Lecture 26: Tries

CSE 373: Data Structures and Algorithms

Memory & B Trees Takeaways

Memory in your computer is organized into layers spanning out from the CPU > L1 Cache > L2 Cache > RAM > Disk

- cache: a place to store some memory that's smaller and closer to the CPU compared to RAM
  - Analogy: refrigerator keeps food closer to kitchen than store
- Getting data from cache to CPU is a lot quicker than from RAM to CPU.
- the more memory a layer can store, the slower it is (generally)
- accessing the disk is **very** slow

Operating system is "the memory boss"

- pulls data from outer layers closer to CPU based on usage
- When do we "evict" memory to make room? How does the OS minimize disk accesses?
- Temporal locality: Once we load something into RAM or cache, keep it around or a while
  - Analogy: top layer of clothing stays on floor, don't wash that knife you just used to cut a sandwich just yet (I am a good adult I swear)
  - EX: if you accessed one index of an array in the cache, keep that array around for a while
- Spatial Locality: Computers try to partition memory you are likely to use close by
  - Analogy: if you're going to your storage unit, might as well fill up your car
  - EX: if you ask for one index of an array from RAM, pull the whole array down

Announcements

P4 due a week from today
- please get started soon

Interview practice tomorrow in section

TA Career Panel

Memorial day next week
- no class, no office hours
- Office hours end on Friday June 4th

Grades posted to canvas
- late penalties have yet to be applied
- please let us know if there are any issues
- regrade requests live on gradescope

Course Evals are out
- please fill out the official one
- supplemental one – if we get over 90.5% everyone gets 1-point ec!

TA Lead Final Review

Final Exam Topics

Sorting

- Sorting algorithm properties (stable, in-place)
- Quadratic sorts: insertion sort, selection sort
- Faster sorts: heap sort, merge sort, quick sort
- Runtimes of all of the above (in the best and worst case)

Graphs

- definitions (e.g., directed, undirected, weighted, unweighted, walks, paths, cycles, self-loops, parallel edges, trees, DAGs, etc.)
- implementations (Adjacency list, Adjacency matrix, and their pros and cons)
- traversals (BFS and DFS)
- Single-source shortest-path algorithms: Dijkstra's algorithm
- Topological sort
- MST algorithms: Prim and Kruskal
- Disjoint sets
- Framing/modeling problems with graphs

Coding Projects

- Implementation of each data structure
- Best / average / worst case runtime for each method of each data structure

Design Decisions

- Given a scenario, what ADT, data structure implementation and/or algorithm is best optimized for your goals?
  - What is unique or specialized about your chosen tool?
  - How do the specialized features of your chosen tool contribute to solving the given problem scenario?
  - How expensive is this tool and its features in terms of runtime and memory?
- Given a scenario, what changes might you make to a design to better serve your goals?

Memory and Locality

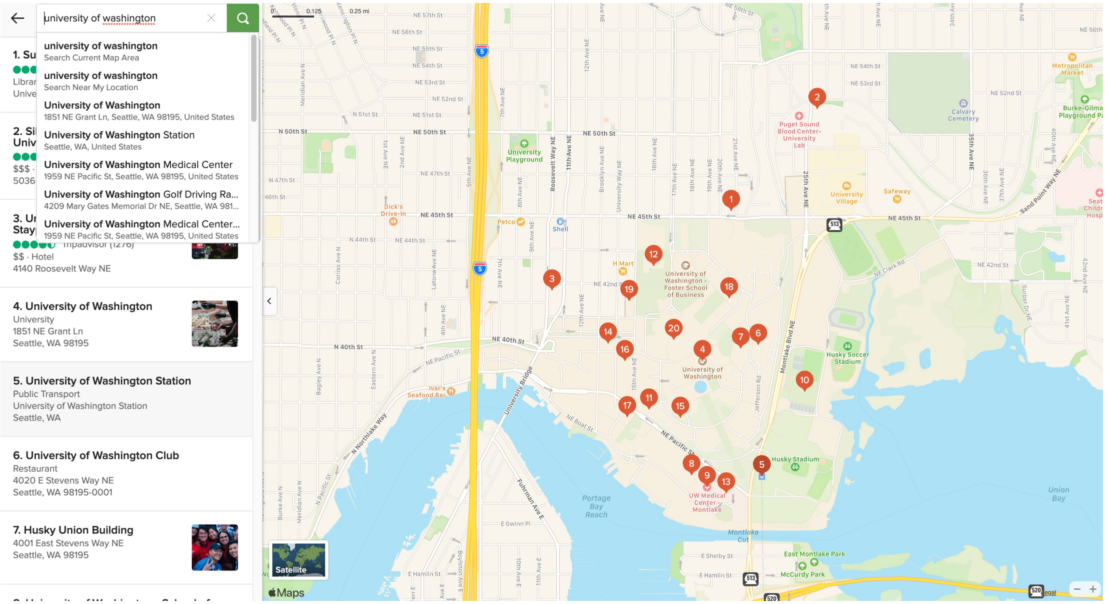
- How to leverage caching

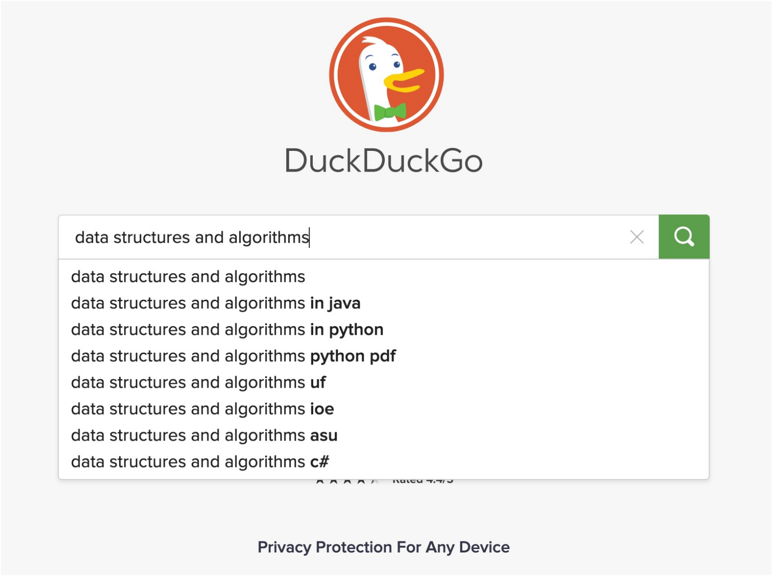
Pre-midterm topics

- all ADTs and data structures
- Asymptotic analysis
  - Code Modeling (including recurrences)
  - Complexity Classes
  - Big O, Omega, Theta
- BSTs, AVL Trees
- Hashing
- Heaps

Autocomplete

- Search Engines support autocomplete.
- How do you efficiently implement autocomplete with the ADTs we know so far?
- Formal Problem: Given a “prefix” of a string, find all strings in a set of possible strings that have the given prefix.

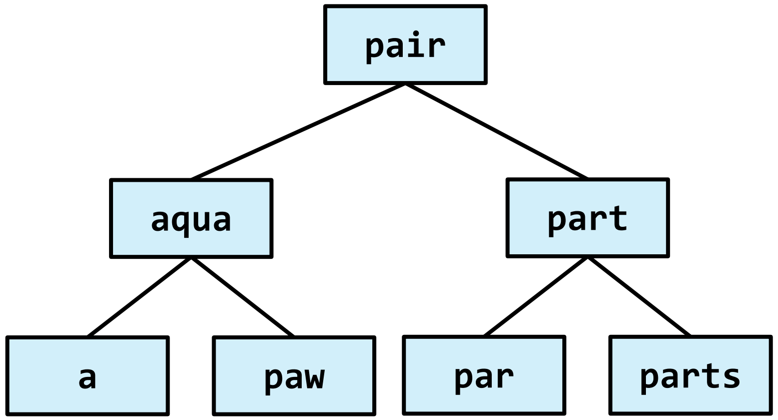




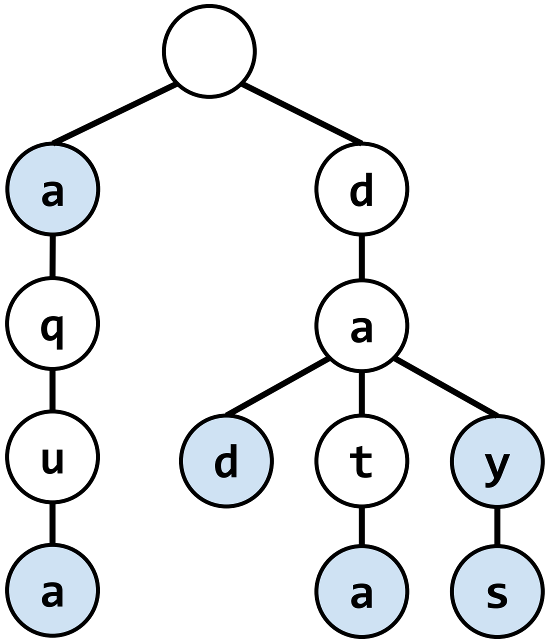
Tries: A *Specialized* Data Structure

Tries are a character-by-character set-of-Strings implementation

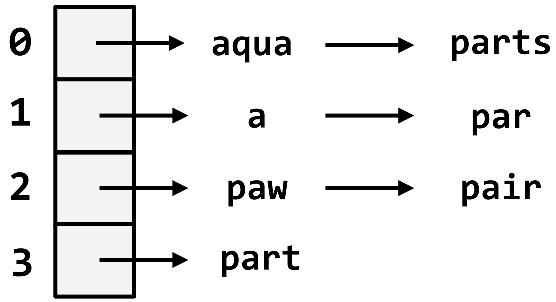
Nodes store *parts of keys* instead of *keys*



Binary Search Tree



Trie



Hash Table

[Trying to Understand Tries]().

