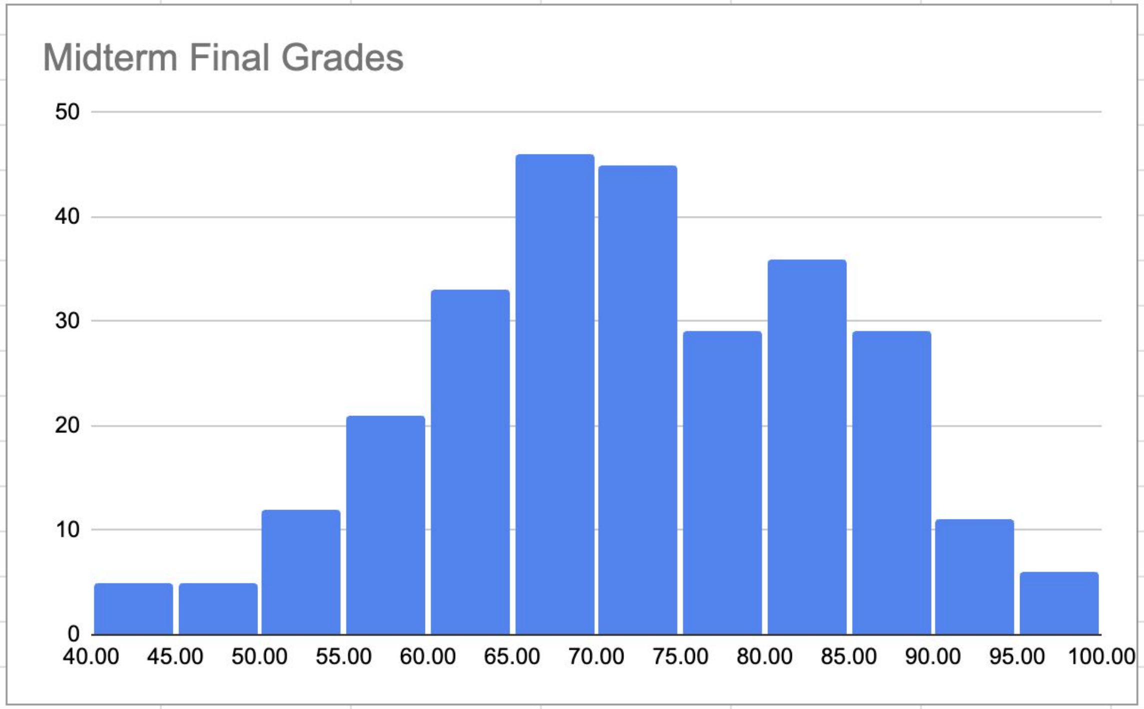
Warm Up

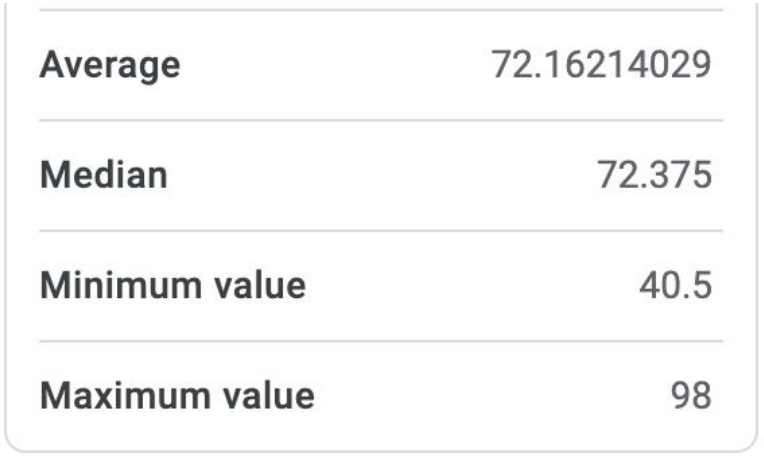
Dynamic Programming is…

A. A programming technique used to dynamically allocate the machine running your logic to allow for larger scale processing
B. A way to make recursion faster
[✓ C. An algorithmic optimization technique that reduces redundant calculations by recognizing the final solution is a summation of smaller subproblems]
D. When you store previous calculations in a memo to use in later recursive calls

Announcements

- Practice final posted (see Ed post)
- TA lead final review this Wednesday after lecture (in lecture hall)





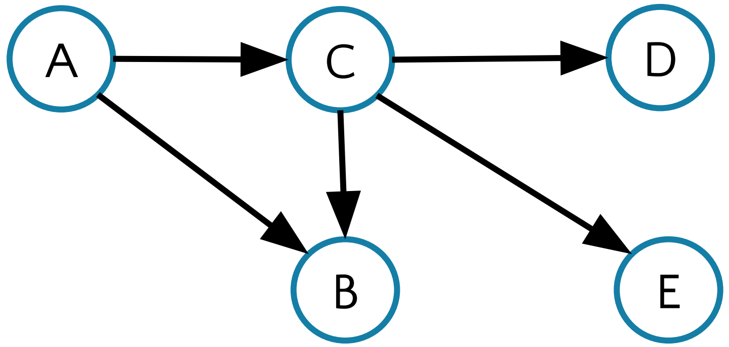
Average	72.16214029
Median	72.375
Minimum value	40.5
Maximum value	98

The 2 Sat Solver

Reductions

Review: Topological Sort

Perform a topological sort of the following DAG



Topological Sort

Given: a directed graph G
Find: an ordering of the vertices so all edges go from left to right.

Directed Acyclic Graph (DAG)

A directed graph without any cycles.

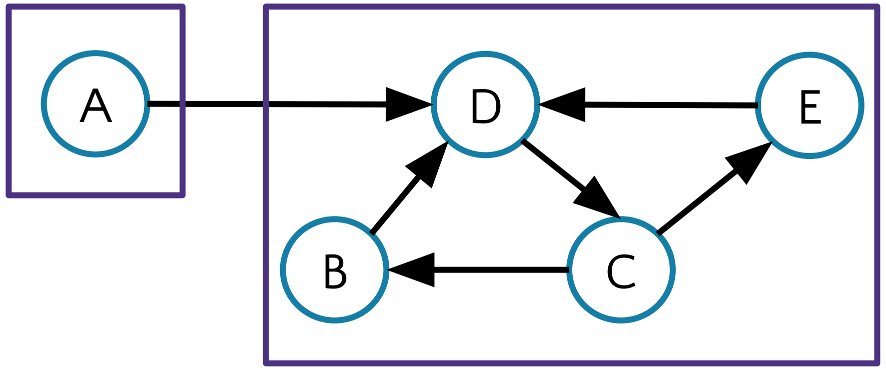
A C B D E

If a vertex doesn't have any edges going into it, we add it to the ordering
If the only incoming edges are from vertices already in the ordering, then add to ordering

Strongly Connected Components

Strongly Connected Component

A subgraph C such that every pair of vertices in C is connected via some path **in both directions,** and there is no other vertex which is connected to every vertex of C in both directions.



Note: the direction of the edges matters!

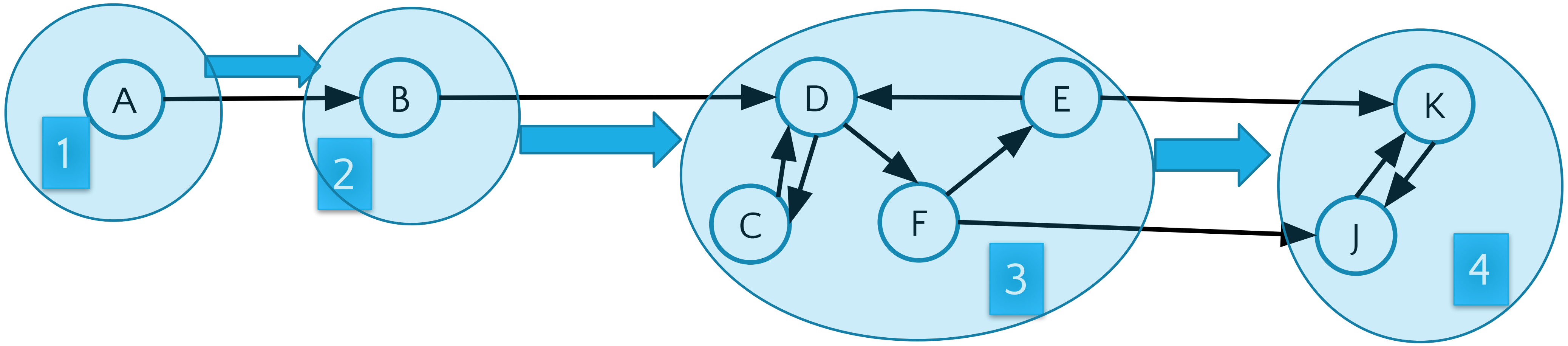
Why Find SCCs?

Graphs are useful because they encode relationships between arbitrary objects.

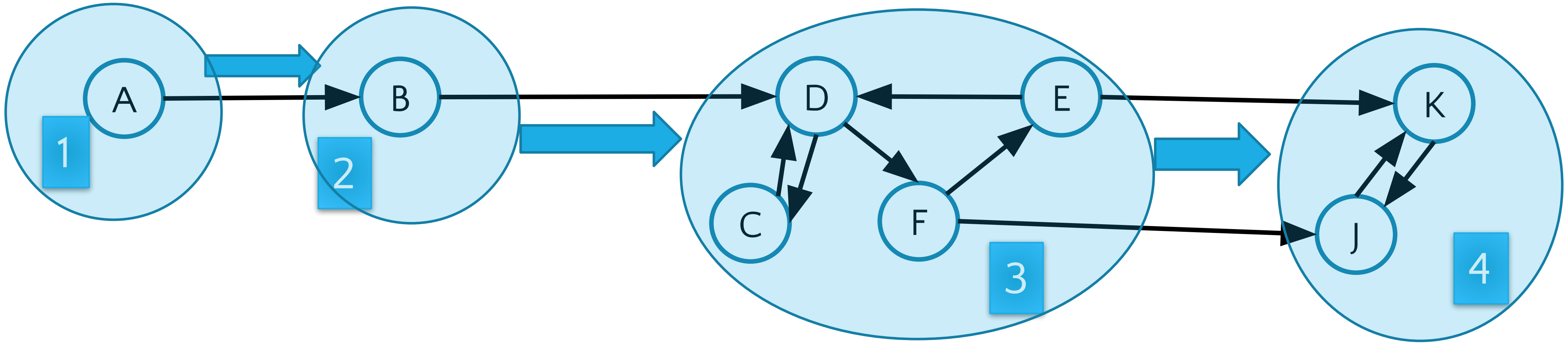
We've found the strongly connected components of G.

Let's build a new graph out of them! Call it H

- Have a vertex for each of the strongly connected components
- Add an edge from component 1 to component 2 if there is an edge from a vertex inside 1 to one inside 2.



Why Find SCCs?



That's awful meta. Why?

This new graph summarizes reachability information of the original graph.

- I can get from A (of G) in 1 to F (of G) in 3 if and only if I can get from 1 to 3 in H.

Why Must H Be a DAG?

H is always a DAG (i.e. it has no cycles). Do you see why?

If there were a cycle, I could get from component 1 to component 2 and back, but then they're actually the same component!

Takeaways

Finding SCCs lets you **collapse** your graph to the meta-structure.
If (and only if) your graph is a DAG, you can find a topological sort of your graph.

