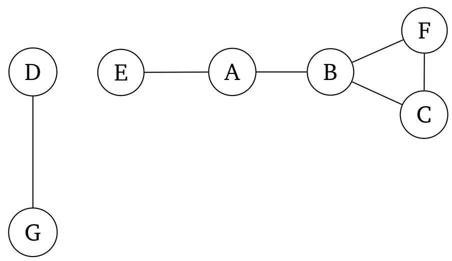
Section 06: Solutions

Section Problems

1. Graph properties

(a) Consider the *undirected, unweighted* graph below.



Answer the following questions about this graph:

(i) Find V, E, $|V|$, and $|E|$.

(ii) What is the maximum *degree* of the graph?

(iii) Are there any cycles? If so, where?

(iv) What is the maximum length simple path in this graph?

(v) What is one edge you could add to the graph that would increase the length of the maximum length simple path of the new graph to 6?

(vi) What are the *connected components* of the graph?

Solution:

(i) $V = \{A, B, C, D, E, F, G\}$ and $E = \{(D, G), (E, A), (A, B), (B, F), (F, C), (C, B)\}$. This means that $|V| = 7$ and $|E| = 6$.

(ii) The vertex with the max degree is B, which has a degree of 3.

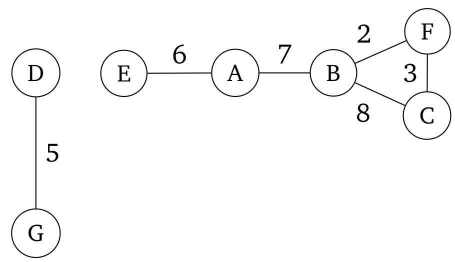
(iii) There is indeed a cycle, between B, C, and F.

(iv) There are multiple simple paths with the maximum length: $(E, A), (A, B), (B, F), (F, C)$ or $(E, A), (A, B), (B, C), (C, F)$ or their inverses (starting at C or F instead of E).

(v) We could add the edge (D, E).

(vi) One connected component is $\{D, G\}$. Another one is $\{E, A, B, C, F\}$.

(b) Consider the *undirected, weighted* graph below.



Answer the following questions about this graph:

(i) What is the path involving the least number of nodes from E to C? What is its cost?

(ii) What is the minimum cost path from E to C? What is its cost?

(iii) What is the minimum length path from E to C? What is its length?

Solution:

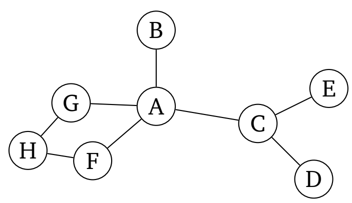
(i) The path with the least number of nodes is $(E, A), (A, B), (B, C)$. The cost is 21.

(ii) The minimum cost path is actually $(E, A), (A, B), (B, F), (F, C)$. The cost is 18.

(iii) The path with the shortest length is $(E, A), (A, B), (B, C)$. The length is 3.

2. Graph traversal

(a) Consider the following graph. Suppose we want to traverse it, starting at node A.



If we traverse this using *breadth-first search*, what are *two* possible orderings of the nodes we visit? What if we use *depth-first search*?

For the first ordering, you **must** run through adding/removing things from the queue/stack. To provide the second ordering for each algorithm, you may simply look at the graph.

Solution:

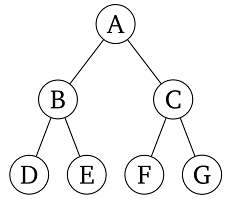
Here are two possible orderings for BFS:

- A, G, F, B, C, H, D, E
- A, C, B, F, G, D, H, E

Here are two possible orderings for DFS:

- A, G, H, F, C, D, E, B
- A, B, C, E, D, F, H, G

(b) Same question, but on this graph:



Solution:

Here are two possible orderings for BFS:

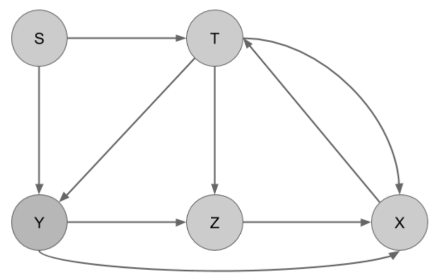
- A, B, C, D, E, F, G
- A, C, B, F, G, D, E

Here are two possible orderings for DFS:

- A, B, D, E, C, F, G
- A, C, G, F, B, E, D

3. Simulating BFS

Consider the following graph:



Run the BFS algorithm to find the shortest paths in this graph starting from vertex s. Draw out the queue of nodes, and also use the table below to keep track of each step in the algorithm. Finally, draw the resulting SPT (shortest path tree) after the algorithm has terminated.

Note: If two nodes enter the queue at the same time, break ties so that the node that comes first alphabetically exits the queue first.

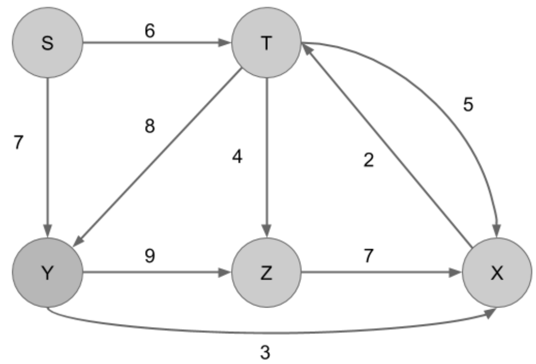
Vertex	Predecessor	Processed
s		
t		
x		
y		
z		

Solution:

Please see the Section slides for a set-by-step walk-through!

4. Simulating Dijkstra's

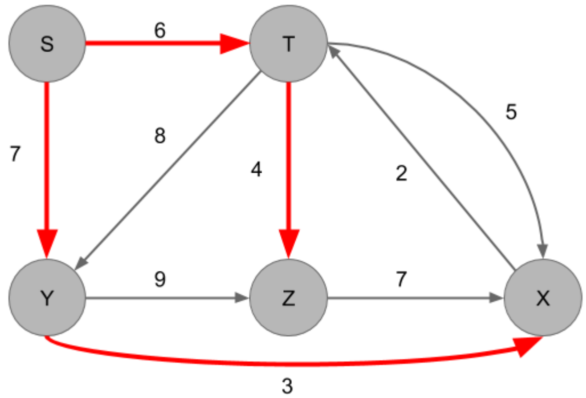
(a) Consider the following graph:



Run Dijkstra's algorithm on this graph starting from vertex s. Use the table below to keep track of each step in the algorithm. Also draw the resulting SPT (shortest path tree) after the algorithm has terminated.

Vertex	**Distance**	**Predecessor**	**Processed**
s			
t			
x			
y			
z			

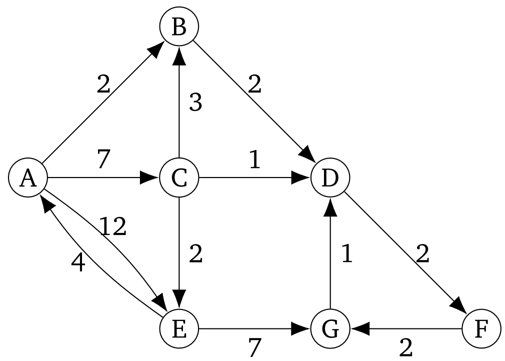
Solution:



The table below shows the final result of running Dijkstra's algorithm.

Vertex	Distance	Predecessor
s	0	–
t	6	s
x	10	y
y	7	s
z	10	t

(b) Here is another graph. What are the final costs and resulting SPT (shortest path tree) if we run Dijkstra's starting on node A?



Solution: