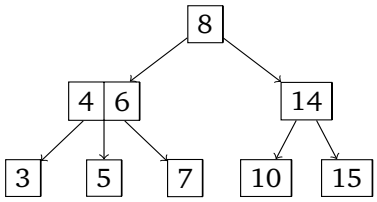
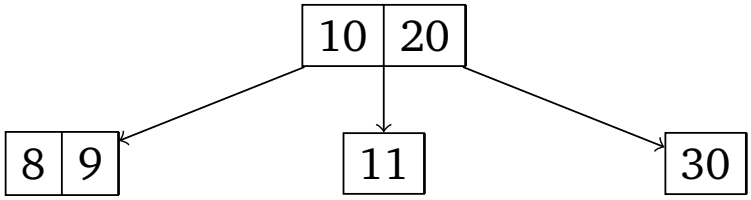
Section 04: Trees and Heaps

1. B-Trees

(a) Draw what the following 2-3 tree would look like after inserting 18, 38, 12, 13, and 20.



(b) Given the following initial 2-3-4 tree, draw the result of performing each operation.



(i) Insert 5 into this tree.

(ii) Insert 7 into the resulting tree.

(iii) Insert 12 into the resulting tree.

(c) Suppose the keys 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 are inserted sequentially into an initially empty 2-3-4 tree. Which insertions cause a split to take place?

2. Big-Θ: Red-Black Trees and BSTs

What is the worst-case big-Θ bound for runtime for each of the following?

(a) Insert and find in a BST.

(b) Insert and find in a Red-Black tree.

(c) Finding the minimum value in a Red-Black tree containing n elements.

(d) Finding the k-th largest item in a Red-Black tree containing n elements.

(e) Listing elements of a Red-Black tree in sorted order

3. Analyzing dictionaries

(a) What are the constraints on the data types you can store in a Red-Black tree? When is a Red-Black tree preferred over another dictionary implementation, such as a HashMap?

(b) When is using a BST preferred over a Red-Black tree?

4. Design

Imagine a database containing information about all trains leaving the Washington Union station on Monday. Each train is assigned a departure time, a destination, and a unique 8-digit train ID number.

What data structures would you use to solve each of the following scenarios? Depending on scenario, you may need to either (a) use multiple data structures or (b) modify the implementation of some data structure.

Justify your choice.

(a) Suppose the schedule contains 200 trains with 52 destinations. You want to easily list out the trains by destination.

(b) In the question above, trains were listed by destination. Now, trains with the same destination should further be sorted by departure time.

(c) A train station wants to create a digital kiosk. The kiosk should be able to efficiently and frequently complete look-ups by train ID number so visitors can purchase tickets or track the location of a train. The kiosk should also be able to list out all the train IDs in ascending order, for visitors who do not know their train ID.

Note that the database of trains is not updated often, so the removal and additions of new trains happen infrequently (aside from when first populating your chosen DS with trains).

5. Heap Insertions

(a) Insert the following sequence of numbers into a *max heap*:

[10, 7, 15, 17, 12, 20, 6, 32]

(b) Now, insert the same values into a *min heap*.

6. Heaps: Sorting and Reversing

(a) Suppose you have an array representation of a heap. Must the array be sorted?

(b) Suppose you have a sorted array (in increasing order). Must it be the array representation of a valid min-heap?

(c) You have an array representation of a min-heap. If you reverse the array, does it become an array representation of a max-heap?