Both of these algorithms run in linear time.

Just about everything you could want to do with your graph will take at least as long.

You should think of these as **"almost free" preprocessing** of your graph.
Your other graph algorithms only need to work on

- topologically sorted graphs
- strongly connected graphs

A Longer Example

The best way to really see why this is useful is to do a bunch of examples.

We don’t have time. The second best way is to see one example right now…

This problem doesn’t *look like* it has anything to do with graphs

- no maps
- no roads
- no social media friendships

Nonetheless, a graph representation is the best one.

I don’t expect you to remember the details of this algorithm.

I just want you to see:

- graphs can show up anywhere
- SCCs and Topological Sort are useful algorithms

Example Problem: Final Review

We have a long list of types of problems we might want to put on the final.

- Heap insertion problem, big-O problems, finding closed forms of recurrences, graph modeling…
- What if we let the students choose the topics?

To try to make you all happy, we might ask for your preferences. Each of you gives us two preferences of the form "I [do/don't] want a [topic] problem on the exam" *

We'll assume you'll be happy if you get at least one of your two preferences.

Final Creation Problem

Given: A list of 2 preferences per student.
Find: A set of questions so every student gets at least one of their preferences (or accurately report no such question set exists).

*This is NOT how Kasey is making the final ;)

Review Creation: Take 1

We have Q kinds of questions and S students.
What if we try every possible combination of questions.
How long does this take? $O(2^Q S)$
If we have a lot of questions, that's **really** slow.

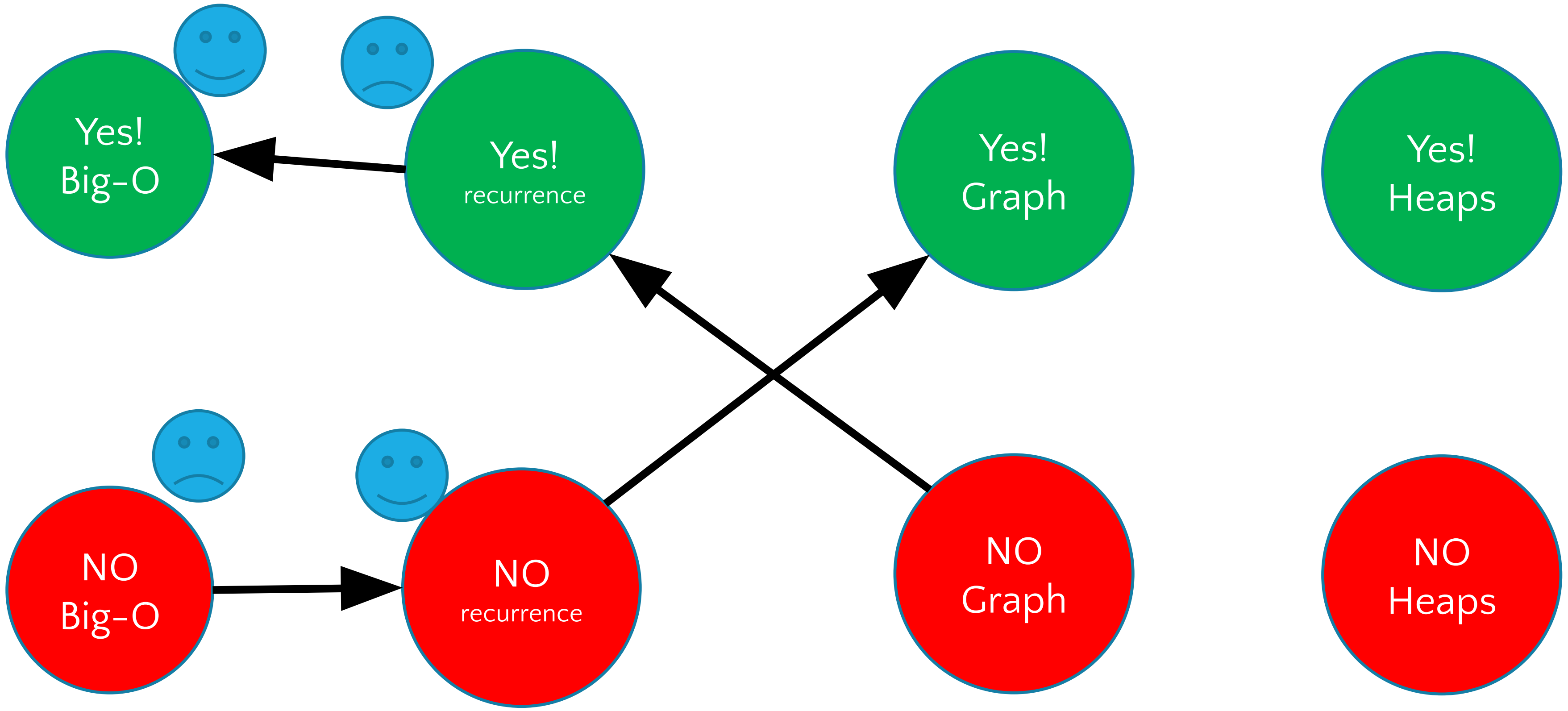
Instead we're going to use a graph
What should our vertices be?

Review Creation: Take 2

Each student introduces new relationships for data:

Let's say your preferences are represented by this table:

Problem	YES	NO
Big-O	X	
Recurrence		X
Graph		
Heaps		



Problem	YES	NO
Big-O		
Recurrence	X	
Graph	X	
Heaps		

If we don't include a big-O proof, can you still be happy?
If we do include a recurrence can you still be happy?

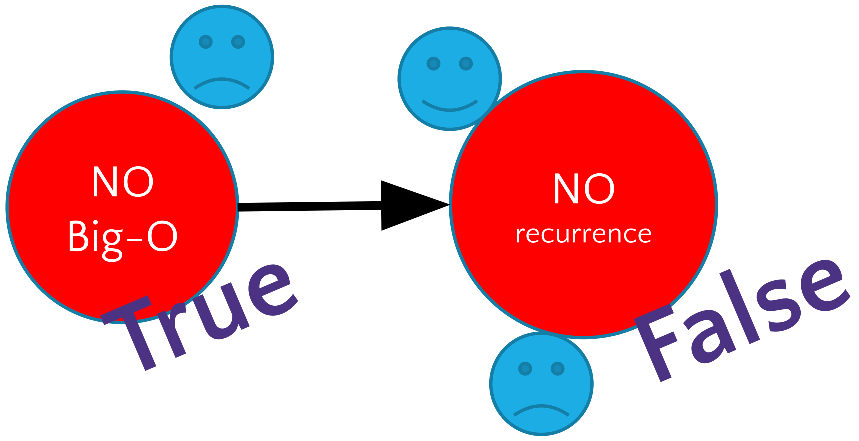
Review Creation: Take 2

Hey we made a graph!

What do the edges mean?

Each edge goes from something making someone unhappy, to the only thing that could make them happy.

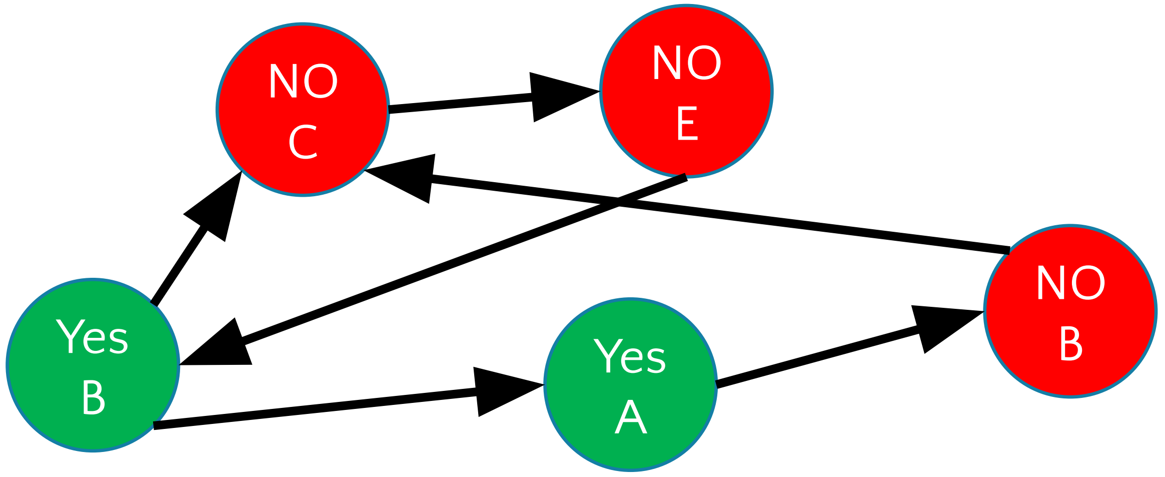
- We need to avoid an edge that goes TRUE THING FALSE THING



Review Creation: Take 2

We need to avoid an edge that goes TRUE THING FALSE THING

Let’s think about a single SCC of the graph.



Can we have a true and false statement in the same SCC?

What happens now that Yes B and NO B are in the same SCC?

Final Creation: SCCs

The vertices of a SCC must either be all true or all false.

Algorithm Step 1: Run SCC on the graph. Check that each question-type-pair are in different SCC.

Now what? Every SCC gets the same value.

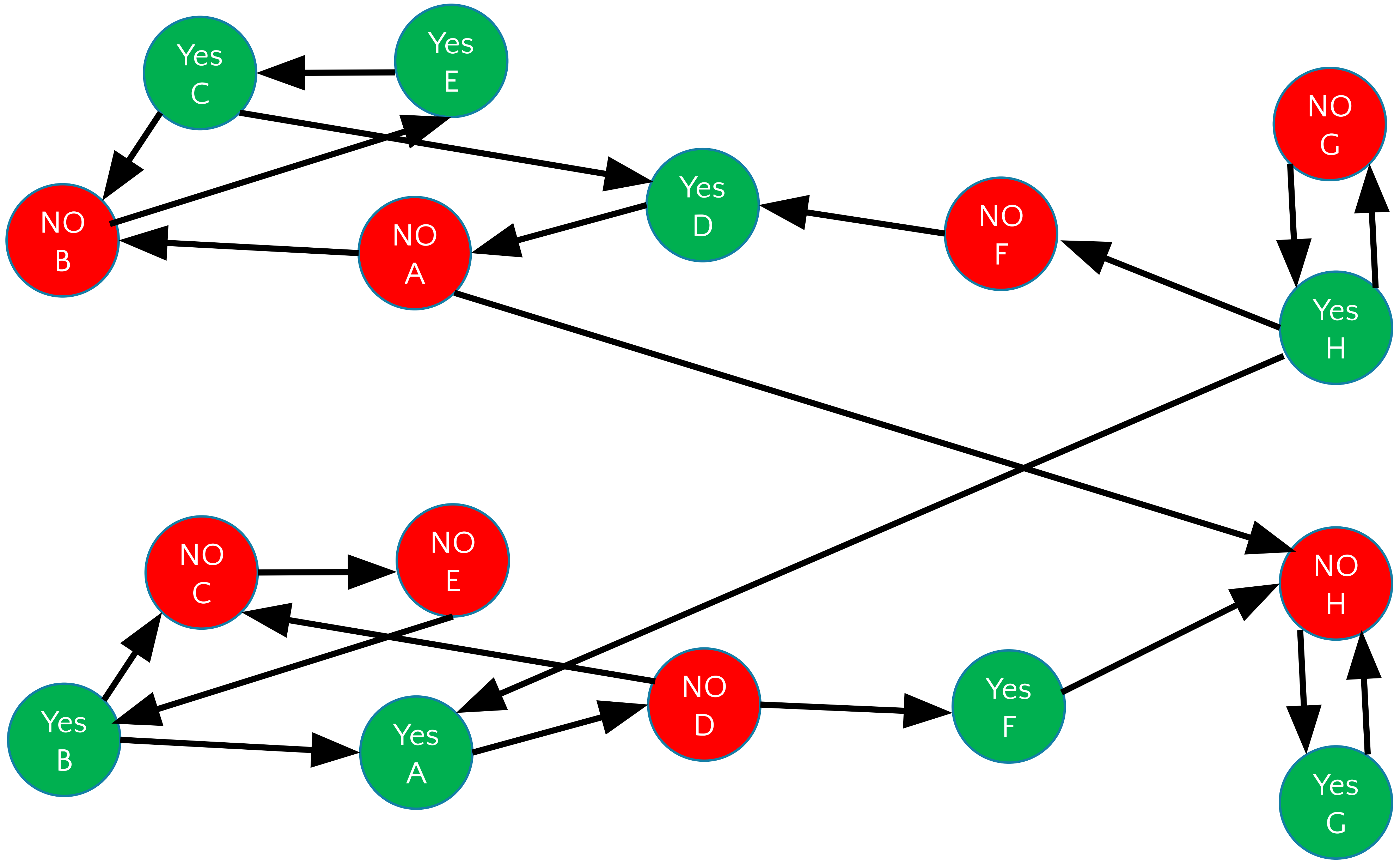
- Treat it as a single object!

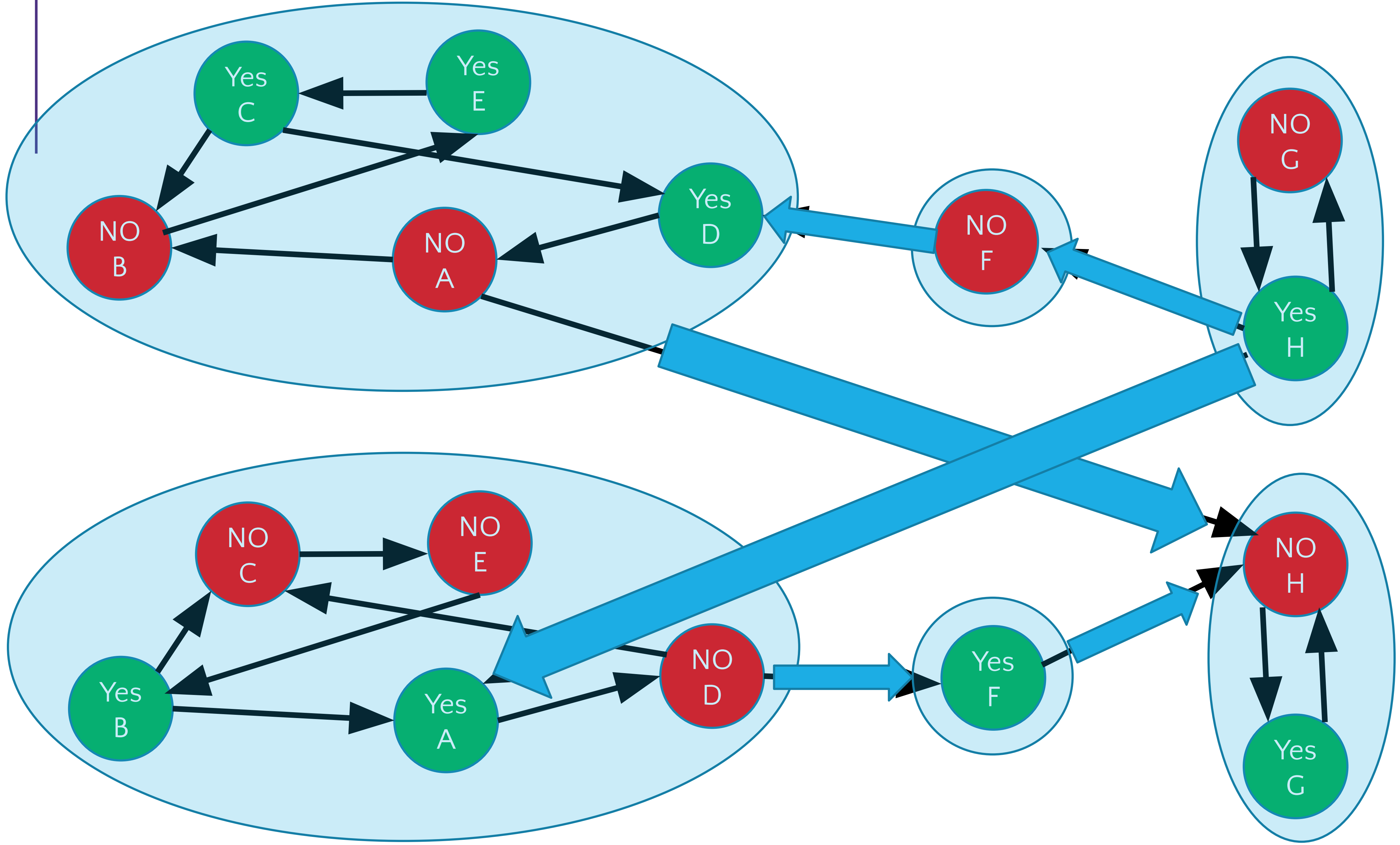
We want to avoid edges from true things to false things.

- "Trues" seem more useful for us at the end.

Is there some way to start from the end?

- YES! Topological Sort


Yes C
Yes E
NO G
NO B
NO A
Yes D
NO F
Yes H
NO C
NO E
NO H
Yes B
Yes A
NO D
Yes F
Yes G


Yes
C
Yes
E
NO
B
NO
A
Yes
D
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F
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G
Yes
H
NO
C
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E
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B
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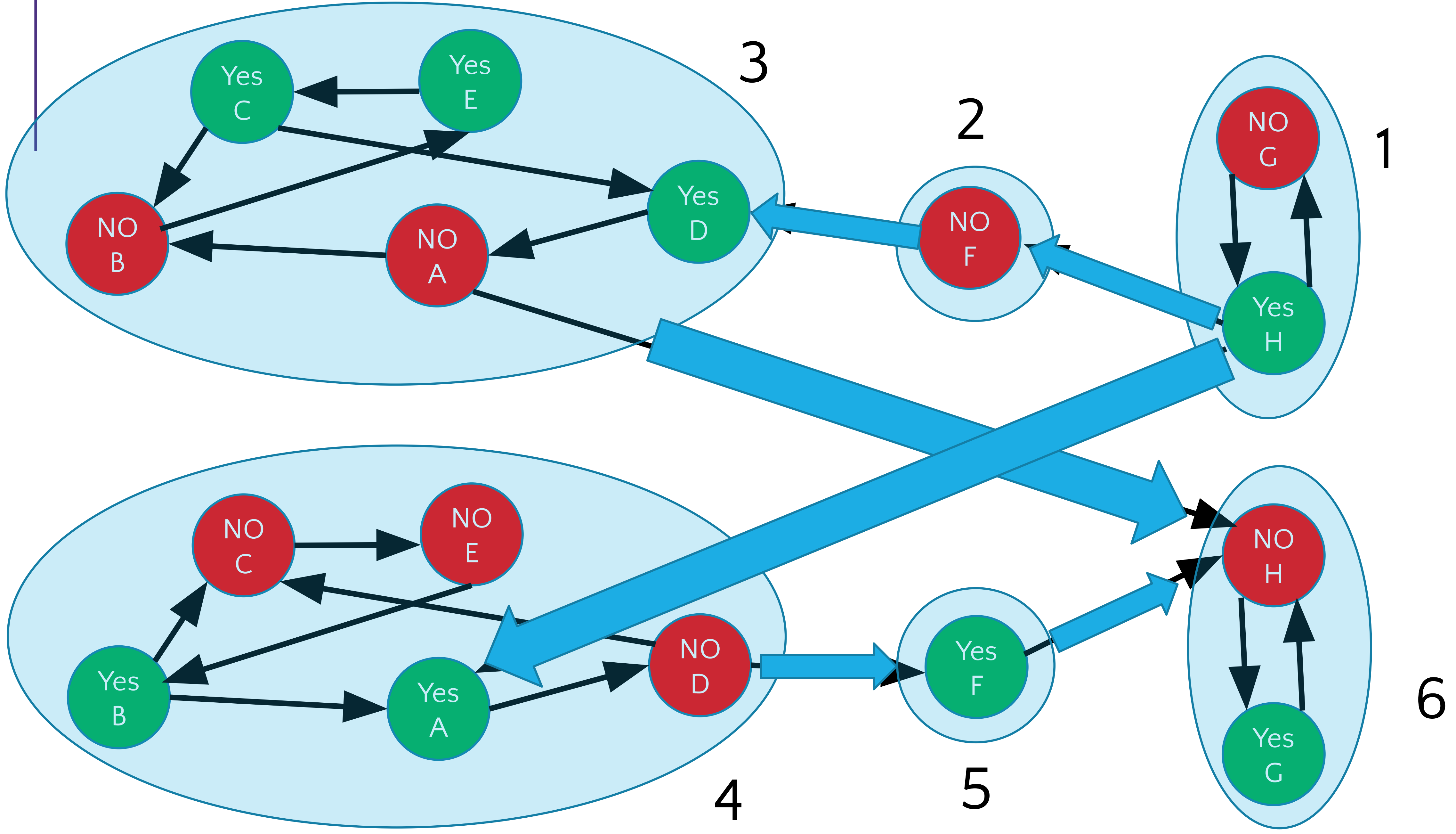
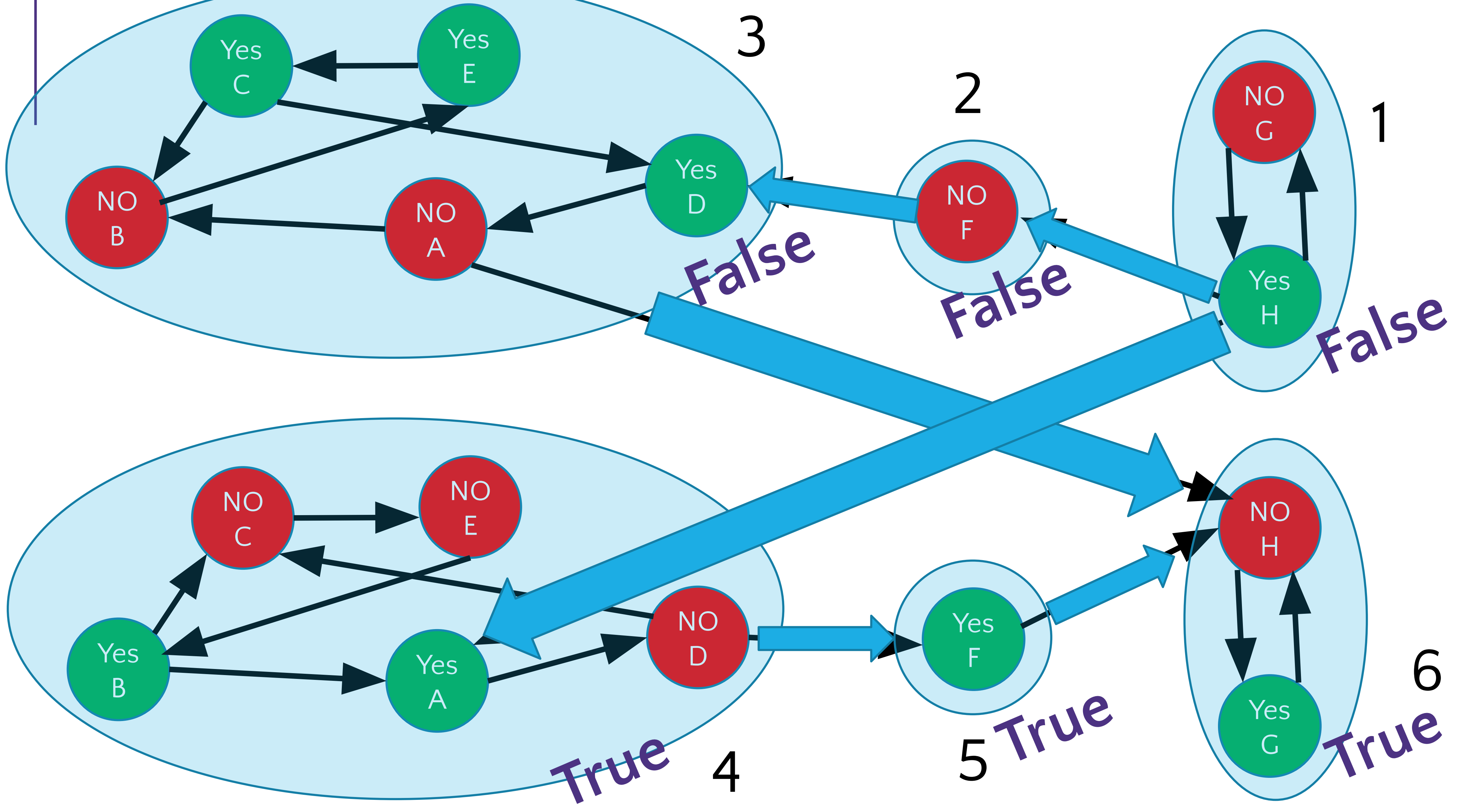
D
Yes
F
NO
H
Yes
G