Abstract Trie

Each level represents an index

- Children represent next possible characters at that index

This Trie stores the following set of Strings:

`a, aqua, dad,`

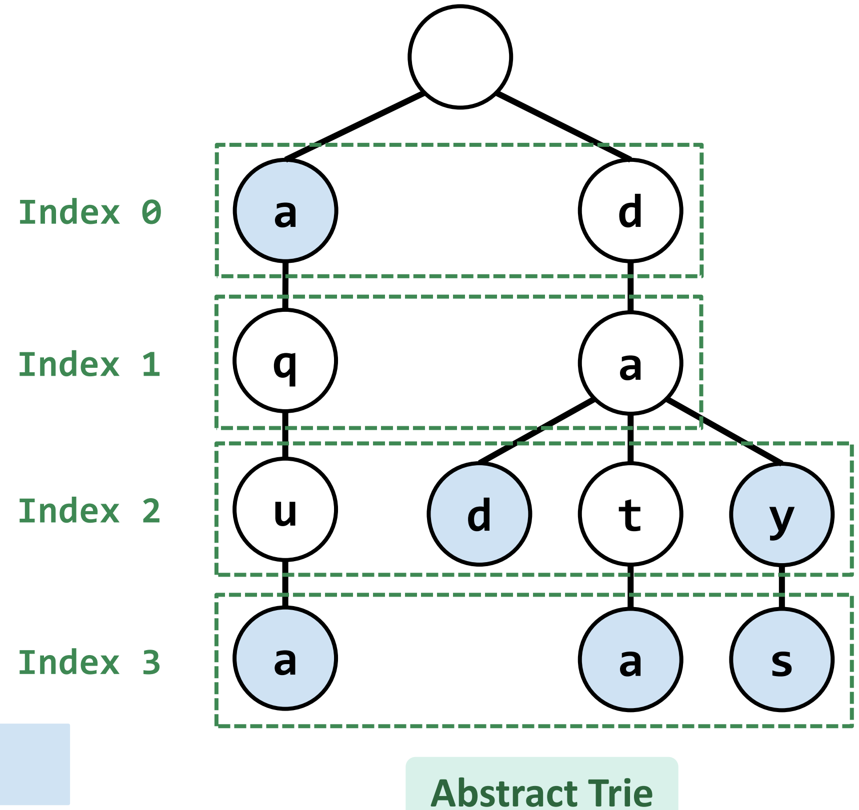
`data, day, days`

0 1 2 3 0 1 2 3 0 1 2

0 1 2 3 0 1 2 0 1 2 3

How do we deal with `a` and `aqua`?

- Mark complete Strings with a `boolean` (shown in blue)
- Complete string: a **String** that belongs in our set



Abstract Trie

Searching in Tries

Search hit: the final node is a key (colored blue)

Search miss: caused in one of two ways

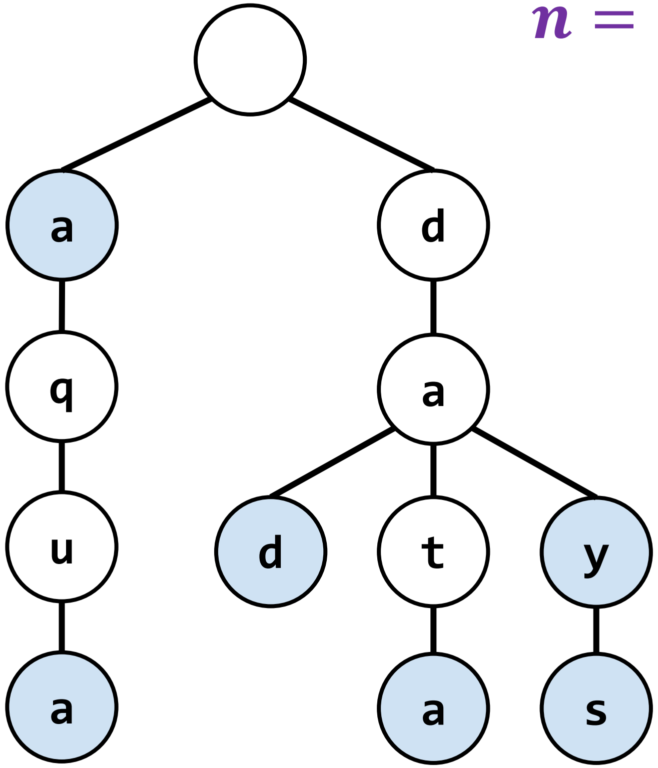
1. The final node is not a key (not colored blue)
2. We "fall" off the Trie

```
contains("data")   // hit,  l = 4
contains("da")     // miss, l = 2
contains("a")      // hit,  l = 1
contains("dubs")   // miss, l = 4
```

`contains` runtime given key of length l with n keys in Trie: $\Theta(l)$

`a, aqua, dad,`
`data, day, days`

$n = 6$



Abstract Trie

Prefix Operations with Tries

`a, aqua, dad, data, day, days`

The main appeal of Tries is its efficient prefix matching!

Prefix: find set of keys associated with given prefix

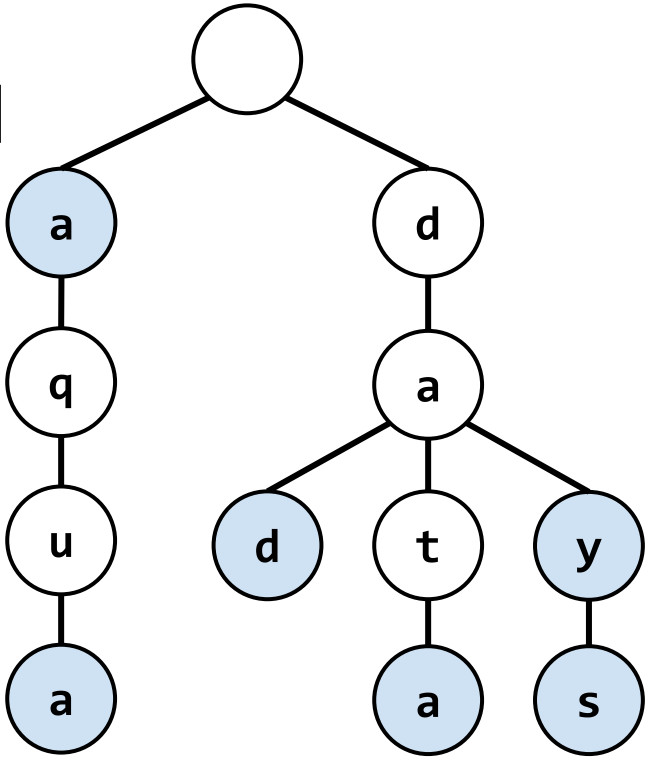
`keysWithPrefix("day")` returns `["day", "days"]`

Longest Prefix From Trie: given a String, retrieve longest prefix of that String that exists in the Trie

