RadixSorts
CSE 373 Winter 2020

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Announcements

- COVID-19 is really something, huh?
  - *HW8*: No change, still due
  - *Drop-in Times*: we'll switch to online DITs next week
  - *Workshops*: cancelled
  - *Quiz sections*: since topic is final prep, may switch to online format or cancel
  - *Final review session*: keeping, but in online format
  - *Final exam*: still happening; online exam during **usual time slot** (Thu, Mar 19 2:30-4:20)
    - Please ensure you have access to a quiet location with good internet connectivity at that time!
  - *Lectures*: today's lecture finishes the topics that'll be on the final exam
    - We'll post pre-recorded videos for next week's 3 lectures
    - Fortunately, topics were review + enrichment. Do your best … or just use the time to finish HW8

- I'm insanely behind on email, but contact us anyway with questions, requests, etc
  - We'll announce details related to format, tools, etc on Piazza
  - You'll probably need to install Zoom (video conferencing)

Feedback from Reading Quiz

- How to handle non-numeric keys like { ♣ , ♠ , ♥ , ♦ }?
  - Map keys to numeric values; exact implementation can vary
  - Eg: ♣ → 0, ♠ → 1, ♥ → 2, ♦ → 3

- We'll answer these in lecture today:
  - What's the runtime of counting sort? Is it $\Theta(N^2)$ or $\Theta(2N)$?
  - What's a radix?
  - How does radix sort maintain stability?
  - Can we use radix sort techniques for comparison sorts?

Lecture Outline

- **Generalizing CountingSort**
- RadixSort
  - LSD RadixSort
  - MSD RadixSort

Comparison-Based Sorting

- **Definition**: A type of sorting algorithm that determines an element's ordering using *comparison operations*
  - More simply: sorting using only compareTo() type operations

- We determined the best we can do with comparison-based sorting is $\Theta(N \log N)$ time complexity

- Can we do better? What if we don't compare at all?

Radix: A Definition

- **Radix**: the number of “characters” in the “alphabet”
  - More formally: the number of elements in the domain

Name	Radix	Characters
Binary	2	0,1
Decimal	10	0,1,2,3,4,5,6,7,8,9
Lowercase Latin Alphabet	26	a,b,c,d,e,f,g,h,I,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z
ASCII	128	http://www.asciitable.com/
Unicode	>137,000	https://en.wikipedia.org/wiki/List_of_Unicode_characters

Reading Review: Generalizing CountingSort

- We want Counting Sort to work for non-unique and/or non-consecutive keys!
  - Count the occurrences for each key value
  - Compute each key's starting index using the counts array
  - For each [item, key] in the input do:
    - Get the destination index by checking the index array for key
    - Copy item into the result using this destination index
    - Increment the index for key
  - Copy items back to initial array (if needed)

- Demo: https://docs.google.com/presentation/d/1FTTxlds-7EqbJ6Md40svCV9zjDL-XxGI00pXp4gXsr8/edit

Poll Everywhere

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- What is the runtime for CountingSort on an input of N items and an alphabet of size (“radix”) R? Treat R as a variable, not a constant.

A. $\Theta(N)$
B. $\Theta(R)$
C. $\Theta(N + R)$
D. $\Theta(NR)$
E. I’m not sure …

CountingSort: Performance Analysis

Time Complexity:

- Θ(N)
- Θ(R)
- Θ(N)
- Overall: Θ(N + R)

```
CountingSort(a):
  map<char, int> counts
  foreach key in a:
    counts[key]++
  map<char, int> indices;
  foreach key in counts:
    indices[key] =
      indices[key - 1] +
      indices[key]
  foreach (key, item) in a:
    output[indices[key]] =
      item;
    indices[key]++
```

CountingSort: Performance Analysis

- CountingSort is stable because it processes then input in order
  - No long-distance swaps like SelectionSort or Hoare Partitioning
- **Runtime** and **memory use** is $\Theta(N + R)$!
  - N = # of items, R = radix of alphabet
- We "beat" comparison sorts by avoiding comparisons!
  - Aaaaccccccttttually … empirical/performance testing is still needed to compare against QuickSort on real-world inputs

Poll Everywhere

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- You have an array of 100 elements, consisting of a city's name and its population. If you want to sort them by population, which algorithm's worst-case runtime *as measured in seconds* (ie, not asymptotically) is lower / faster?