3
Yes C
Yes E
NO B
NO A
Yes D
2
NO F
1
NO G
Yes H
NO C
NO E
Yes B
Yes A
NO D
Yes F
NO H
Yes G
6
4
5



3
2
1
Yes C
Yes E
Yes D
NO B
NO A
NO F
NO G
Yes H
False
False
False
NO C
NO E
NO D
Yes B
Yes A
Yes F
NO H
Yes G
4
5
6
True
True
True

Making the Final

Algorithm:
Make the requirements graph.

Find the SCCs.

If any SCC has including and not including a problem, we can't make the final.

Run topological sort on the graph of SCC.

Starting from the end:

- If everything in a component is unassigned, set them to true, and set their opposites to false.

This works!!

How fast is it?

O(Q + S). That's a HUGE improvement.

Some More Context

The Final Making Problem was a type of "Satisfiability" problem.

We had a bunch of variables (include/exclude this question), and needed to satisfy everything in a list of requirements.

2-Satisfiability ("2-SAT")

Given: A set of Boolean variables, and a list of requirements, each of the form:

```
  variable1==[True/False] || variable2==[True/False]
```

Find: A setting of variables to "true" and "false" so that **all** of the requirements evaluate to "true"

The algorithm we just made for Final Creation works for any 2-SAT problem.

The 2 Sat Solver

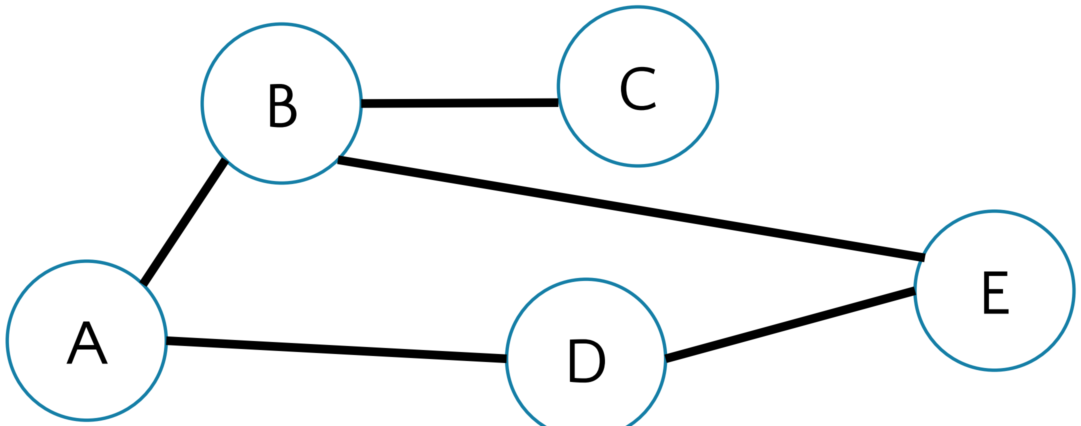
Reductions

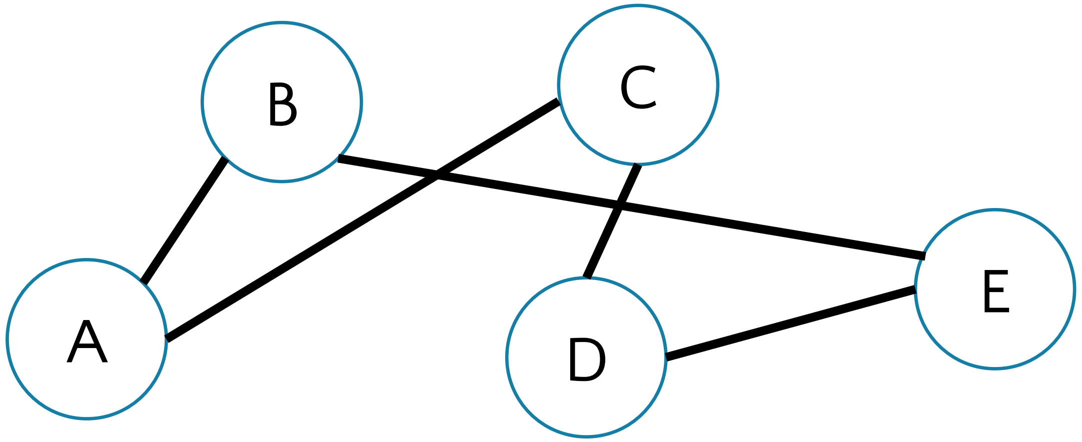
2-Coloring

2-Coloring

Given an undirected, unweighted graph ***G***, color each vertex "red" or "blue" such that the endpoints of every edge are different colors (or report no such coloring exists).

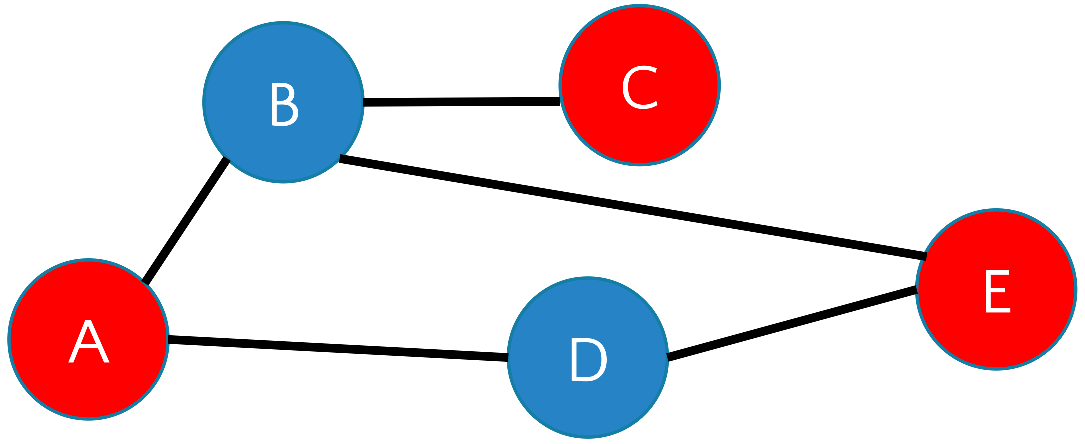
Can these graphs be 2-colored? If so find a 2-coloring. If not try to explain why one doesn't exist.

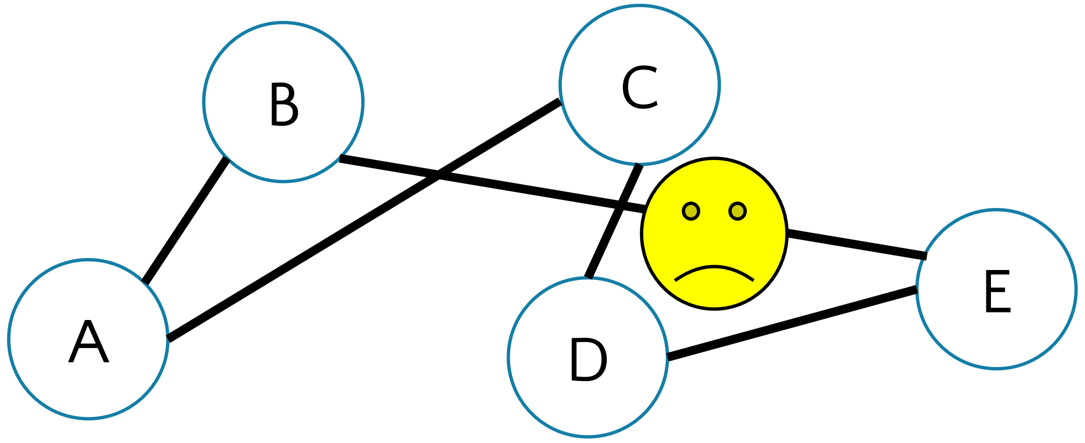




2-Coloring

Can these graphs be 2-colored? If so find a 2-coloring. If not try to explain why one doesn't exist.





What are we doing?

To wrap up the course we want to take a big step back.

This whole quarter we've been taking problems and solving them faster.

We want to spend the last few lectures going over more ideas on how to solve problems faster, and why we don't expect to solve everything extremely quickly.