`longestPrefixOf("aquarium")` returns `"aqua"`

`longestPrefixOf("aqueous")` returns `"aqu"`

`longestPrefixOf("dawgs")` returns `"da"`



Abstract Trie

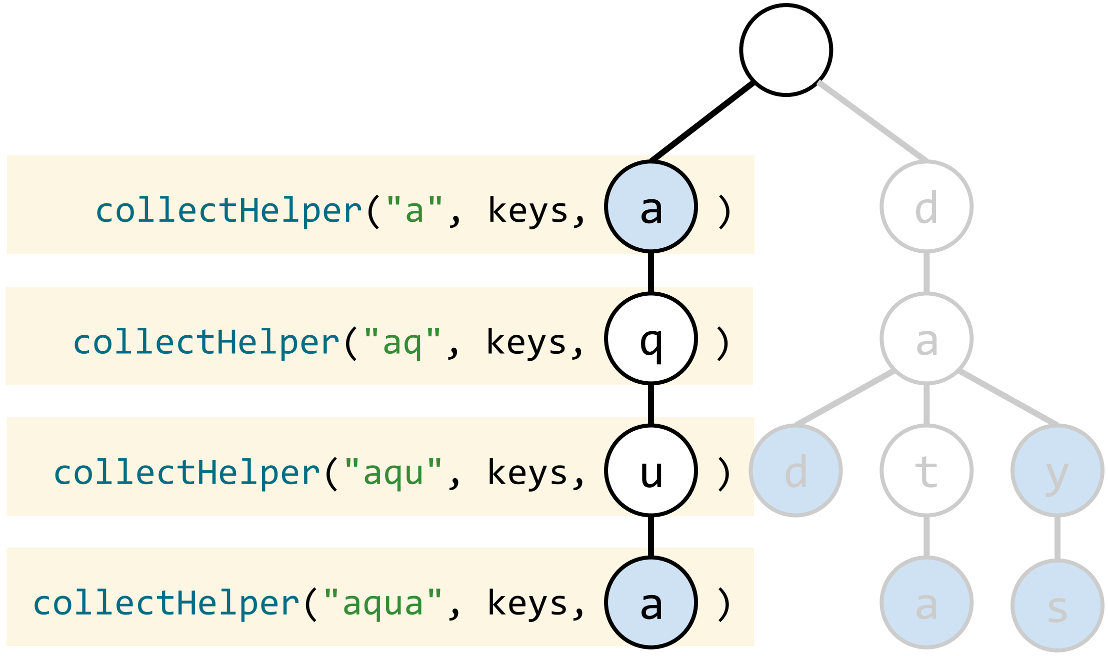
Collecting Trie Keys

- **Collect:** return set of all keys in the Trie (like `keySet()`)

  `collect(trie) = ["a", "aqua", "dad", "data", "day", "days"]`

```
List collect() {
    List keys;
    for (Node c : root.children) {
        collectHelper(n.char, keys, c);
    }
    return keys;
}

void collectHelper(String str, List keys, Node n) {
    if (n.isKey()) {
        keys.add(s);
    }
    for (Node c : n.children) {
        collectHelper(str + c.char, keys, c);
    }
}
```



`keysWithPrefix` Implementation

- `keysWithPrefix(String prefix)`
  - Find all the keys that corresponds to the given prefix

```
List keysWithPrefix(String prefix) {

    Node root;  // Node corresponding to given prefix
    List keys;  // Empty list to store keys

    for (Node n : root.children) {
        collectHelper(prefix + n.char, keys, c);
    }
}

void collectHelper(String str, List keys, Node n) {
    if (n.isKey()) {
        keys.add(s);
    }
    for (Node c : n.children) {
        collectHelper(str + c.char, keys, c);
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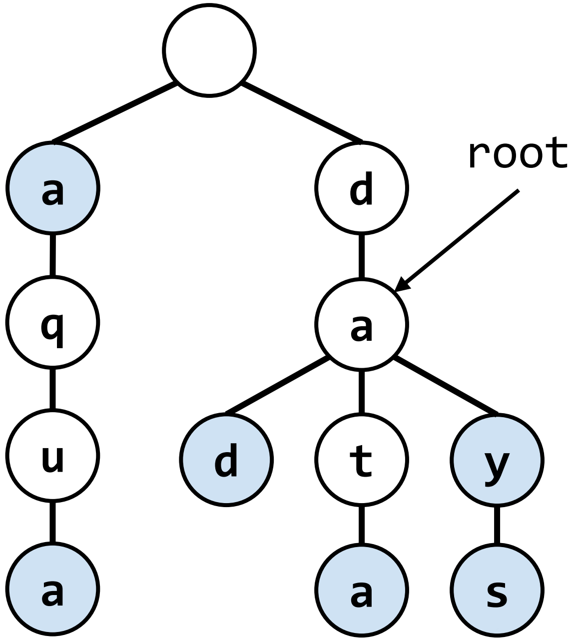
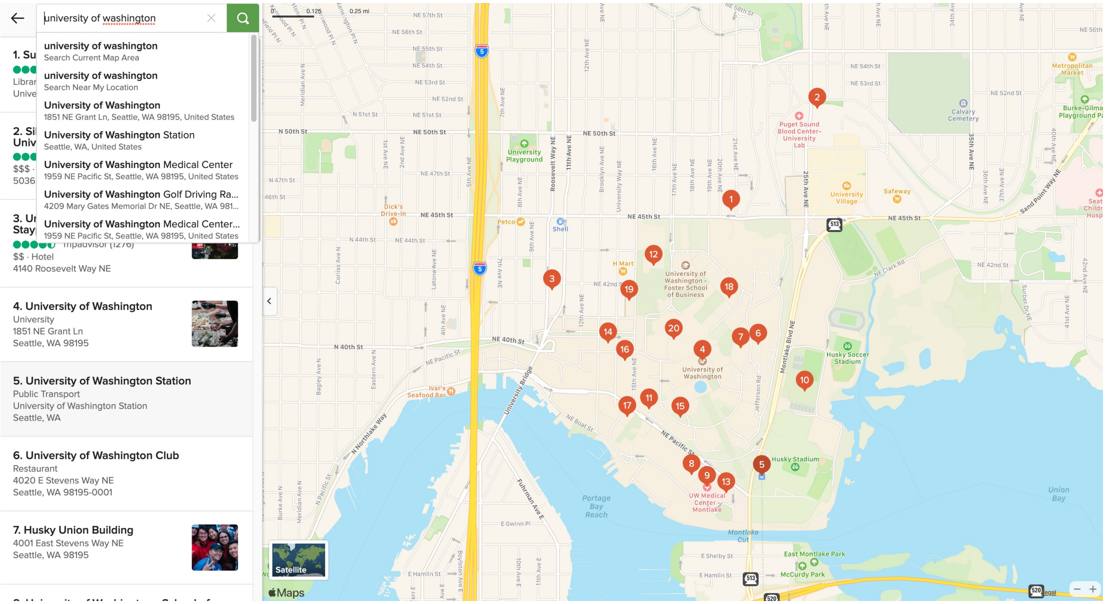
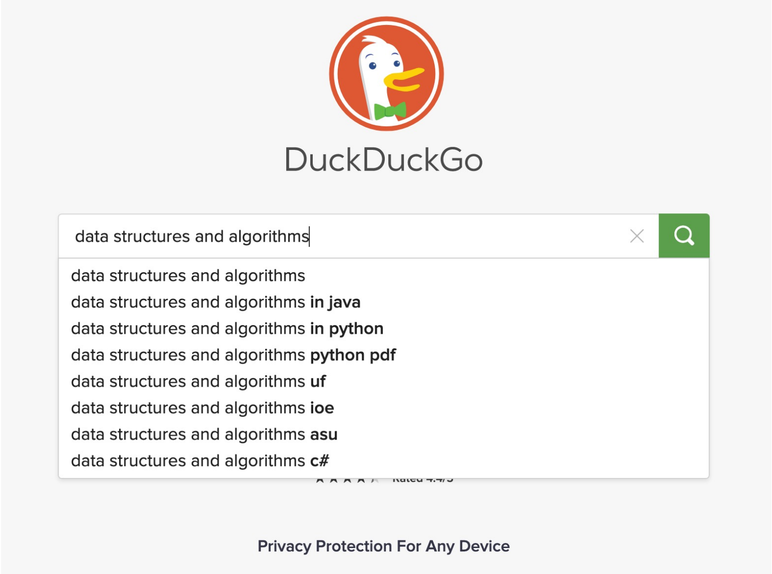


Autocomplete with Tries

- Autocomplete should return the **most relevant results**

- One method: a Trie-based `Map<String, Relevance>`
  - When a user types in a string `"hello"`, call `keysWithPrefix("hello")`
  - Return the 10 Strings with the highest relevance





Lecture Outline

Tries Introduction

Implementing a Trie using arrays

Advanced Implementations: dealing with sparsity

- Hash Tables, BSTs, and Ternary Search Trees

Trie Implementation Idea: *Encoding*

ASCII Table

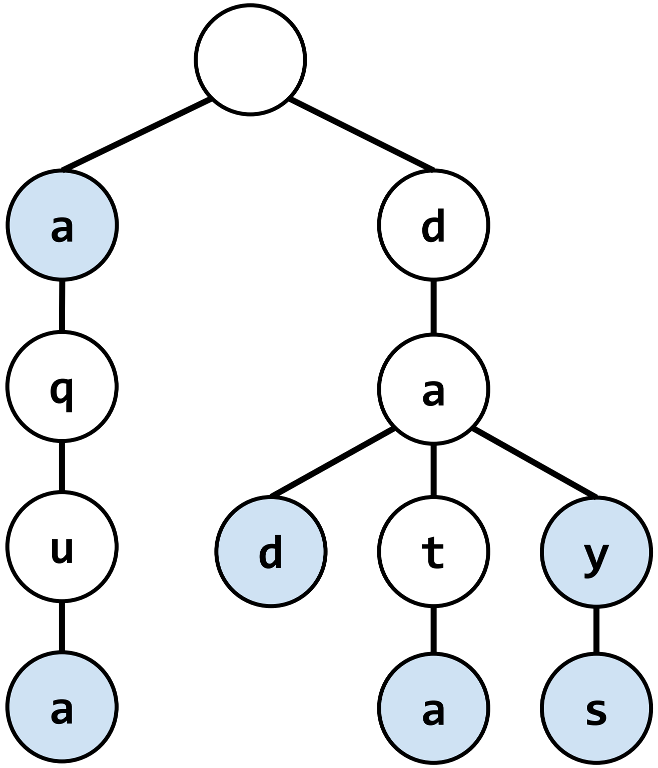
Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char
0	0	0		32	20	40	[space]	64	40	100	@	96	60	140	`
1	1	1		33	21	41	!	65	41	101	A	97	61	141	a
2	2	2		34	22	42	"	66	42	102	B	98	62	142	b
3	3	3		35	23	43	#	67	43	103	C	99	63	143	c
4	4	4		36	24	44	$	68	44	104	D	100	64	144	d
5	5	5		37	25	45	%	69	45	105	E	101	65	145	e
6	6	6		38	26	46	&	70	46	106	F	102	66	146	f
7	7	7		39	27	47	'	71	47	107	G	103	67	147	g
8	8	10		40	28	50	(	72	48	110	H	104	68	150	h
9	9	11		41	29	51	)	73	49	111	I	105	69	151	i
10	A	12		42	2A	52	*	74	4A	112	J	106	6A	152	j
11	B	13		43	2B	53	+	75	4B	113	K	107	6B	153	k
12	C	14		44	2C	54	,	76	4C	114	L	108	6C	154	l
13	D	15		45	2D	55	-	77	4D	115	M	109	6D	155	m
14	E	16		46	2E	56	.	78	4E	116	N	110	6E	156	n
15	F	17		47	2F	57	/	79	4F	117	O	111	6F	157	o
16	10	20		48	30	60	0	80	50	120	P	112	70	160	p
17	11	21		49	31	61	1	81	51	121	Q	113	71	161	q
18	12	22		50	32	62	2	82	52	122	R	114	72	162	r
19	13	23		51	33	63	3	83	53	123	S	115	73	163	s
20	14	24		52	34	64	4	84	54	124	T	116	74	164	t
21	15	25		53	35	65	5	85	55	125	U	117	75	165	u
22	16	26		54	36	66	6	86	56	126	V	118	76	166	v
23	17	27		55	37	67	7	87	57	127	W	119	77	167	w
24	18	30		56	38	70	8	88	58	130	X	120	78	170	x
25	19	31		57	39	71	9	89	59	131	Y	121	79	171	y
26	1A	32		58	3A	72	:	90	5A	132	Z	122	7A	172	z
27	1B	33		59	3B	73	;	91	5B	133	[	123	7B	173	{
28	1C	34		60	3C	74	<	92	5C	134	\	124	7C	174	|
29	1D	35		61	3D	75	=	93	5D	135	]	125	7D	175	}
30	1E	36		62	3E	76	>	94	5E	136	^	126	7E	176	~
31	1F	37		63	3F	77	?	95	5F	137	_	127	7F	177	

DataIndexedCharMap Pseudocode

```
class TrieSet {
    final int R = 128;  // # of ASCII encodings
    Node overallRoot;

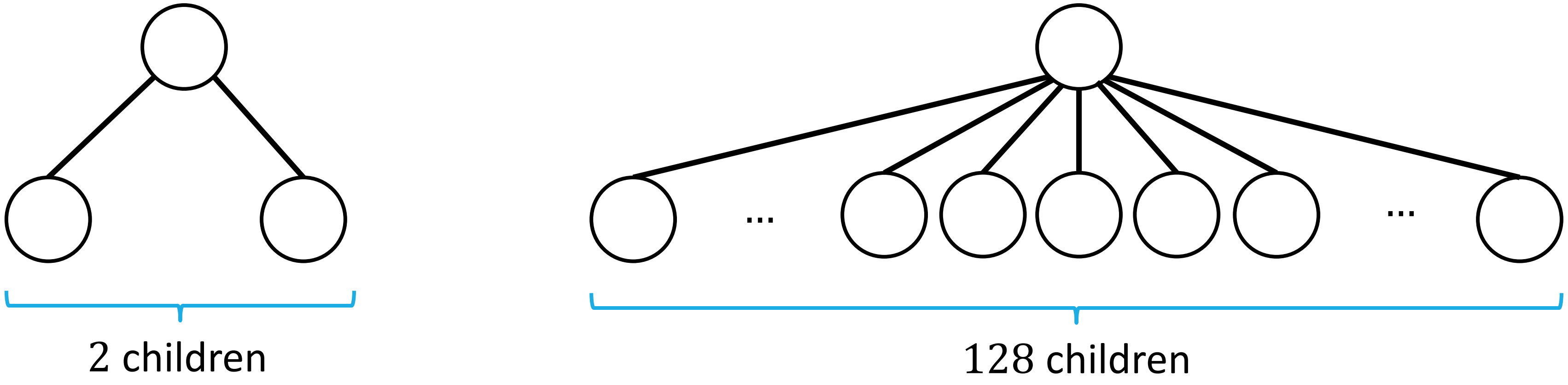
    // Private internal class
    class Node {
     // Field declarations
        char ch;
        boolean isKey;
        DataIndexedCharMap<Node> next;  // array encoding

        // Constructor
        Node(char c, boolean b, int R) {
            ch = c;
            isKey = b;
            next = new DataIndexedCharMap<Node>(R);
        }
    }
}
```



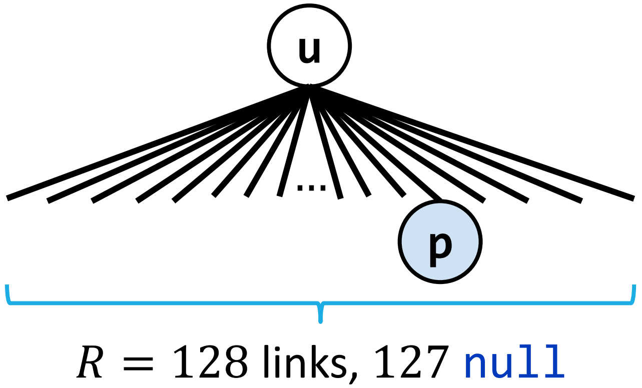
Data Structure for Trie Implementation

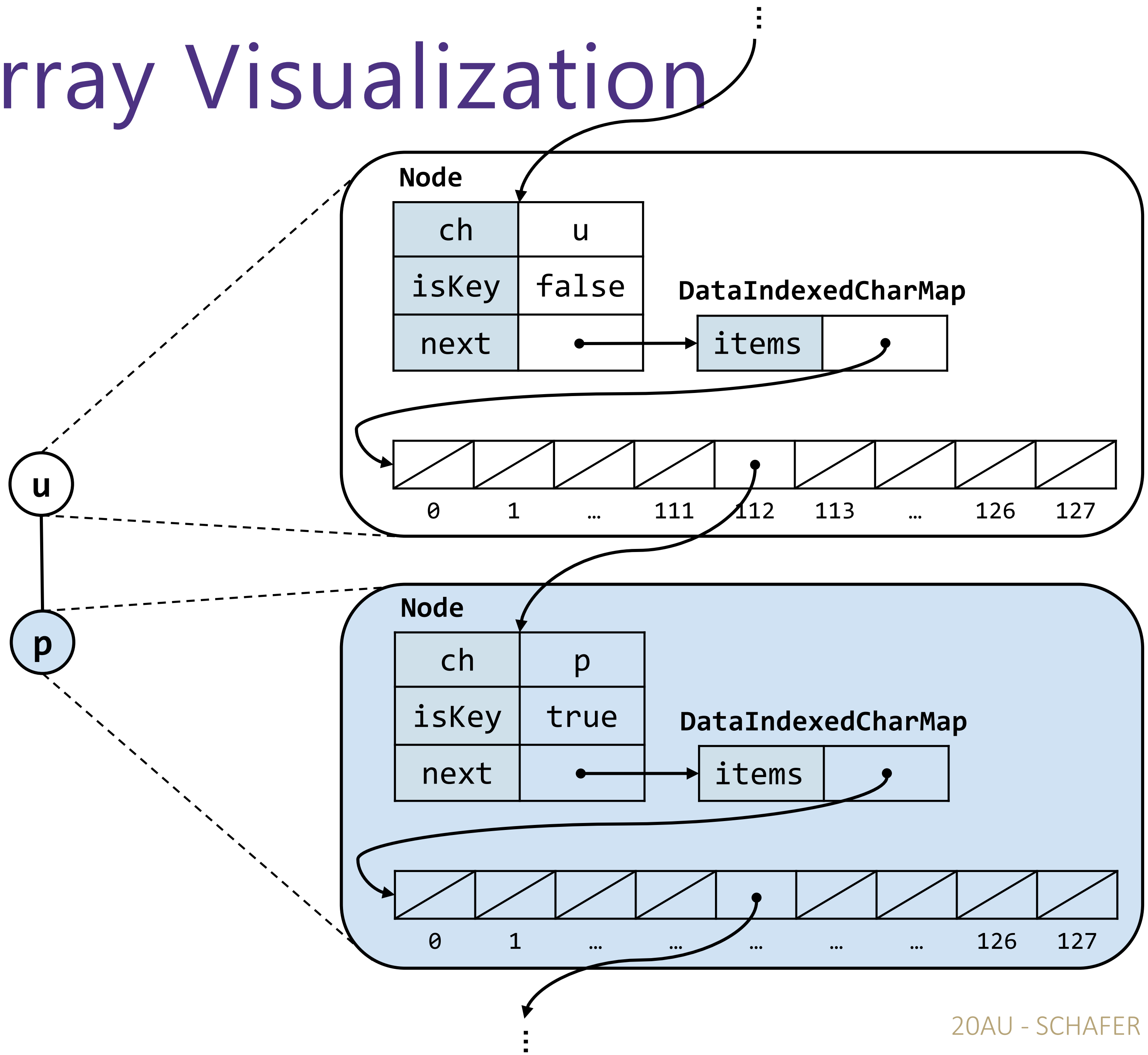
- Think of a Binary Tree
  - Instead of two children, we have 128 possible children
  - Each child represents a possible next character of our Trie
- How could we store these 128 children?



Data-Indexed Array Visualization

```
// Private internal class
class Node {
    // Field declarations
    char ch;
    boolean isKey;
    DataIndexedCharMap<Node> next;
}
```



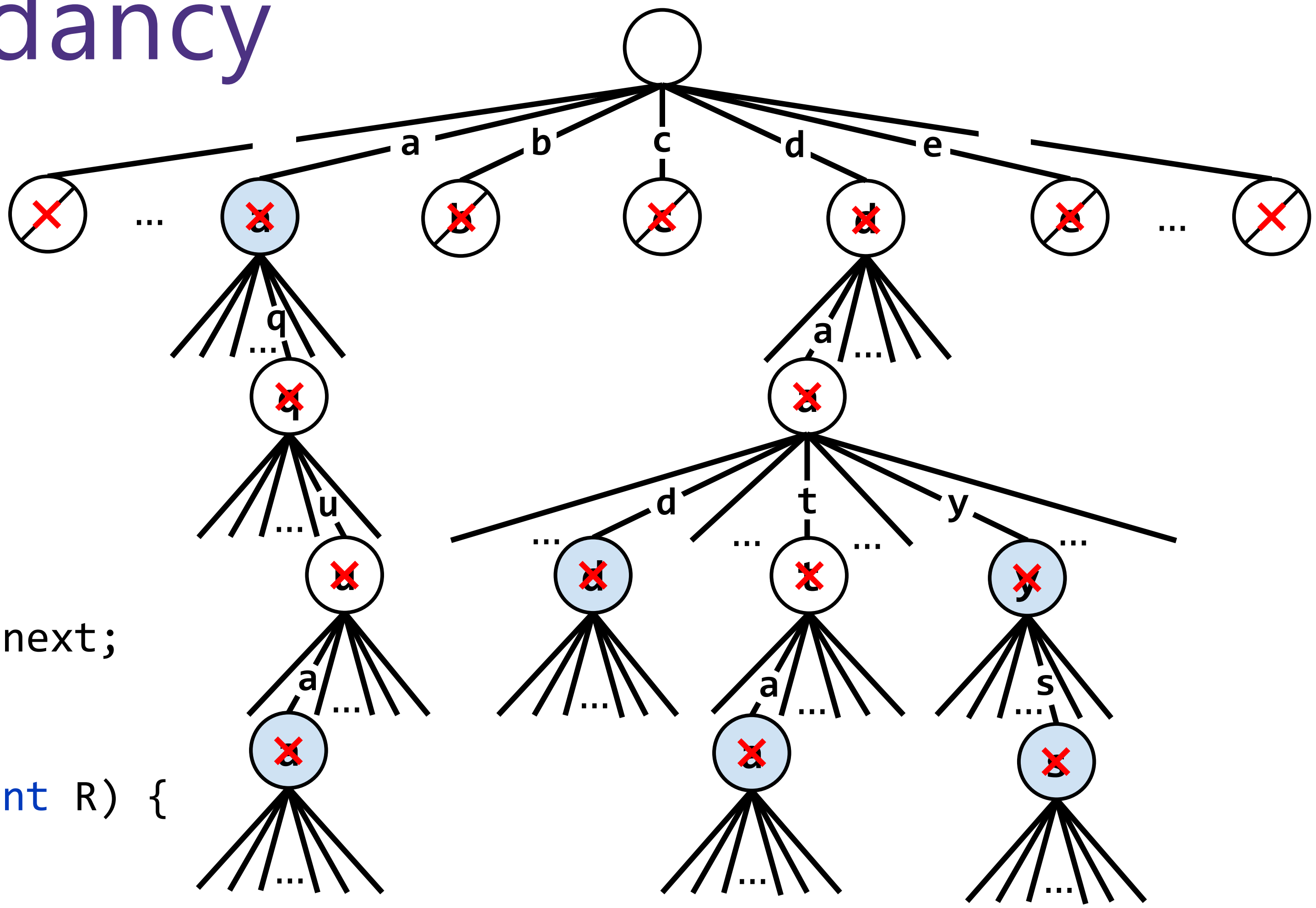


Removing Redundancy

```
class TrieSet {
    final int R = 128;
    Node overallRoot;

    // Private internal class
    class Node {
     // Field declarations
        char ch;
        boolean isKey;
        DataIndexedCharMap<Node> next;