A. CountingSort

B. QuickSort

C. I'm not sure …

CountingSort: Performance Analysis

- **Runtime** and **memory use** is $\Theta(N + R)$!
  - N = # of items, R = radix of alphabet

- But did we *actually* beat comparison sorts?
  - If N >= R: performance is reasonable
  - If N >> R: R is negligible, performance is great!
  - What if N << R?
    - In other words: When is our alphabet large?
    - Integers, strings, …

Sorting Cities by Population

- CountingSort builds an array of size ~30,000,000 -- the largest city's population -- to sort the input

- ... which is a very large and very sparse array
  - Most indices are unused because we are sorting only 100 cities!

Lecture Outline

- Generalizing CountingSort

- RadixSort
  - **LSD RadixSort**
  - MSD RadixSort

RadixSort's Raison D'être

- We want to be able to sort keys that don't belong to a finite alphabet, such as strings
  - Strings don't belong to a finite alphabet, but they **consist of characters** from a finite alphabet!
  - Numbers do too

- RadixSort's idea is similar to tries':
  - Subdivide the key; it's not an atomic indivisible "whole"?
  - Sort each chunk/character/digit independently using CountingSort

- How should we "chunk"? In what order should we process the chunks?

Least Significant Digit (LSD) RadixSort

- LSD RadixSort: Sort each chunk independently, from rightmost to leftmost
- Example:

Alphabet: {1, 2, 3}

Key	Name
22	Stitch
12	Gantu
31	Nani
23	Lilo
11	David

→

Key	Name
3**1**	Nani
1**1**	David
2**2**	Stitch
1**2**	Gantu
2**3**	Lilo

→

Key	Name
11	David
12	Gantu
22	Stitch
23	Lilo
31	Nani

LSD RadixSort: Correctness

- Does LSD RadixSort create correct results?
  - What property of CountingSort enables that?
  - Can you give an example of what could go wrong?

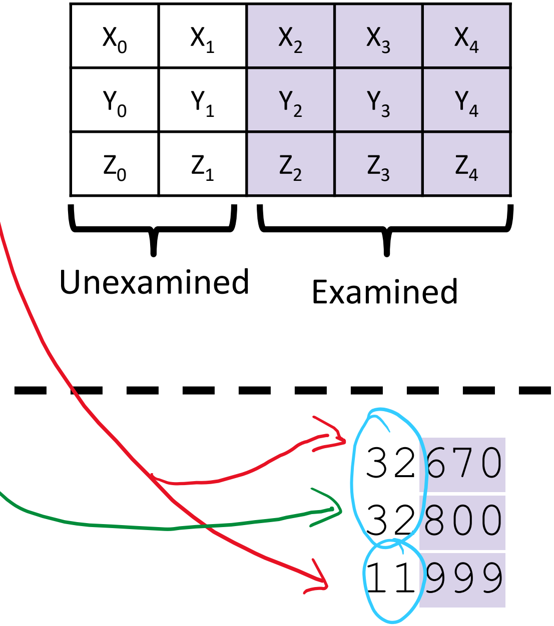
Key	Name
22	Stitch
12	Gantu
31	Nani
23	Lilo
11	David

Key	Name
3**1**	Nani
1**1**	David
2**2**	Stitch
1**2**	Gantu
2**3**	Lilo

Key	Name
11	David
12	Gantu
23	Lilo
22	Stitch
31	Nani

LSD RadixSort Correctness: More Formally

- If the **unexamined chunks** are **different**, the examined chunks don't matter!
  - A later pass will sort correctly on more significant chunks
- If the **unexamined chunks** are **identical**, the keys are already properly ordered
  - Since the sort is stable, they will remain so



LSD RadixSort: Non-equal Key Lengths 🤔

- If keys are of unequal length, treat empty spaces as less-than all other chunks in the alphabet/domain

- Example:

Alphabet: {1, 2, 3}

Key	Value
3	is
31	fun!
23	duper
12	super
1	sorting

Key	Name
3**1**	fun!
·**1**	sorting
1**2**	super
·**3**	is
2**3**	duper

Key	Name
·1	sorting
·3	is
12	super
23	duper
31	fun!