We're going to

- Recall reductions
- Classify problems into those we can solve in a reasonable amount of time, and those we can't
- Explain the biggest open problem in Computer Science

Reductions: Take 2

Reduction (informally)

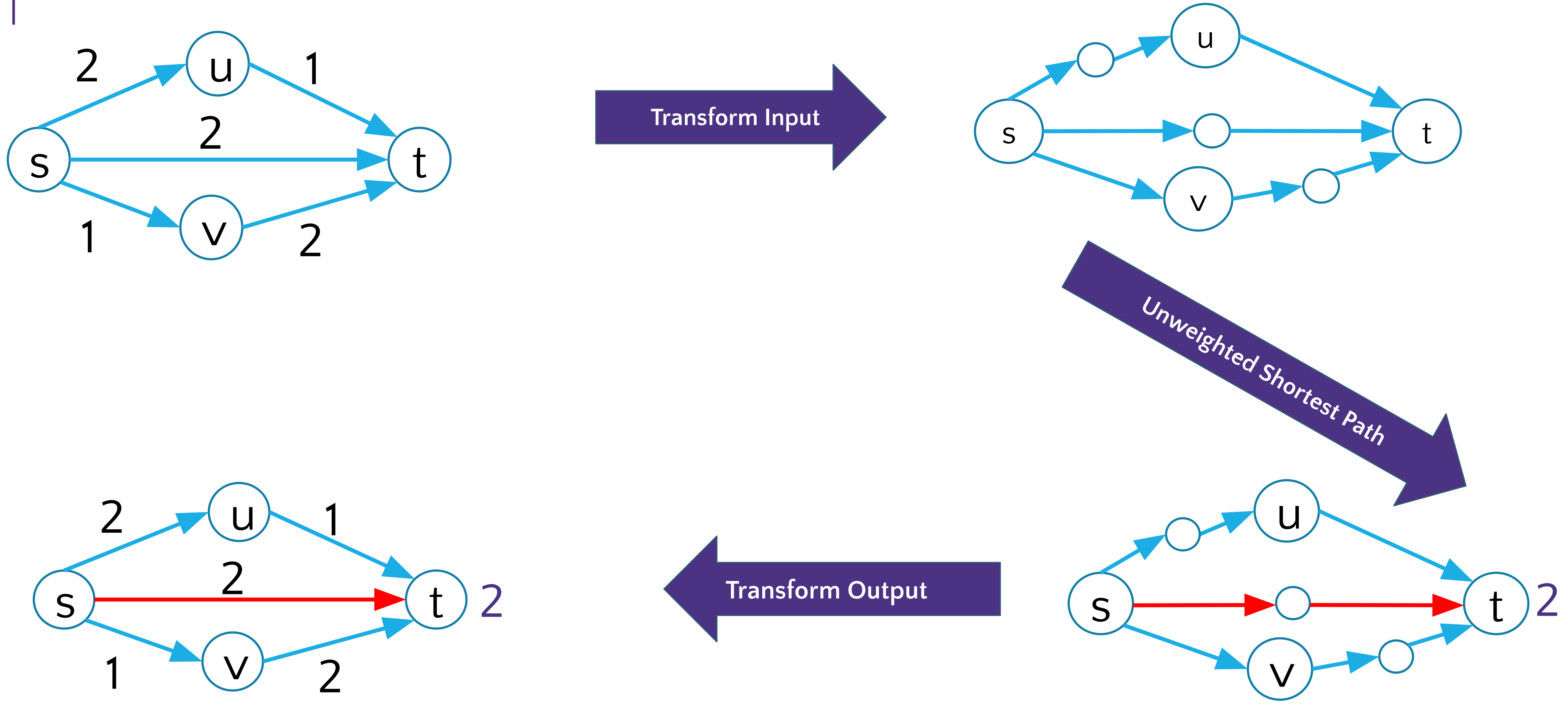
Using an algorithm for Problem B to solve Problem A.

You already do this all the time.

In Homework 2, you reduced implementing a hashset to implementing a hashmap.

Any time you use a library, you're reducing your problem to the one the library solves.

Weighted Graphs: A Reduction



Reductions

It might not be too surprising that we can solve one shortest path problem with the algorithm for another shortest path problem.

The real power of reductions is that you can sometimes reduce a problem to another one that looks very very different.

We're going to reduce a graph problem to 2-SAT.

2-Coloring

Given an undirected, unweighted graph ***G***, color each vertex "red" or "blue" such that the endpoints of every edge are different colors (or report no such coloring exists).

2-Coloring

Why would we want to 2-color a graph?

- We need to divide the vertices into two sets, and edges represent vertices that **can't** be together.

You can modify BFS to come up with a 2-coloring (or determine none exists)

- This is a good exercise!

But coming up with a whole new idea sounds like **work.**

And we already came up with that cool 2-SAT algorithm.

- Maybe we can be lazy and just use that!
- Let's **reduce** 2-Coloring to 2-SAT!

Use our 2-SAT algorithm to solve 2-Coloring

A Reduction

We need to describe 2 steps

1. How to turn a graph for a 2-color problem into an input to 2-SAT

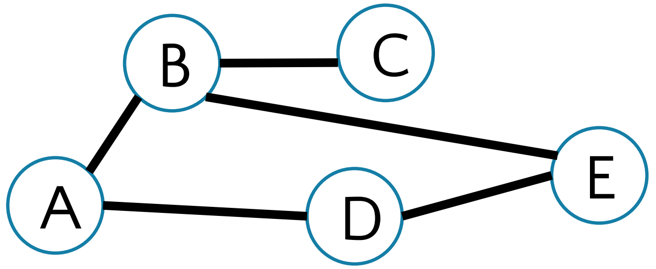
2. How to turn the ANSWER for that 2-SAT input into the answer for the original 2-coloring problem.

How can I describe a two coloring of my graph?

Have a variable for each vertex – is it red?

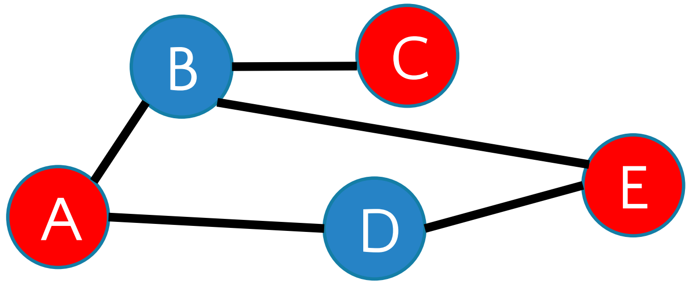
How do I make sure every edge has different colors? I need one red endpoint and one blue one, so this better be true to have an edge from v1 to v2:

```
(v1IsRed || v2isRed) && (!v1IsRed || !v2IsRed)
```



```
(AisRed||BisRed)&&(!AisRed||!BisRed)
(AisRed||DisRed)&&(!AisRed||!DisRed)
(BisRed||CisRed)&&(!BisRed||!CisRed)
(BisRed||EisRed)&&(!BisRed||!EisRed)
(DisRed||EisRed)&&(!DisRed||!EisRed)
```

```
AisRed = True
BisRed = False
CisRed = True
DisRed = False
EisRed = True
```



Questions?

That’s all!

Lecture 25: P vs NP

CSE 373: Data Structures and Algorithms

Announcements

Slido Event #7165864
https://app.sli.do/event/h2TaZahvBEHWg2h79kNHxC

- Final exam TA lead review after class today
- Please fill out the section final review survey to help your TAs play for tomorrow's section
- P4 – please get started on Seam Carving if you haven't already
  - TAs posted a P4 walk through to help clarify
  - Note that office hours will end on week 10
  - You cannot use late days for P4 (I already made the turn in as late as I can accept assignments)
- Please nominate your TAs for an award!
  - TO NOMINATE GO TO: https://www.cs.washington.edu/students/ta/bandes

P vs NP Explained (YouTube)

P vs. NP

A brief history of computer science problem solving

- The field of “computer science” is the pursuit of determining how to use “computers” to help solve human problems
- 1843 the first “computer” is designed to solve bernouli’s numbers, a very difficult calculation
- 1943 the MARC II is designed to solve missile trajectory and other military calculations
- 1960s computers begin to become more generalized, researchers start looking for ways to use computers beyond basic math computations
- 1970s researchers are exploring what types of problems computers can help with, and finding themselves stuck and unsure if there exists a computer assisted solution or not…

1970s computer science research

- Researchers were collecting problems to solve
  - Some problems resulted in algorithms that could “efficiently” find a solution
- When researchers were stuck on a problem they turned to “reductions” to see if they could apply a newly discovered algorithm to their own problem
- To help one another understand if they were working on an unsolved problem or not researchers started to categorize problems into complexity classes…
  - Enter “complexity research”

“Efficiency”

So far you’ve only met problems that have an “efficient” solution

For our purposes “efficient” essentially means “can be executed by current day computers”

Formally we will consider any code that can run in **polynomial** or “**P**” time to be “efficient”

P complexity class

The set of all decision problems that have an algorithm that runs in time $O(n^k)$ for some constant k

Are these algorithms always actually efficient?

Well… no

Your n^{10000} algorithm or $10000n^3$ algorithms probably aren’t going to finish anytime soon, but these edge cases are rare, and polynomial time is good as a low bar

“Efficiency” at scale

We have seen some inefficient algorithms

- Recursive backtracking (k^n where k represents number of choices)
- Recursive fibonacci (2^n)

But as long as n is small we can still compute them

N^3 solution where n= 100 takes ~3 hrs

2^n solution where n = 10 takes ~milliseconds,

but n = 100 takes 300 quintillion years (longer than the age of the universe)

Running Times

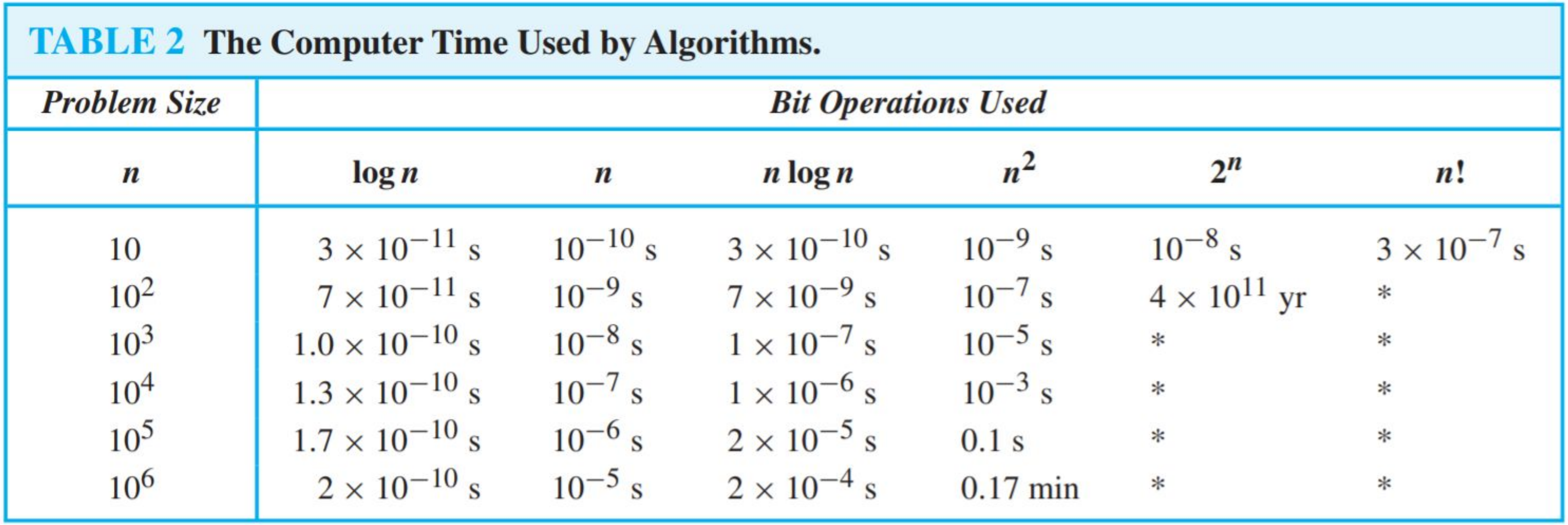


TABLE 2 The Computer Time Used by Algorithms.

