EX1: Algorithmic Analysis I

Due date: Friday April 15, 2022 at 11:59 pm PDT
Latest turn-in date: Monday April 18, 2022 at 11:59 pm PDT
Instructions:
High-level collaboration is allowed, but exercises are to be completed and submitted individually. Submit your responses to the "EX1: Algorithmic Analysis" on Gradescope here: https://www.gradescope.com/courses/379339/assignments/1938580/.
Make sure to log in to your Gradescope account using your UW email to access our course.

Runtime Bound Review:

O: **Upper bound** on how quickly function grows.	Informally: "Less than or equal to".
Ω: **Lower bound** on how quickly function grows.	Informally: "Greater than or equal to".
Θ: **Tight bound** on how quickly function grows.	Informally: "Equal to".

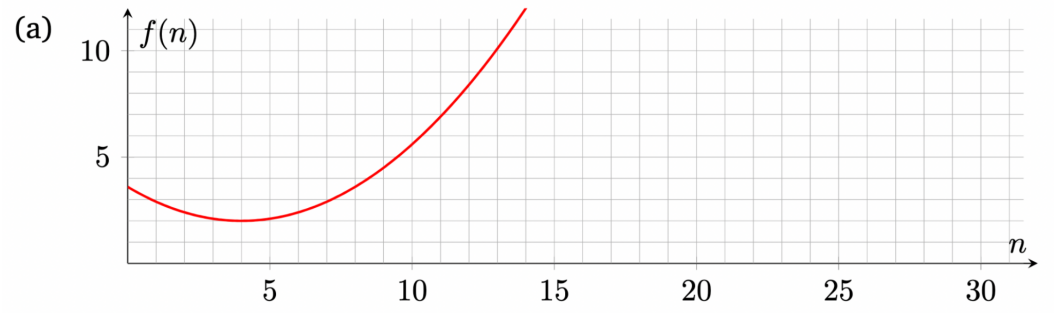
1. Asymptotic Analysis: Visually

For each of the following plots, list **all** possible Ω, Θ, and O bounds from the following choices, not just the tight bounds.

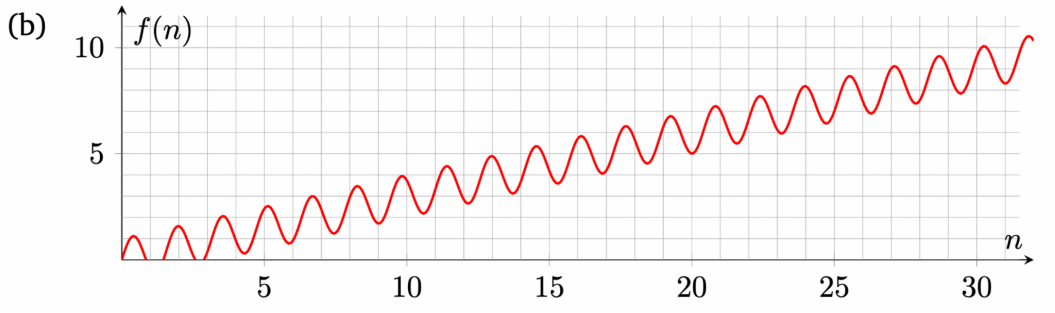
$$n^2,\ 1,\ n,\ log(n),\ \frac{1}{n}$$

You do not need to show your work; just list the bounds. If a particular bound doesn't exist for a given plot, briefly explain why. Assume that the plotted functions continue to follow the same trend shown in the plots as n increases. Each provided bound must either be a constant or a simple polynomial.

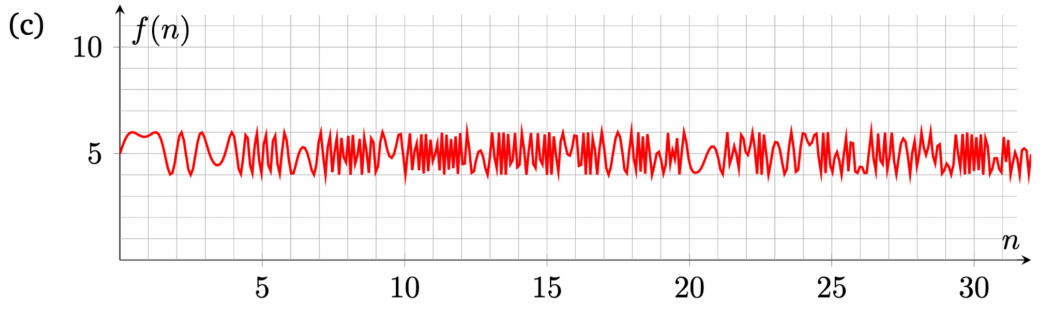
(a)