LSD RadixSort: Runtime

- N = # items, R = radix, L = # chunks in longest item
  - We have to run CountingSort for each chunk
  - CountingSort has runtime on the order of $\Theta(N + R)$
  - Therefore, LSD RadixSort's runtime: **$\Theta(LN + LR)$**

LSD RadixSort: Summary

- Use CountingSort on each chunk, from right to left
  - Now we can sort non-alphabetic keys that consist of alphabetic keys!

- Performance (N = # items, R = radix, L = # chunks in longest item):
  - Runtime: **Θ(LN + LR)**
  - Memory use: **Θ(N + R)**
    - Output array: N
    - Need L counts array (R) and L starting indices array (R), but can reuse them between chunks

- If R is small (CountingSort's restriction) and L is small (an LSD RadixSort restriction), the runtime isn't shabby!

If only the runtime didn't depend on the longest key … 🤔

Most Significant Digit (MSD) RadixSort

- By definition, LSD RadixSort examines the **least significant** chunk first!
  - ie, may do more computation than necessary

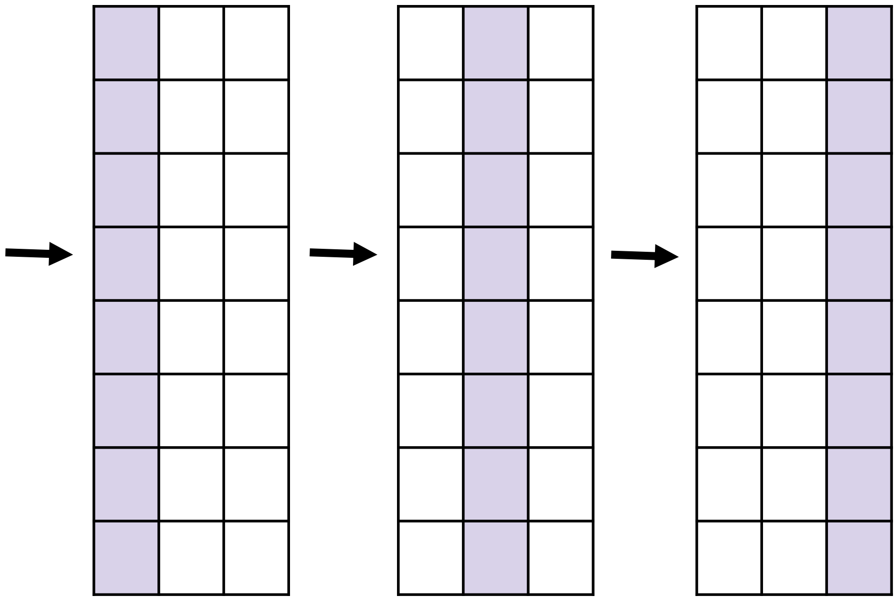
- MSD RadixSort Idea: similar to LSD, but leftmost to rightmost
  - Handles keys that are much longer than the rest, eg:

```
          349499234
            4589245
1329543516382737 62
           62302213
            2934592
          432035235
```

MSD RadixSort: Example

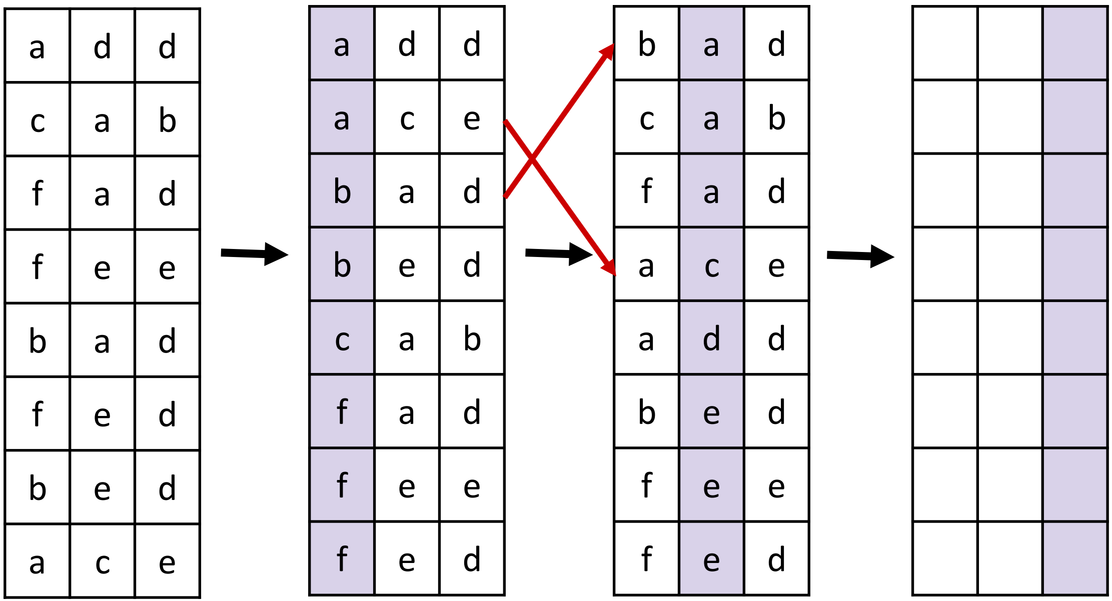
- Suppose we sort each chunk left to right. Will we arrive at the correct result? Why or why not?

a	d	d
c	a	b
f	a	d
f	e	e
b	a	d
f	e	d
b	e	d
a	c	e



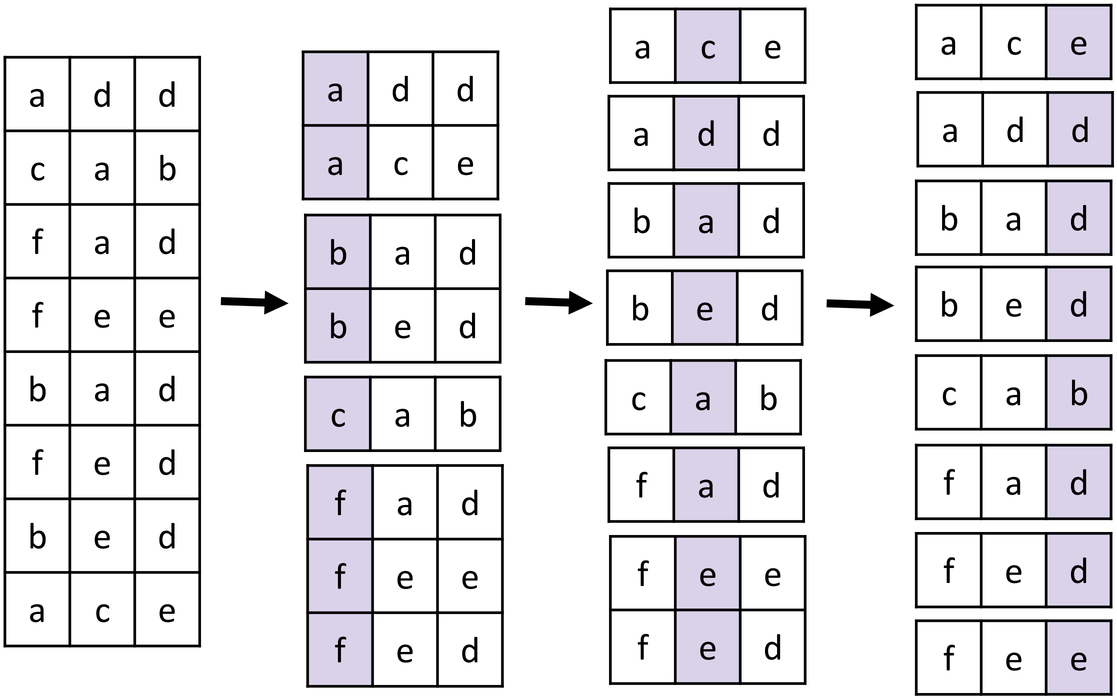
MSD RadixSort: Example

- No! Items that were previously ordered by a more-significant chunk may get swapped!