Problem Size	*Bit Operations Used*					
n	$\log n$	n	$n \log n$	n^2	2^n	$n!$
10	3×10^{-11} s	10^{-10} s	3×10^{-10} s	10^{-9} s	10^{-8} s	3×10^{-7} s
10^2	7×10^{-11} s	10^{-9} s	7×10^{-9} s	10^{-7} s	4×10^{11} yr	*
10^3	1.0×10^{-10} s	10^{-8} s	1×10^{-7} s	10^{-5} s	*	*
10^4	1.3×10^{-10} s	10^{-7} s	1×10^{-6} s	10^{-3} s	*	*
10^5	1.7×10^{-10} s	10^{-6} s	2×10^{-5} s	0.1 s	*	*
10^6	2×10^{-10} s	10^{-5} s	2×10^{-4} s	0.17 min	*	*

Table from Rosen's Discrete Mathematics textbook
How big of a problem can we solve for an algorithm with the given running times?
"*" means more than 10^{100} years.

Aside: Decision Problems

Today’s goal is to break problems into solvable/not solvable categories

For today, we’re going to talk about **decision problems**.

- Problems that have a “yes” or “no” answer.

Why?

Theory reasons / how we translate problems for computer understanding

But it’s not too bad

- most problems can be rephrased as very similar decision problems

E.g. instead of “find the shortest path from s to t” ask,

- Is there a path from s to t length at most *k*?

NP Complexity Class

NP (stands for "nondeterministic polynomial")

The set of all decision problems such that if the answer is YES, there is a proof of that which can be verified in polynomial time

Decision Problems such that:

- If the answer is YES, you can prove the answer is yes by
  - A given "proof" or a "certificate" can be verified in polynomial time
  - Puzzle problems where a given answer can be either confirmed or rejected
- What certificate would be convenient for short paths?
  - The path itself. Easy to check the path is really in the graph and really short.

Light Spanning Tree:
IS there a spanning tree of graph G of weight at most k?

The spanning tree itself.
Verify by checking it really connects every vertex and its weight.

2-Coloring:
Can you color vertices of a graph red and blue so every edge has differently colored endpoints?

The coloring.
Verify by checking each edge.

2-SAT:
Given a set of variables and a list of requirements:
(variable==[T/F] || variable==[T/F])
Find a setting of the variables to make every requirement true.

The assignment of variables.
Verify by checking each requirement.

P vs. NP, the conundrum

P vs. NP

Are P and NP the same complexity class?
That is, can every problem that can be verified in polynomial time also be solved in polynomial time.

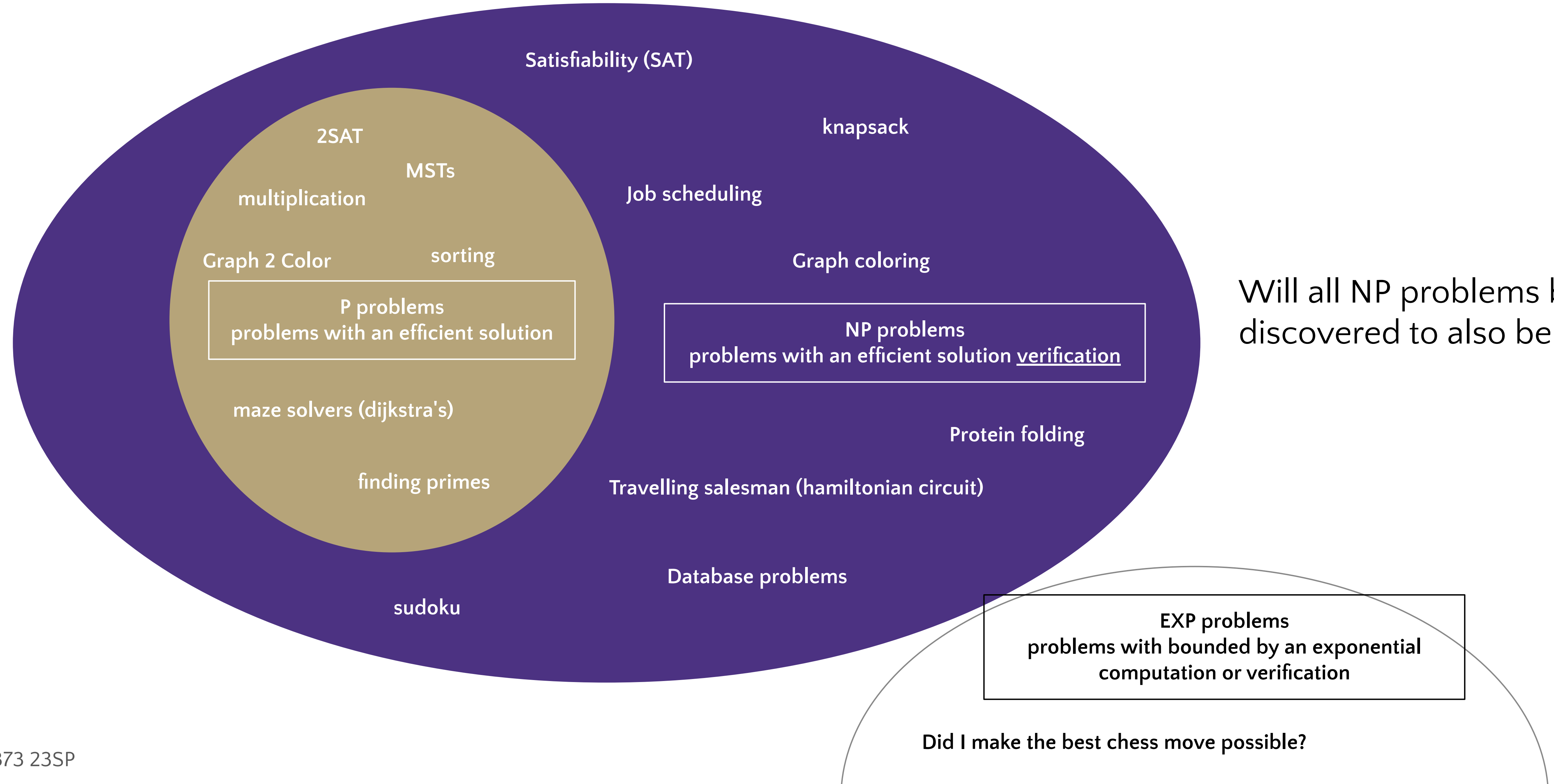
Does being able to quickly validate a correct solution also mean you can quickly find a correct solution?

No one knows the answer to this question.

In fact, it's the biggest open problem in Computer Science.

P vs NP

Can we **PROVE** that all problems with an efficiently verifiable solution can be solved efficiently?



Satisfiability (SAT)
knapsack
Job scheduling
Graph coloring
2SAT
MSTs
multiplication
Graph 2 Color
sorting
P problems
problems with an efficient solution
NP problems
problems with an efficient solution verification
maze solvers (dijkstra's)
Protein folding
finding primes
Travelling salesman (hamiltonian circuit)
Database problems
sudoku
EXP problems
problems with bounded by an exponential computation or verification
Did I make the best chess move possible?

Will all NP problems be discovered to also be in P?

Searching for a solution to P v NP

Hard Problems

Let's say we want to prove that every problem in NP can actually be solved efficiently.

We might want to start with a really hard problem in NP.

What is the hardest problem in NP?

What does it mean to be a hard problem?

Reductions are a good definition:

- If A reduces to B then "A ≤ B" (in terms of difficulty)
  - Once you have an algorithm for B, you have one for A automatically from the reduction!

Does there exist an algorithm that all NP problems reduce to?

NP-Completeness

NP-complete

The problem B is NP-complete if B is in NP and
for all problems A in NP, A reduces to B.

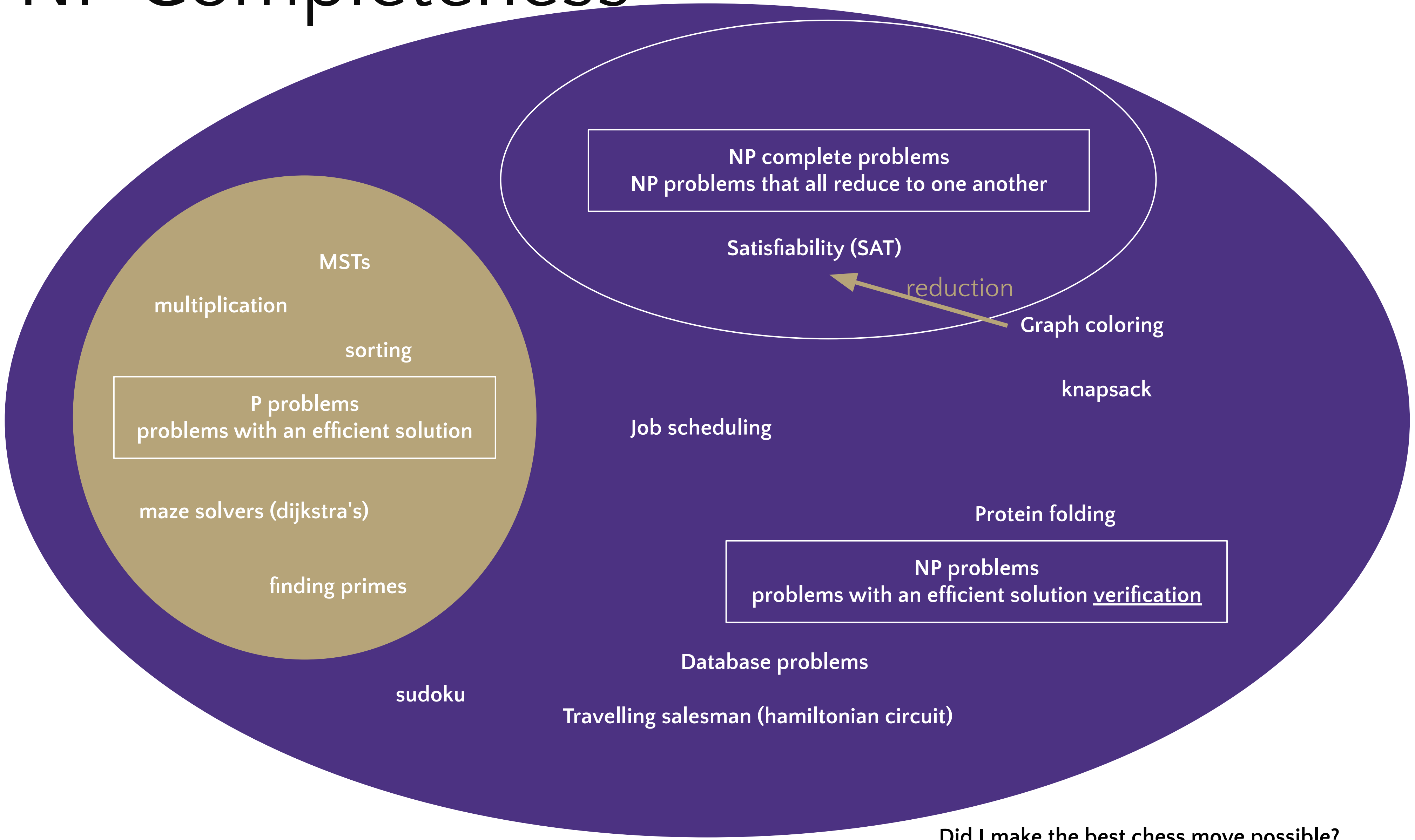
An NP-complete problem is a “hardest” problem in NP.

If you have an algorithm to solve an NP-complete problem, you have an algorithm for **every** problem in NP.

An NP-complete problem is a **universal language** for encoding “I’ll know it when I see it” problems.

Does one of these exist?

NP Completeness



NP complete problems
NP problems that all reduce to one another
Satisfiability (SAT)
reduction
Graph coloring
knapsack
Job scheduling
Protein folding
NP problems
problems with an efficient solution verification
Database problems
Travelling salesman (hamiltonian circuit)
sudoku
MSTs
multiplication
sorting
P problems
problems with an efficient solution
maze solvers (dijkstra's)
finding primes

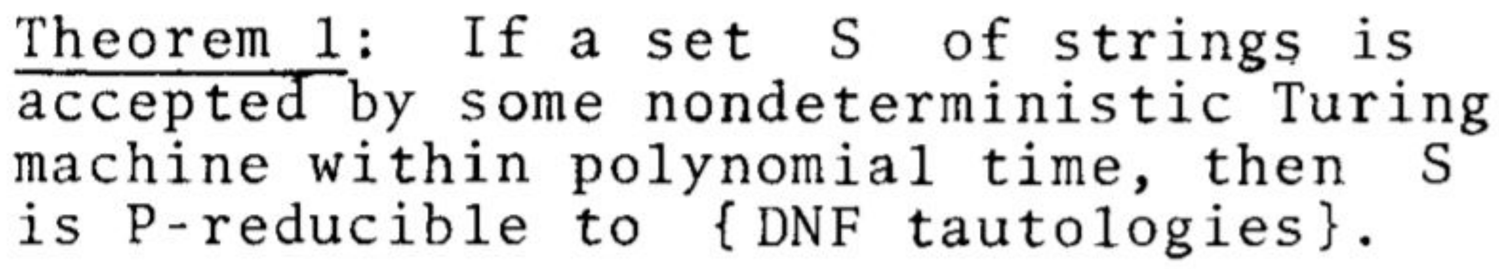
Did I make the best chess move possible?

NP-Completeness

An NP-complete problem does exist!

Cook-Levin Theorem (1971)

3-SAT is NP-complete


Theorem 1: If a set S of strings is accepted by some nondeterministic Turing machine within polynomial time, then S is P-reducible to {DNF tautologies}.

This sentence (and the proof of it) won Cook the Turing Award.

2-SAT vs. 3-SAT

2-Satisfiability ("2-SAT")

Given: A set of Boolean variables, and a list of requirements, each of the form:

```
variable1==[True/False] || variable2==[True/False]
```

Find: A setting of variables to "true" and "false" so that **all** of the requirements evaluate to "true"

3-Satisfiability ("3-SAT")

Given: A set of Boolean variables, and a list of requirements, each of the form:

```
variable1==[True/False]||variable2==[True/False]||variable3==[True/False]
```

Find: A setting of variables to "true" and "false" so that **all** of the requirements evaluate to "true"

2-SAT vs. 3-SAT

2-Satisfiability ("2-SAT")

Given: A set of Boolean variables, and a list of requirements, each of the form:

```
        variable1==[True/False] || variable2==[True/False]
```

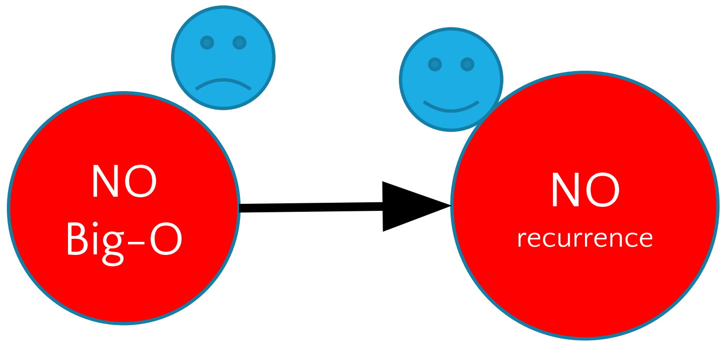
Find: A setting of variables to "true" and "false" so that **all** of the requirements evaluate to "true"

Our first try at 2-SAT (just try all variable settings) would have taken $O(2^Q S)$ time

But we came up with a really clever graph that reduced the time to $O(Q + S)$ time

2-SAT vs. 3-SAT

Can we do the same for 3-SAT?



For 2-SAT we thought we had 2^Q options, but we realized that we didn't have as many choices as we thought - once we made a few choices, out hand was forced and we didn't have to check all possibilities.

3-Satisfiability ("3-SAT")

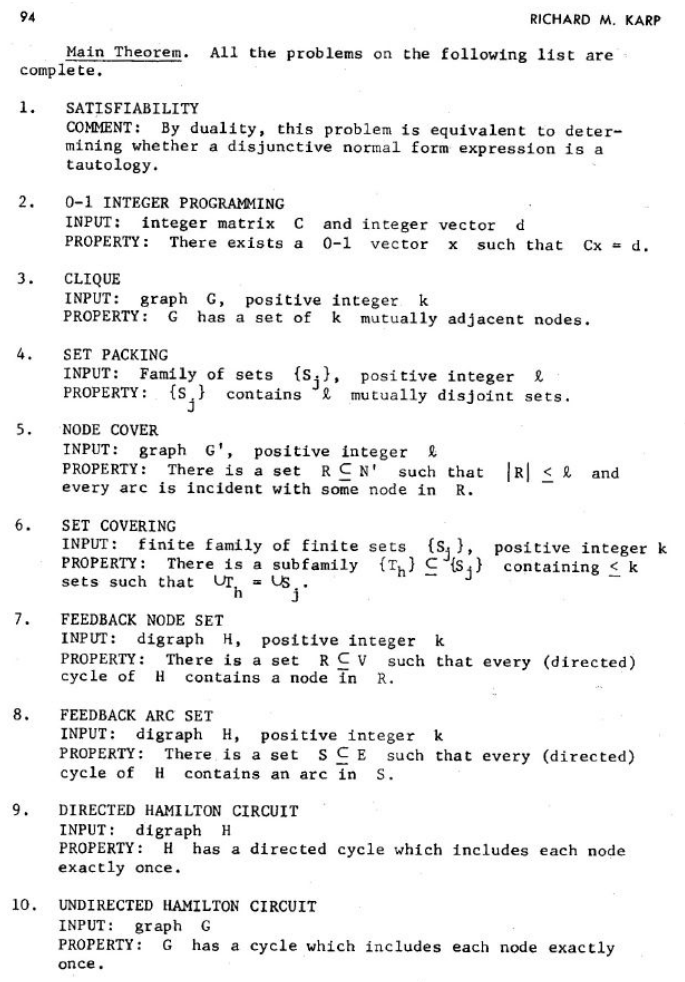
Given: A set of Boolean variables, and a list of requirements, each of the form:

`variable1==[True/False]||variable2==[True/False]||variable3==[True/False]`

Find: A setting of variables to "true" and "false" so that **all** of the requirements evaluate to "true"

NP-Complete Problems

But Wait! There's more!



Main Theorem. All the problems on the following list are complete.

1. SATISFIABILITY
COMMENT: By duality, this problem is equivalent to determining whether a disjunctive normal form expression is a tautology.

2. 0-1 INTEGER PROGRAMMING
INPUT: integer matrix C and integer vector d
PROPERTY: There exists a 0-1 vector x such that $Cx = d$.

3. CLIQUE
INPUT: graph G, positive integer k
PROPERTY: G has a set of k mutually adjacent nodes.

4. SET PACKING
INPUT: Family of sets $\{S_j\}$, positive integer ℓ
PROPERTY: $\{S_j\}$ contains ℓ mutually disjoint sets.

5. NODE COVER
INPUT: graph G', positive integer ℓ
PROPERTY: There is a set $R \subseteq N'$ such that $|R| \leq \ell$ and every arc is incident with some node in R.

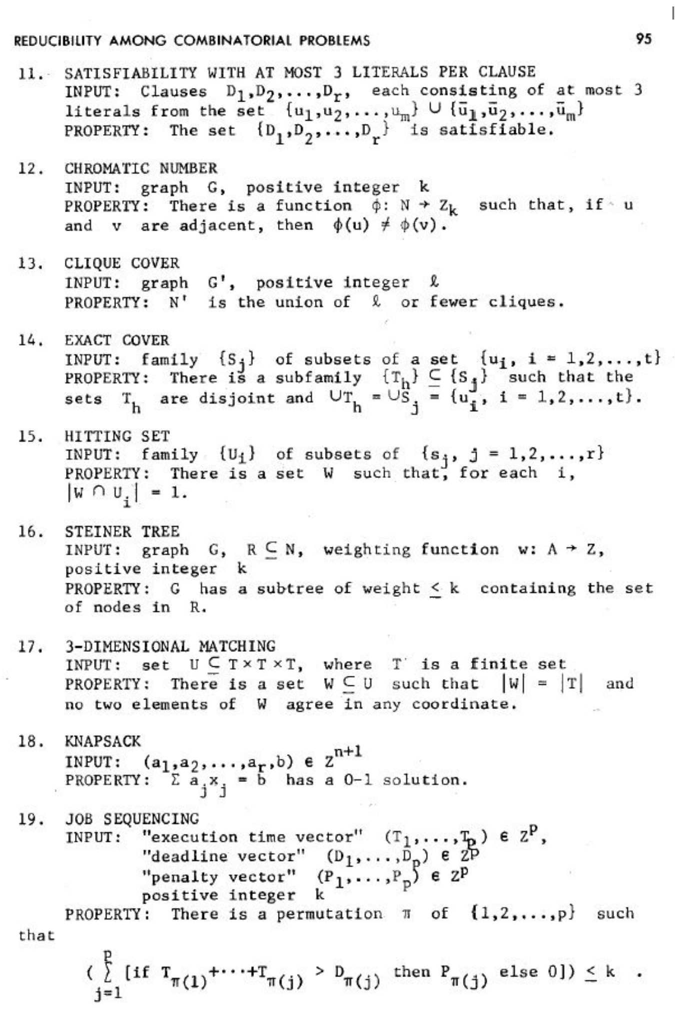
6. SET COVERING
INPUT: finite family of finite sets $\{S_j\}$, positive integer k
PROPERTY: There is a subfamily $\{T_h\} \subseteq \{S_j\}$ containing $\leq k$ sets such that $\cup T_h = \cup S_j$.

7. FEEDBACK NODE SET
INPUT: digraph H, positive integer k
PROPERTY: There is a set $R \subseteq V$ such that every (directed) cycle of H contains a node in R.

8. FEEDBACK ARC SET
INPUT: digraph H, positive integer k
PROPERTY: There is a set $S \subseteq E$ such that every (directed) cycle of H contains an arc in S.

9. DIRECTED HAMILTON CIRCUIT
INPUT: digraph H
PROPERTY: H has a directed cycle which includes each node exactly once.

10. UNDIRECTED HAMILTON CIRCUIT
INPUT: graph G
PROPERTY: G has a cycle which includes each node exactly once.



11. SATISFIABILITY WITH AT MOST 3 LITERALS PER CLAUSE
INPUT: Clauses $D_1, D_2, \ldots, D_r$, each consisting of at most 3 literals from the set $\{u_1, u_2, \ldots, u_m\} \cup \{\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_m\}$
PROPERTY: The set $\{D_1, D_2, \ldots, D_r\}$ is satisfiable.

12. CHROMATIC NUMBER
INPUT: graph G, positive integer k
PROPERTY: There is a function $\phi: N \to Z_k$ such that, if u and v are adjacent, then $\phi(u) \neq \phi(v)$.