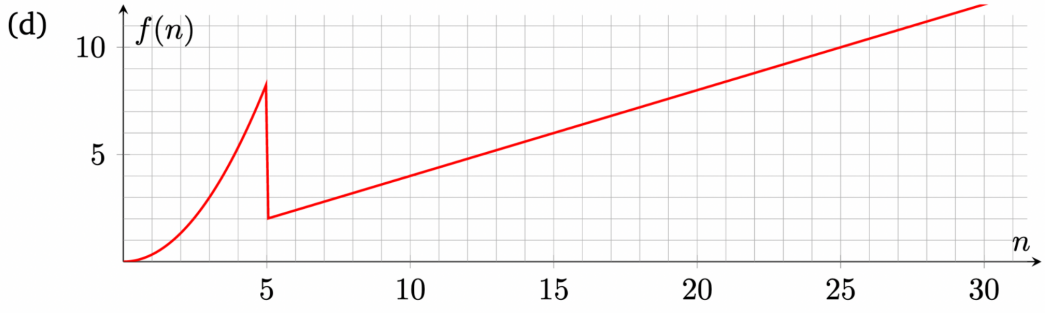
(b)


$f(n)$
10
5
n
5
10
15
20
25
30

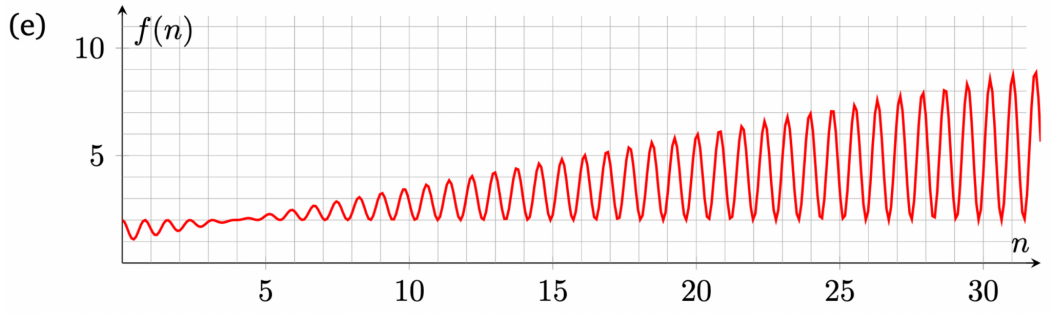
(c)


$f(n)$
10
5
n
5
10
15
20
25
30

(d)


$f(n)$
10
5
n
5
10
15
20
25
30

(e)


$f(n)$
10
5
n
5
10
15
20
25
30

2. Asymptotic Analysis: Conceptual

The following two statements are true. Explain why each of them are true in 1-2 sentences.

- If a function is in $\Theta(logN)$, then it could also be in $O(N)$.

- If a function is in $\Omega(N^2)$, then it could also be in $O(N^2 logN)$.

The following statement is false. Provide a counterexample function that shows why this statement is false.

- If a function is in $\Omega(N^2)$, then it is **never** in $O(N^2)$.

3. Asymptotic and Case Analysis: Through Code

Congratulations! You got the job offer at Hash Tea. Today marks your first day on the job and your employers help catch you up to speed by giving you a function to complete *n* boba orders. Analyze the code snippet below and answer the following questions.

Note: the method call to `blend()` is known to take time $\Theta(logk)$, where k is the size of `blend's` input. The method `blend` outputs `true` or `false` depending on if blending the `sugar` and `milkTea` made the delicious foam correctly.

```
boolean makeBobaOrder(int n) {
    int sugar = 2 * n;
    int milkTea = n;
    if (n <= 0) {
        return false;
    }
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < 8; j++) {
            if (!blend(sugar * milkTea)) {
                return false;
            }
        }
    }
    return true;
}
```

For the following questions, express your answer in terms of n (the input size).

In what case will the `makeBobaOrder()` function be a constant time operation?

What is the worst-case upper bound (O) of the `makeBobaOrder()` function?
(**Note:** multiple solutions exist for this question, please provide one)

What is the worst-case tight bound (Θ) of the `makeBobaOrder()` function?
(**Hint:** In algorithmic analysis, logarithms are in base 2. Recall, $log_2(3^2) = 2log_2(3)$)