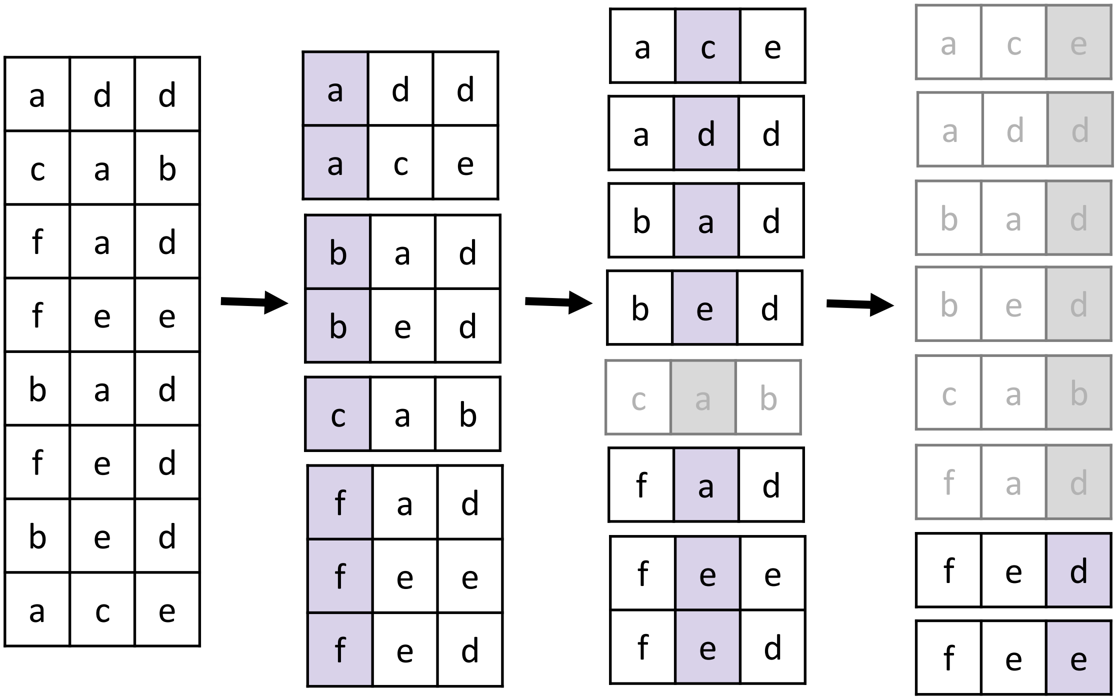
MSD RadixSort: Example

- Solution: sort each subproblem separately, rejoin at the end



MSD RadixSort: Example

- Optimization: don't subdivide or sort already-sorted singletons



MSD RadixSort: Runtime

- Best-case runtime of MSD RadixSort, expressed in N, R, L?
- What type of input leads to this best-case?
  - One CountingSort pass, looking only at the first chunk: **Θ(N + R)**
  - Every input has a unique most-significant chunk

- Worst-case runtime of MSD RadixSort, expressed in N, R, L?
  - L CountingSort passes to look at every chunk (ie, degenerates to LSD RadixSort): **Θ(LN + LR)**
  - Every key is the same or only differs in the least-significant chunk

MSD RadixSort: Memory

- Memory usage: **Θ(N + R)**
  - Output array: N
  - Each chunk requires <=R CountingSorts for each subproblem, and each CountingSort requires N+R memory. However, we can reuse that memory between each CountingSort

MSD RadixSort: Analysis

- Runtime:
  - Best case: **Θ(N + R)**
  - Worst case: **Θ(LN + LR)**
- Memory usage: **Θ(N + R)**

- In practice, long strings are rarely random; they may contain structure
  - Eg, HTML has tags: <html>, <p>,

- Structured strings may benefit from specialized sorting algorithms or, minimally, specialized "chunkers"
  - Eg, a HTML-tag-aware chunking

Random (sublinear)	Non-random with duplicates (nearly linear)	Worst case (linear)
1EIO402	are	1DNB377
1HYL490	by	1DNB377
1ROZ572	sea	1DNB377
2HXE734	seashells	1DNB377
2IYE230	seashells	1DNB377
2XOR846	sells	1DNB377
3CDB573	sells	1DNB377
3CVP720	she	1DNB377
3IGJ319	she	1DNB377
3KNA382	shells	1DNB377
3TAV879	shore	1DNB377
4CQP781	surely	1DNB377
4QGI284	the	1DNB377
4YHV229	the	1DNB377

From Algorithms, 4th edition by Sedgewick and Wayne

tl;dr

	Time Complexity	Space Complexity
CountingSort	Θ(N+R)	Θ(N+R)
LSD RadixSort	Θ(LN + LR)	Θ(N + R)
MSD RadixSort	Best: Θ(N + R) Worst: Θ(LN + LR)	Θ(N + LR)

And, finally …

- Thank you for your understanding and patience re: COVID-19
- Thank you for being a great class!
- Good luck on HW8 and the final. Stay in touch!