Topological Sort; Reductions

CSE 373 Winter 2020

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Poll Everywhere

pollev.com/uwcse373

- Approximately how long did Homework 6 (A* Search) take?

A. 0-3 hours
B. 4-7 hours
C. 8-11 hours
D. 12-15 hours
E. 16-19 hours
F. 19+ hours
G. I don't want to say …

Announcements

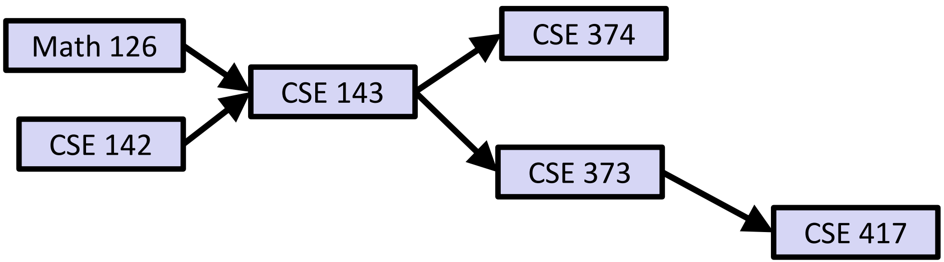
- We are converting Friday's Workshop (12:30-1:20 @ CSE 203) into DIT
- My DIT on Friday is in CSE 303 (not my office)
- Extra time for today's reading quiz due to a broken link

Lecture Outline

- **Topological Sorting**
- General Technique: Reductions
  - Reducing Shortest Path in a negative DAG to TopoSort
  - Reducing Seam Carving to A* Search

Sorting Dependencies

- Given a set of courses and their prerequisites, find an order to take the courses in, assuming you can only take one course per quarter



Topological Sort (aka Topological Ordering)

- Example: dependency graphs
  - An edge (u, v) means u must happen before v
  - A topological sort of a dependency graph gives an ordering that **respects dependencies**

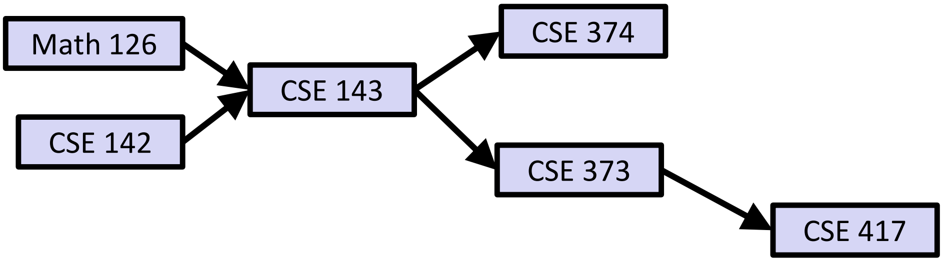
- Applications:
  - Graduating
  - Compiling multiple Java files
  - Multi-job Workflows

Topological Sort

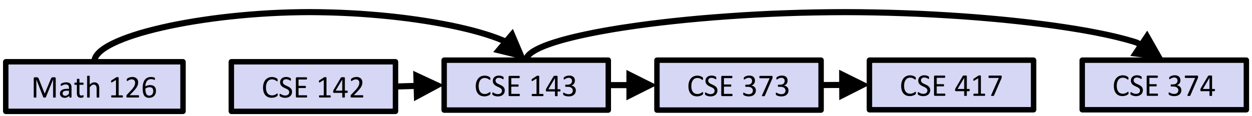
- Given a directed graph G
- Find an ordering of the vertices so all edges go from "left to right"

Sorting Dependencies

- Our course prerequisite chart:

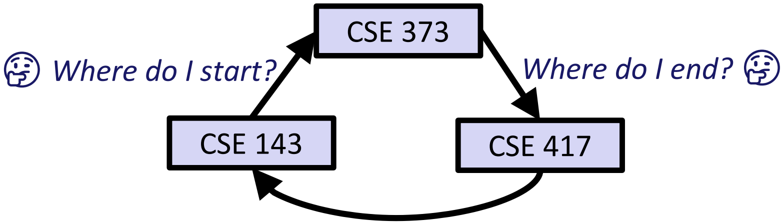


- Possible ordering:



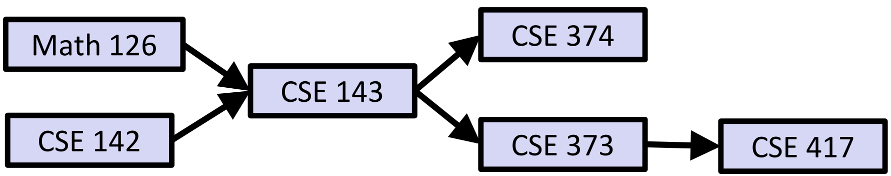
Can We Always TopoSort a Graph?

- Can you topologically sort this graph?



No

- What's the difference between this graph and our first graph?



