Lecture 22: Introduction to Sorting

CSE 373: Data Structures and Algorithms
Administrivia

Assignment Reminders
- Project 4 due Wednesday June 2nd
- Exercise 4 due Friday May 21st
- Grades coming this week
INEFFECTIVE SORTS

DEFINE  HALLHEARTEDMergesort(list):
    IF LENGTH(list) < 2:
        RETURN list
    PIVOT = INT(LENGTH(list) / 2)
    A = HALLHEARTEDMergesort(list[:PIVOT])
    B = HALLHEARTEDMergesort(list[PIVOT:])
    // UHM HMM
    RETURN [A, B] // HERE. SORRY.

DEFINE  FIRSTBogosort(list):
    // AN OPTIMIZED BOGOSORT
    // RUNS IN O(N LOGN)
    FOR n FROM 1 TO LOG(LENGTH(list)):
        Shuffle(list)
        IF ISORTED(list):
            RETURN list
        RETURN "KERNEL PAGE FAULT (ERROR CODE: 2)"

DEFINE  JOHNNIEABQUICKSORT(list):
    OK SO YOU CHOOSE A PIVOT
    THEN DIVIDE THE LIST IN HALF
    FOR EACH HALF:
        CHECK TO SEE IF IT'S SORTED
        NO LIMIT IT DOESN'T MATTER
        COMPARE EACH ELEMENT TO THE PIVOT
        THE BIGGER ONES GO IN A NEW LIST
        THE EQUAL ONES GO INTO OH
        THE SECOND LIST FROM BEFORE
        HANG ON, LET ME NAME THE LIST
        THIS IS LIST A
        THE NEW ONE IS LIST B
        PUT THE BIG ONES INTO LIST A
        NOW TAKE THE SECOND LIST
        CALL IT LIST OH, A2
        WHICH ONE WAS THE PIVOT IN?
        SCRATCH ALL THAT
        IT JUST RECURSIVELY CALLS ITSELF
        UNTIL BOTH LISTS ARE EMPTY
        RIGHT?
        NOT EMPTY, BUT YOU KNOW WHAT I MEAN
        AM I ALLOWED TO USE THE STANDARD LIBRARIES?

DEFINE  PHANTOMSORT(list):
    IF ISORTED(list):
        RETURN list
    FOR n FROM 1 TO 10000:
        PIVOT = RANDOM(0, LENGTH(list))
        LIST = LIST[PIVOT:] + LIST[:PIVOT]
        IF ISORTED(list):
            RETURN LIST
        IF NOT ISORTED(list):
            RETURN LIST
        IF NOT ISORTED(list):
            // THIS CAN'T BE HAPPENING
            RETURN LIST
        IF NOT ISORTED(list):
            // COME ON COME ON
            RETURN LIST
        // OH JEEZ
        // I'M GONNA BE IN SO MUCH TROUBLE
        LIST = []
        SYSTEM("SHUTDOWN -H -S")
        SYSTEM("RM -RF .")
        SYSTEM("RM -RF ~*")
        SYSTEM("RM -RF ~/")
        SYSTEM("RD /S /Q C:\*") // PORTABILITY
        RETURN [1, 2, 3, 4, 5]
Where are we?

This course is “data structures and algorithms”

Data structures
- Organize our data so we can process it effectively

Algorithms
- Actually process our data!

We’re going to start focusing on algorithms

We’ll start with sorting
- A very common, generally-useful preprocessing step
- And a convenient way to discuss a few different ideas for designing algorithms.
Types of Sorts

Comparison Sorts

Compare two elements at a time

General sort, works for most types of elements

What does this mean? `compareTo()` works for your elements
- And for our running times to be correct, `compareTo` must run in $O(1)$ time.

Niche Sorts aka “linear sorts”

Leverages specific properties about the items in the list to achieve faster runtimes

niche sorts typically run $O(n)$ time

For example, we’re sorting small integers, or short strings.

In this class we’ll focus on comparison sorts
Sorting: Definitions (Knuth’s TAOCP)

An **ordering relation** < for keys a, b, and c has the following properties:

- Law of Trichotomy: Exactly one of a < b, a = b, b < a is true
- Law of Transitivity: If a < b, and b < c, then a < c

A **sort** is a permutation (re-arrangement) of a sequence of elements that puts the keys into non-decreasing order, relative to the ordering relation

- \( x_1 \leq x_2 \leq x_3 \leq \ldots \leq x_N \)

```java
class Movie {
    String name;
    int year;
}
```

- More complex: Whenever we sort, *we also must decide* what ordering relation to use for that application
  - Sort by name?
  - Sort by year?
  - Some combination of both?
A sort is **stable** if the relative order of *equivalent* keys is maintained after sorting.

|-------|-------------|------------|------------|-------------|-------------|-------------|

**Stable** sort using name as key:

|-------------|-------------|------------|------------|-------------|-------------|

**Unstable** sort using name as key:

|-------------|-------------|------------|------------|-------------|-------------|

- Stability and Equivalency only matter for complex types
  - i.e. when there is more data than just the key

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Sorting: Performance Definitions

Runtime performance is sometimes called the **time complexity**
- Example: Dijkstra’s has time complexity $O(E \log V)$.

Extra memory usage is sometimes called the **space complexity**
- Dijkstra’s has space complexity $\Theta(V)$
  - Priority Queue, distTo and edgeTo maps
- The input graph takes up space $\Theta(V+E)$, but we don’t count this as part of the space complexity since the graph itself already exists and is an input to Dijkstra’s
Sorting Goals

**In Place sort**

A sorting algorithm is in-place if it allocates $O(1)$ extra memory

Modifies input array (can’t copy data into new array)

Useful to minimize memory usage

**Stable sort**

A sorting algorithm is stable if any equal items remain in the same relative order before and after the sort

Why do we care?