        // Constructor
        Node(char c, boolean b, int R) {
            ch = c;
            isKey = b;
            next = new DataIndexedCharMap<Node>(R);
        }
    }
}
```



Itempool

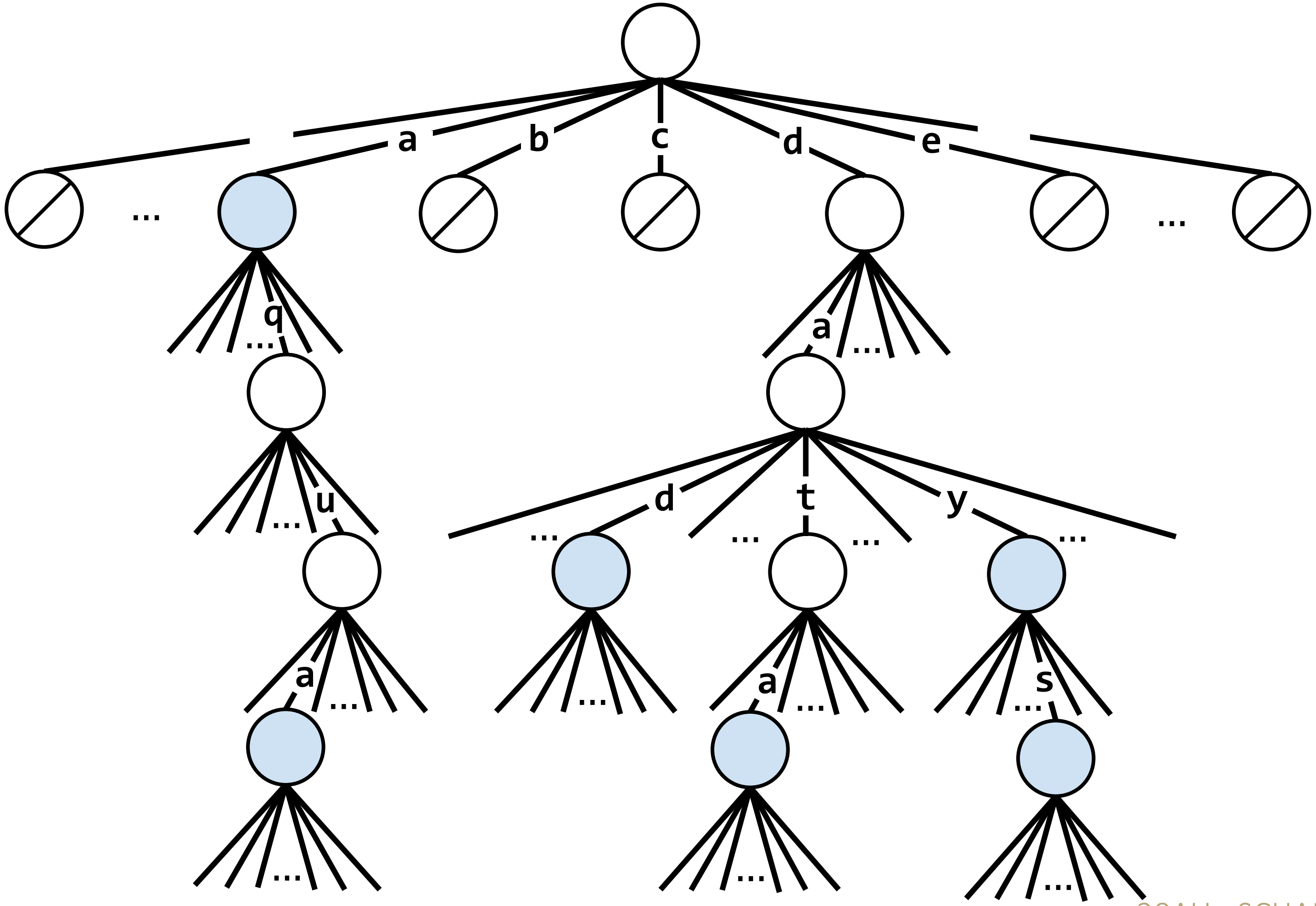
Pause Video when Prompted

Does the structure of a Trie depend on the order of insertion?

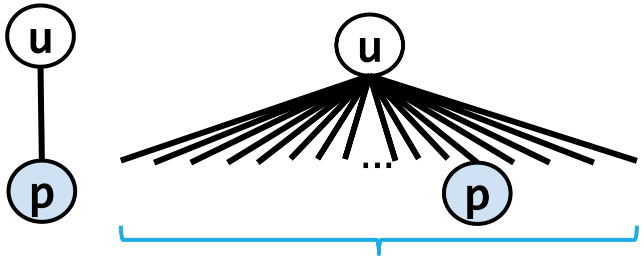
a) Yes

b) No

c) I'm not sure…



Runtime Comparison



R = 128 links, 127 `null` (unused)

- Typical runtime when treating length l of keys as a constant:

Data Structure	Key Type	`contains`	`add`	`keysWithPrefix`
Balanced BST	Comparable	$\Theta(\log(n))$	$\Theta(\log(n))$	$\Theta(n)$
Hash Map	Hashable	$\Theta(1)$*	$\Theta(1)$*	$\Theta(n)$
Trie (Data-Indexed Array)	String (Character)	$\Theta(1)$	$\Theta(1)$	$\Theta(p)$ **

* In-practice runtime

** Where p is the number of strings with the given prefix. Usually p << n.

- Takeaways:
  - \+ When keys are Strings, Tries give us a better `add` and `contains` runtime
  - – `DataIndexedCharMap` takes up a lot of space by storing R links per node

Lecture Outline

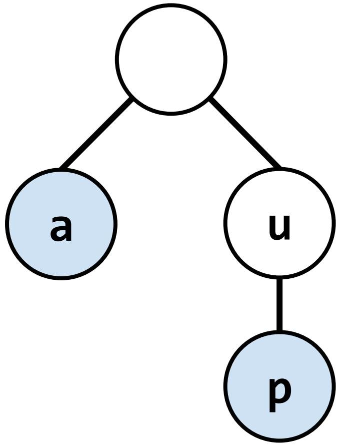