13. CLIQUE COVER
INPUT: graph G', positive integer ℓ
PROPERTY: N' is the union of ℓ or fewer cliques.

14. EXACT COVER
INPUT: family $\{S_j\}$ of subsets of a set $\{u_i, i = 1,2,\ldots,t\}$
PROPERTY: There is a subfamily $\{T_h\} \subseteq \{S_j\}$ such that the sets T_h are disjoint and $\cup T_h = \cup S_j = \{u_i, i = 1,2,\ldots,t\}$.

15. HITTING SET
INPUT: family $\{U_i\}$ of subsets of $\{s_j, j = 1,2,\ldots,r\}$
PROPERTY: There is a set W such that, for each i, $|W \cap U_i| = 1$.

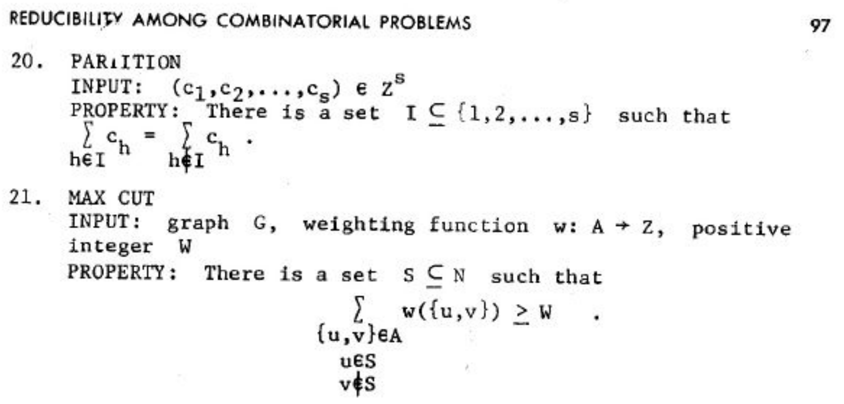
16. STEINER TREE
INPUT: graph G, $R \subseteq N$, weighting function $w: A \to Z$, positive integer k
PROPERTY: G has a subtree of weight $\leq k$ containing the set of nodes in R.

17. 3-DIMENSIONAL MATCHING
INPUT: set $U \subseteq T \times T \times T$, where T is a finite set
PROPERTY: There is a set $W \subseteq U$ such that $|W| = |T|$ and no two elements of W agree in any coordinate.

18. KNAPSACK
INPUT: $(a_1, a_2, \ldots, a_r, b) \in Z^{n+1}$
PROPERTY: $\Sigma a_j x_j = b$ has a 0-1 solution.

19. JOB SEQUENCING
INPUT: "execution time vector" $(T_1, \ldots, T_p) \in Z^p$,
"deadline vector" $(D_1, \ldots, D_p) \in Z^p$
"penalty vector" $(P_1, \ldots, P_p) \in Z^p$
positive integer k
PROPERTY: There is a permutation π of $\{1,2,\ldots,p\}$ such that

$$\left(\sum_{j=1}^{p} [\text{if } T_{\pi(1)} + \cdots + T_{\pi(j)} > D_{\pi(j)} \text{ then } P_{\pi(j)} \text{ else } 0]\right) \leq k \quad .$$



20. PARTITION
INPUT: $(c_1, c_2, \ldots, c_s) \in Z^s$
PROPERTY: There is a set $I \subseteq \{1,2,\ldots,s\}$ such that $\sum_{h \in I} c_h = \sum_{h \notin I} c_h$.

21. MAX CUT
INPUT: graph G, weighting function $w: A \to Z$, positive integer W
PROPERTY: There is a set $S \subseteq N$ such that

$$\sum_{\substack{\{u,v\} \in A \\ u \in S \\ v \notin S}} w(\{u,v\}) \geq W \quad .$$

Karp's Theorem (1972)

A lot of problems are NP-complete

NP-Complete Problems

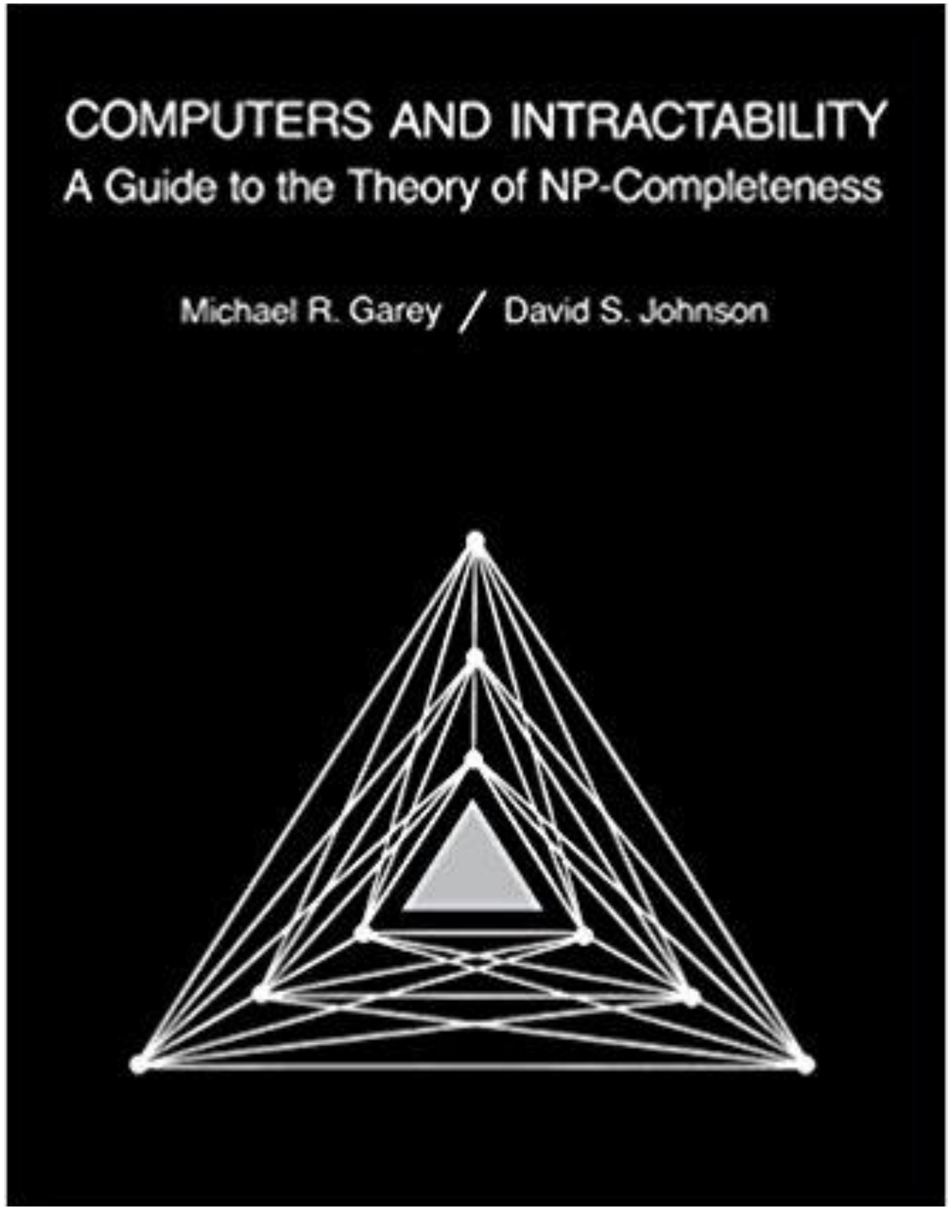
But Wait! There's more!

By 1979, at least 300 problems had been proven NP-complete.

Garey and Johnson put a list of all the NP-complete problems they could find in this textbook.

Took almost 100 pages to just list them all.

No one has made a comprehensive list since.


COMPUTERS AND INTRACTABILITY
A Guide to the Theory of NP-Completeness
Michael R. Garey / David S. Johnson

NP-Complete Problems

But Wait! There's more!

In the last month, mathematicians and computer scientists have put papers on the arXiv claiming to show (at least) 25 more problems are NP-complete.

There are literally thousands of NP-complete problems known.

And some of them look weirdly similar to problems we've already studied.

Examples

There are literally thousands of NP-complete problems.
And some of them look weirdly similar to problems we do know efficient algorithms for.

In P

Short Path

Given a directed graph, report if there is a path from s to t of length at most *k*

NP-Complete

Long Path

Given a directed graph, report if there is a path from s to t of length at least *k*

Examples

In P

Light Spanning Tree

Given a weighted graph, find a spanning tree (a set of edges that connect all vertices) of weight at most *k*

NP-Complete

Traveling Salesperson

Given a weighted graph, find a tour (a walk that visits every vertex and returns to its start) of minimum weight

The electric company just needs a greedy algorithm to lay its wires.
Amazon doesn't know a way to optimally route its delivery trucks.

Dealing with NP-Completeness

Option 1: Maybe it's a special case we understand

Maybe you don't need to solve the general problem, just a special case

Option 2: Maybe it's a special case we *don't* understand (yet)

There are algorithms that are known to run quickly on "nice" instances. Maybe your problem has one of those.

One approach: Turn your problem into a SAT instance, find a solver and cross your fingers.

Dealing with NP-Completeness

Option 3: Approximation Algorithms

You might not be able to get an exact answer, but you might be able to get close.

Optimization version of Traveling Salesperson

Given a weighted graph, find a tour (a walk that visits every vertex and returns to its start) of weight at most *k*.

Algorithm:

Find a minimum spanning tree.

Have the tour follow the visitation order of a DFS of the spanning tree.

Theorem: This tour is at most twice as long as the best one.

Why should you care about P vs. NP

Most computer scientists are convinced that P≠NP.

Why should you care about this problem?

It's your chance for:

- $1,000,000. The Clay Mathematics Institute will give $1,000,000 to whoever solves P vs. NP (or any of the 5 remaining problems they listed)
- To get a Turing Award

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Why Should You Care if P=NP?

Suppose P=NP.

Specifically that we found a genuinely in-practice efficient algorithm for an NP-complete problem. What would you do?

- $1,000,000 from the Clay Math Institute obviously, but what's next?

Why Should You Care if P=NP?

We found a genuinely in-practice efficient algorithm for an NP-complete problem. What would you do?

- Another $5,000,000 from the Clay Math Institute
- Put mathematicians out of work.
- Decrypt (essentially) all current internet communication.
- No more secure online shopping or online banking or online messaging…or online *anything.*
- Cure cancer with efficient protein folding

A world where P=NP is a very very different place from the world we live in now.

Why Should You Care if P≠NP?

We already expect P ≠ NP. Why should you care when we finally prove it?

P ≠ NP says something fundamental about the universe.

For some questions there is not a clever way to find the right answer

- Even though you'll know it when you see it
- Some problems require "creative leaps" to find a solution that <u>cannot be programmed</u>

There is actually a way to obscure information, so it cannot be found quickly no matter how clever you are.

Why Should You Care if P≠NP?

To prove P≠NP we need to better understand the differences between problems.

- Why do some problems allow easy solutions and others don't?
- What is the structure of these problems?

We don't care about P vs NP just because it has a huge effect about what the world looks like.

We will learn a lot about computation along the way.

If P = NP, then the world would be a profoundly different place than we usually assume it to be. There would be no special value in "creative leaps", no fundamental gap between solving a problem and recognizing the solution once it's found. Everyone who could appreciate a symphony would be Mozart. Everyone who could follow a step by step argument would be Gauss" –Scott Aaronson, MIT complexity researcher

Questions?

That’s all!