- "data exploration" Client code will want to sort by multiple features and "break ties" with secondary features

$[(8, \text{"fox"}), (9, \text{"dog"}), (4, \text{"wolf"}), (8, \text{"cow"})]$  

$[(4, \text{"wolf"}), (8, \text{"fox"}), (8, \text{"cow"}), (9, \text{"dog"})]$ – Stable

$[(4, \text{"wolf"}), (8, \text{"cow"}), (8, \text{"fox"}), (9, \text{"dog"})]$ – Unstable

**Speed**

Of course, we want our algorithms to be fast.

Sorting is so common, that we often start caring about constant factors.
SO MANY SORTS

Quicksort, Merge sort, in-place merge sort, heap sort, insertion sort, intro sort, selection sort, timsort, cubesort, shell sort, bubble sort, binary tree sort, cycle sort, library sort, patience sorting, smoothsort, strand sort, tournament sort, cocktail sort, comb sort, gnome sort, block sort, stackoverflow sort, odd-even sort, pigeonhole sort, bucket sort, counting sort, radix sort, spreadsort, burstsort, flashsort, postman sort, bead sort, simple pancake sort, spaghetti sort, sorting network, bitonic sort, bogosort, stooge sort, insertion sort, slow sort, rainbow sort...
Goals

Algorithm Design (like writing invariants) is more art than science.

We’ll do a little bit of designing our own algorithms
- Take CSE 417 (usually runs in Winter) for more

Mostly we’ll understand how existing algorithms work

Understand their pros and cons
- Design decisions!

Practice how to apply those algorithms to solve problems
Algorithm Design Patterns

Algorithms don’t just come out of thin air.

There are common patterns we use to design new algorithms.

Many of them are applicable to sorting (we’ll see more patterns later in the quarter)

Invariants/Iterative improvement
  - Step-by-step make one more part of the input your desired output.

Using data structures
  - Speed up our existing ideas

Divide and conquer
  - Split your input
  - Solve each part (recursively)
  - Combine solved parts into a single
Principle 1

Invariants/Iterative improvement
- Step-by-step make one more part of the input your desired output.

We’ll write iterative algorithms to satisfy the following invariant:
After $k$ iterations of the loop, the first $k$ elements of the array will be sorted.
Selection Sort

Sorted Items

Current Item

Unsorted Items

https://www.youtube.com/watch?v=Ns4TPTC8whw
Selection Sort

public void selectionSort(collection) {
    for (entire list)
        int newIndex = findNextMin(currentItem);
        swap(newIndex, currentItem);
}

public int findNextMin(currentItem) {
    min = currentItem
    for (unsorted list)
        if (item < min)
            min = currentItem
    return min
}

public int swap(newIndex, currentItem) {
    temp = currentItem
    currentItem = newIndex
    newIndex = currentItem
}

Worst case runtime?  \( \Theta(n^2) \)

Best case runtime?  \( \Theta(n^2) \)

In-practice runtime?  \( \Theta(n^2) \)

Stable?  No

In-place?  Yes
Selection Sort Stability

Swapping non-adjacent items can result in instability of sorting algorithms
Insertion Sort

Sorted Items

Unsorted Items

Current Item

https://www.youtube.com/watch?v=ROalU379I3U
```
public void insertionSort(collection) {
    for (entire list)
        if (currentItem is smaller than largestSorted)
            int newIndex = findSpot(currentItem);
            shift(newIndex, currentItem);
}
public int findSpot(currentItem) {
    for (sorted list going backwards)
        if (spot found) return
}
public void shift(newIndex, currentItem) {
    for (i = currentItem > newIndex)
        item[i+1] = item[i]
        item[newIndex] = currentItem
}
```

Worst case runtime?  $\Theta(n^2)$

Best case runtime?  $\Theta(n)$

In-practice runtime?  $\Theta(n^2)$

Stable?  Yes

In-place?  Yes
Insertion Sort Stability

Insertion sort is stable
- All swaps happen between adjacent items to get current item into correct relative position within sorted portion of array
- Duplicates will always be compared against one another in their original orientation, thus it can be maintained with proper if logic
Principle 2

Selection sort:

After $k$ iterations of the loop, the $k$ smallest elements of the array are (sorted) in indices $0, ..., k - 1$

Runs in $\Theta(n^2)$ time no matter what.

Using data structures
- Speed up our existing ideas

If only we had a data structure that was good at getting the smallest item remaining in our dataset...
- We do!
Heap Sort

1. run Floyd’s buildHeap on your data
2. call removeMin n times

```java
public void heapSort(input) {
    E[] heap = buildHeap(input)
    E[] output = new E[n]
    for (n)
        output[i] = removeMin(heap)
}
```

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<th>Answer</th>
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<td>$\Theta(n \log n)$</td>
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</tr>
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<td>Stable?</td>
<td>No</td>
</tr>
<tr>
<td>In-place?</td>
<td>If we get clever...</td>
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In Place Heap Sort

Heap

Sorted Items

Current Item

percolateDown(22)
In Place Heap Sort

```
public void inPlaceHeapSort(input) {
    buildHeap(input) // alters original array
    for (n : input)
        input[n - i - 1] = removeMin(heap)
}
```

Complication: final array is reversed! Lots of fixes:
- Run reverse afterwards ($O(n)$)
- Use a max heap
- Reverse compare function to emulate max heap

Worst case runtime? $\Theta(n \log n)$
Best case runtime? $\Theta(n)$
In-practice runtime? $\Theta(n \log n)$
Stable? No
In-place? Yes