Tries Introduction

Implementing a Trie using arrays

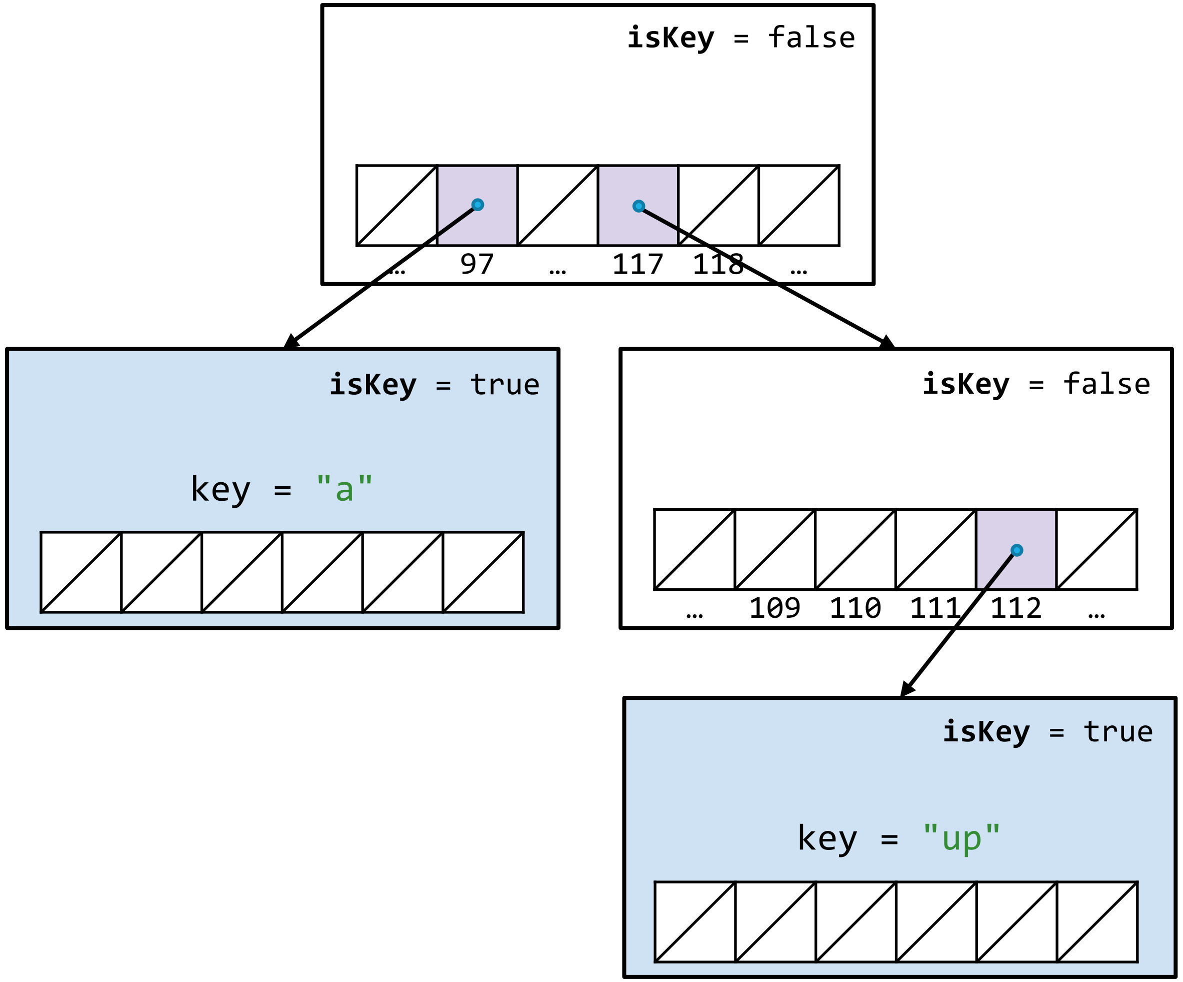
Advanced Implementations: dealing with sparsity

- Hash Tables, BSTs, and Ternary Search Trees

DataIndexedCharMap Implementation



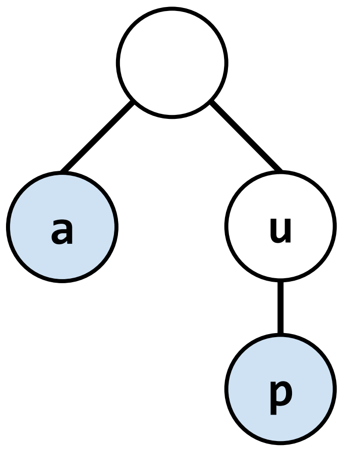
Abstract Trie



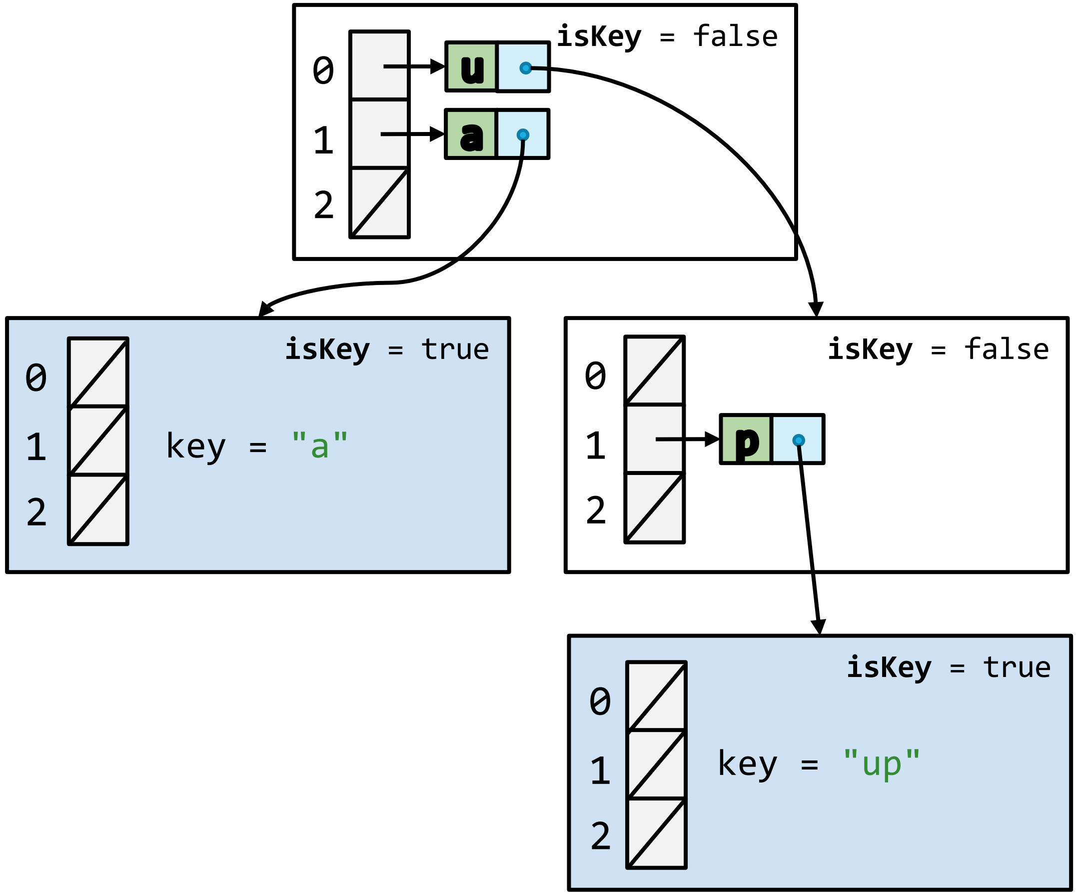
Data-Indexed Array Trie

Hash Table-based Implementation

- Use Hash Table to find character at a given index



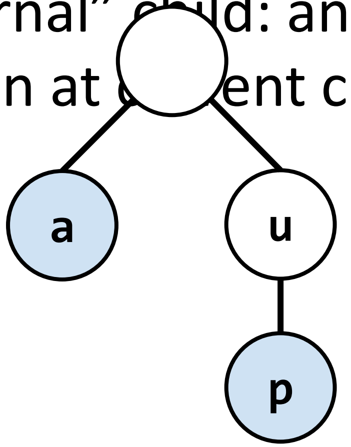
Abstract Trie



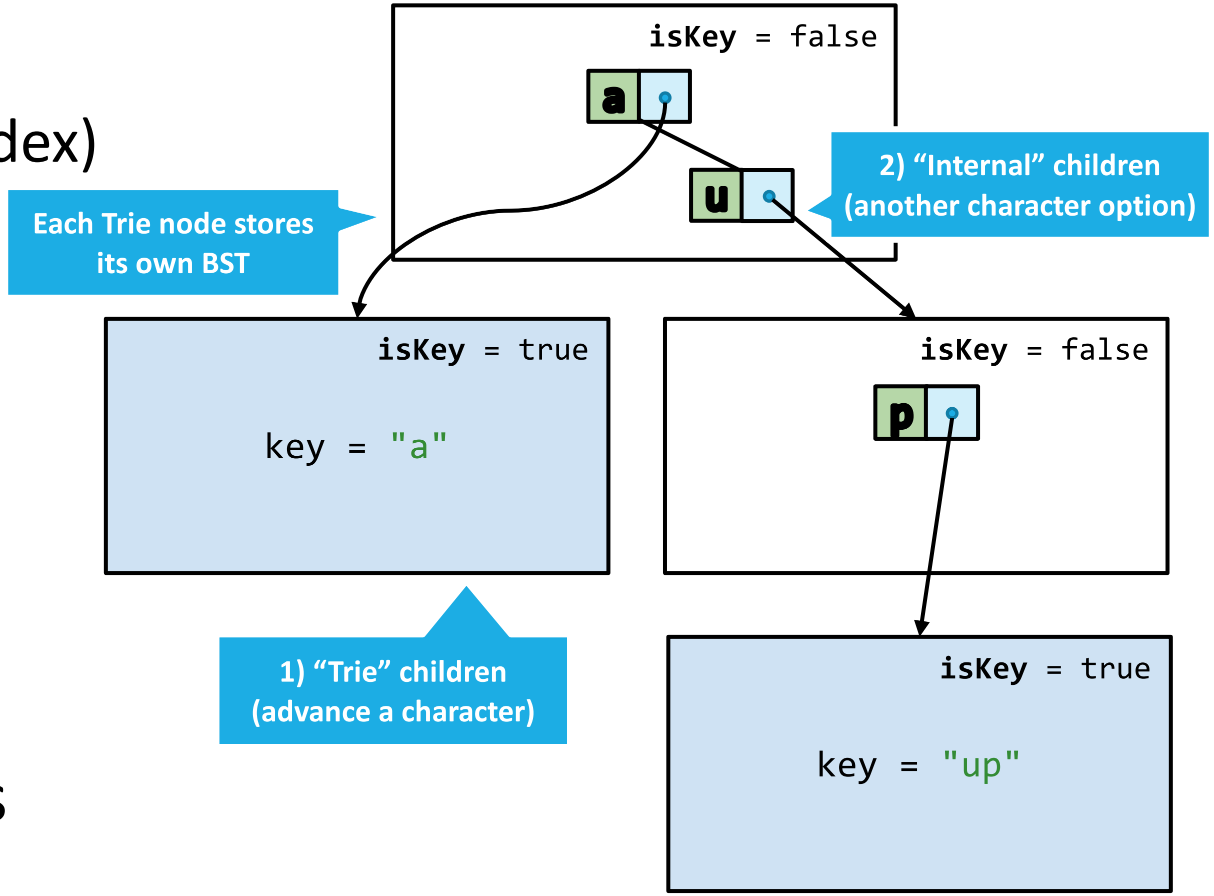
Hash Table-based Trie

BST-based Implementation

- Use Binary Search Tree to find character at a given index
- Two types of children:
  1) “Trie” child: advance a character (index)
  2) “Internal” child: another character option at current character (index)



Abstract Trie

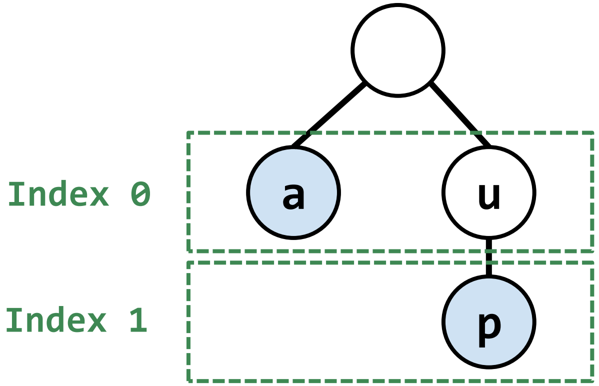


BST-based Trie

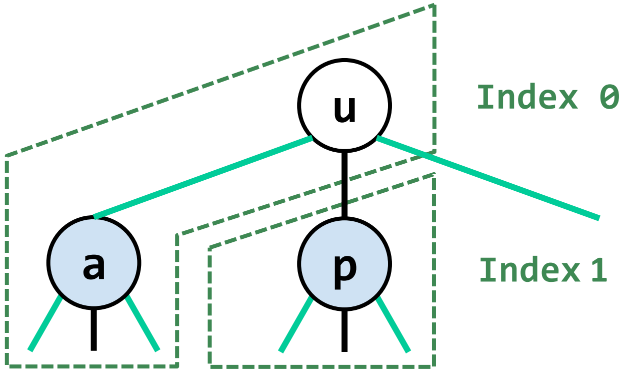
- Both are essentially child references
  - Could we simplify this design?

Ternary Search Tree (TST) Implementation

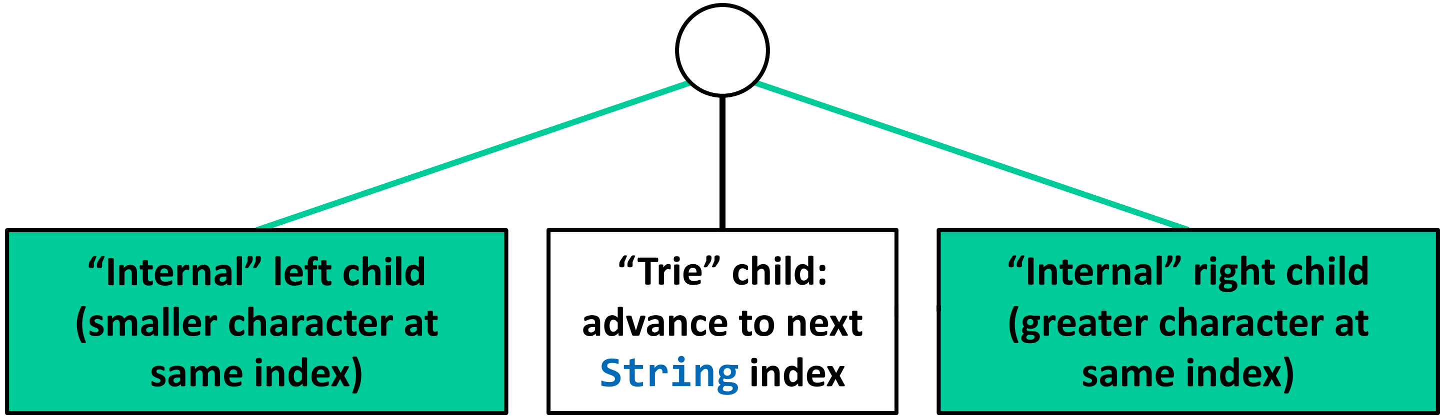
- Combines character mapping with Trie itself



Abstract Trie



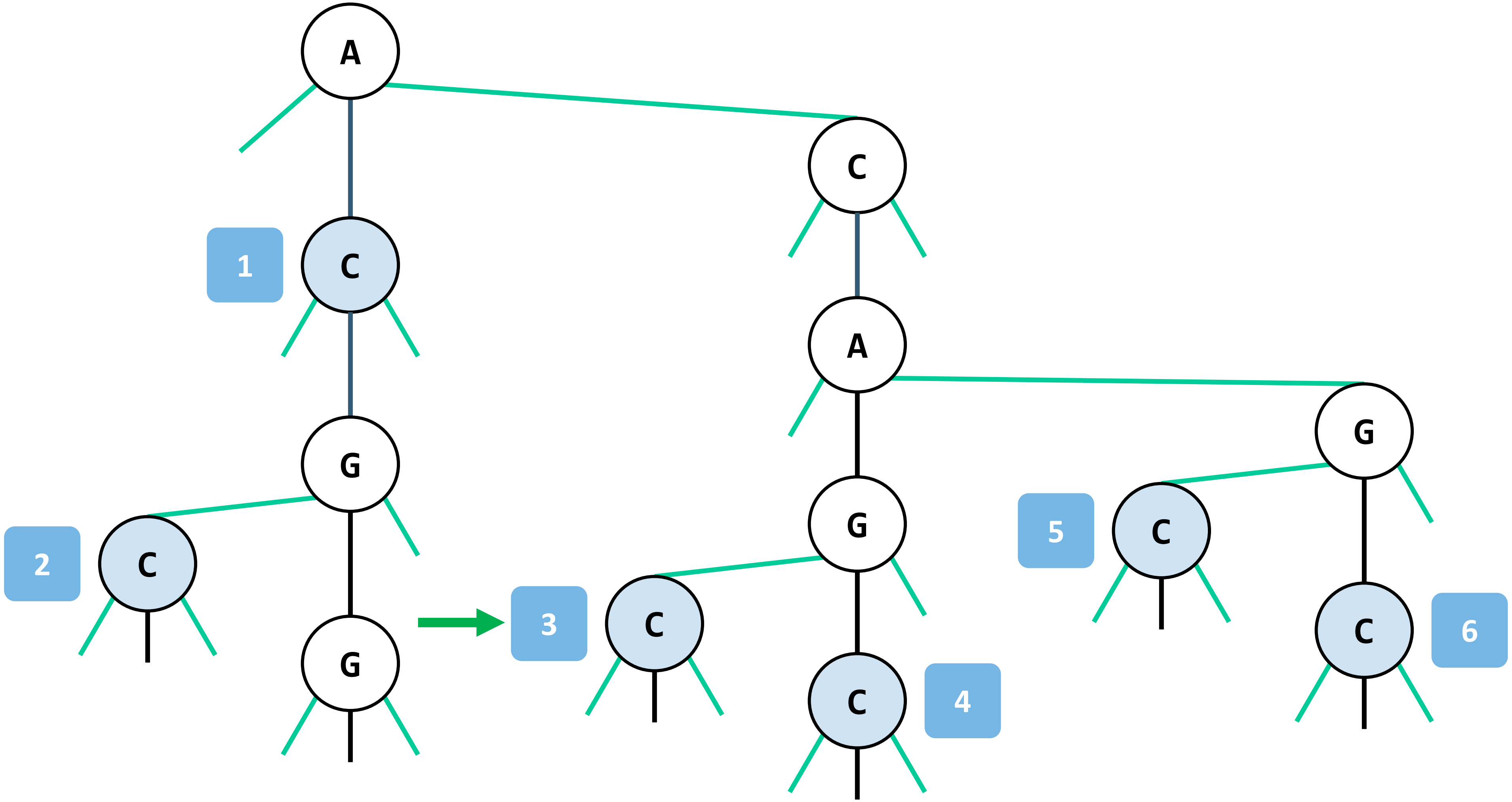
Ternary Search Tree (TST)



Itempool

Pause Video when Prompted

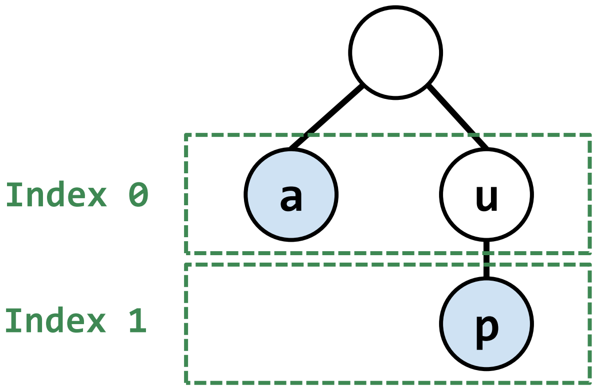
Which node is associated with the key "CAC"?



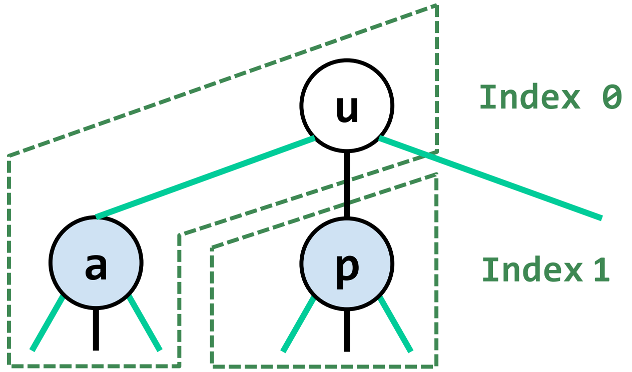
Searching in a TST

- Searching in a TST
  - If smaller, take left link
  - If greater, take right link
  - If equal, take the middle link and move to next character
- **Search hit:** final node yields a key that belongs in our set
- **Search miss:** reach `null` link or final node is yields a key not in our set

Keys: [a , u p]
Index: 0 0 1



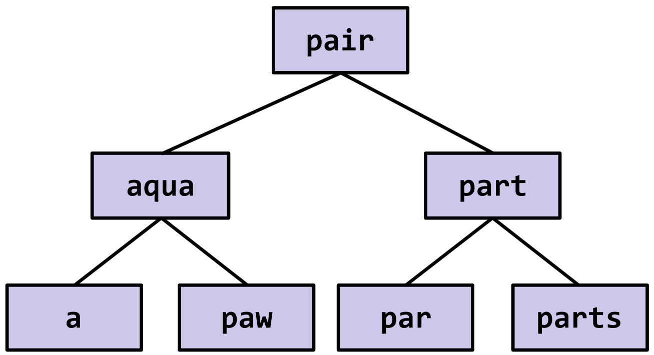
Abstract Trie



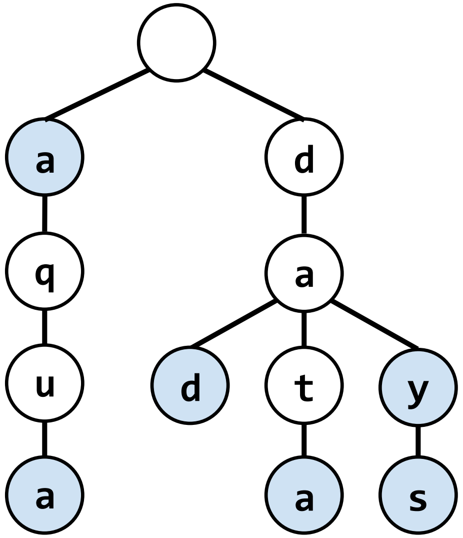
Ternary Search Tree (TST)

Trie Takeaways

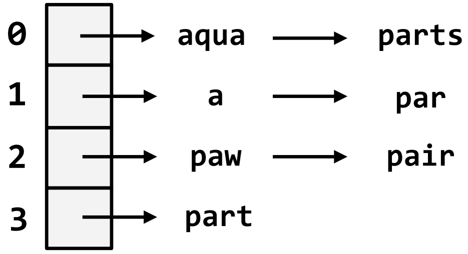
- Tries can be used for storing Strings (or any sequential data)
- Real-world performance often better than Hash Table or Search Tree
- Many different implementations: `DataIndexedCharMap`, Hash Tables, BSTs (and more possible data structures within nodes), and TSTs
- Tries enable efficient prefix operations like `keysWithPrefix`



Binary Search Tree



Trie



Hash Table