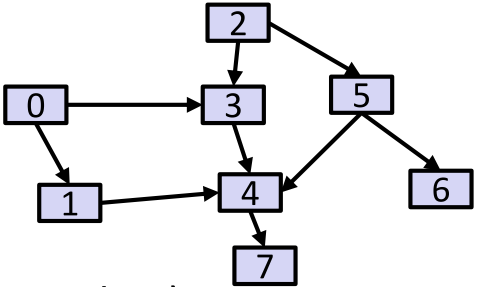
- A graph has a topological ordering iff it is a DAG
  - But a DAG can have multiple orderings

Directed Acyclic Graph

- A directed graph without any cycles
- Edges may or may not be weighted

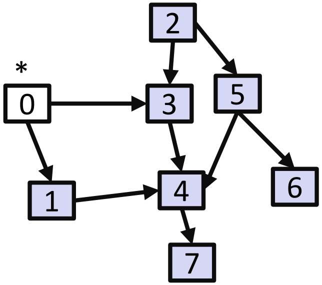
Sorting a DAG

- Does this graph have a topological ordering? If so find one

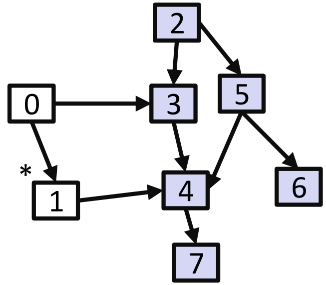


- Algorithm (from reading):
  - Perform DFS post-order traversal(s) from every vertex with in-degree 0, *remembering marked vertices between traversals*

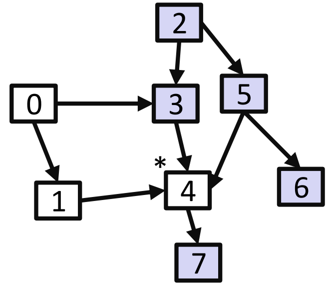
Sorting a DAG



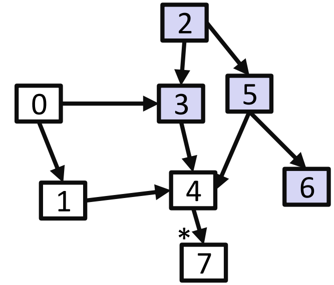
Post-order: []



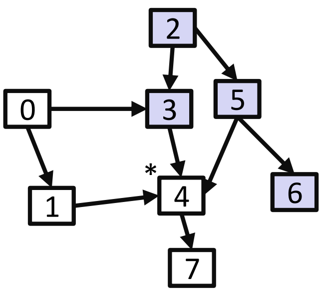
Post-order: []



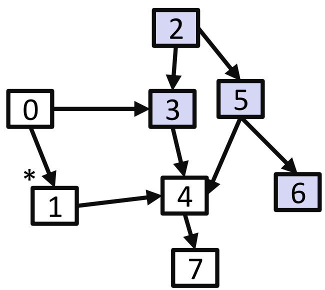
Post-order: []



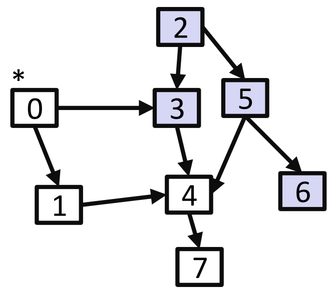
Post-order: [7]



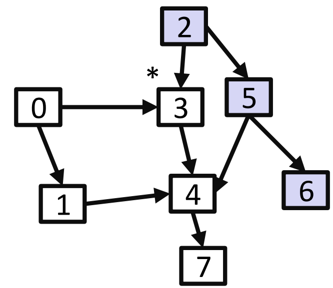
Post-order: [7, 4]



Post-order: [7, 4, 1]



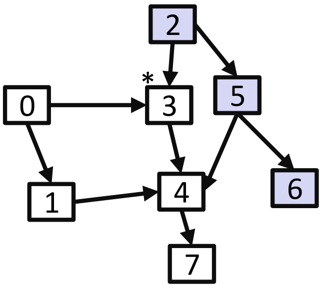
Post-order: [7, 4, 1]



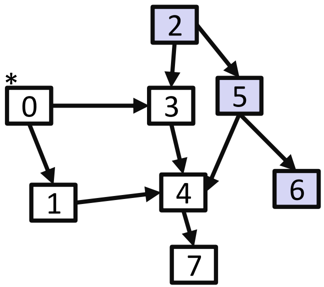
Post-order: [7, 4, 1, 3]

Sorting a DAG

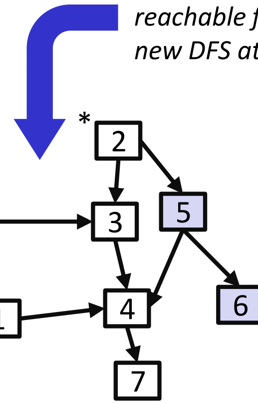
At this step, we've visited everything reachable from node 0 and are starting a new DFS at a node with an in-degree of 0



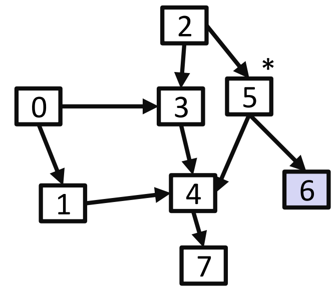
Post-order: [7, 4, 1, 3]



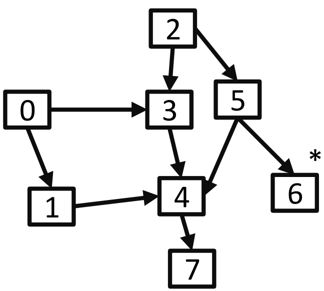
Post-order: [7, 4, 1, 3, 0]



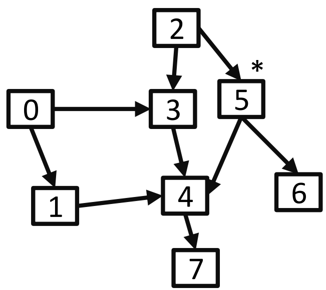
Post-order: [7, 4, 1, 3, 0]



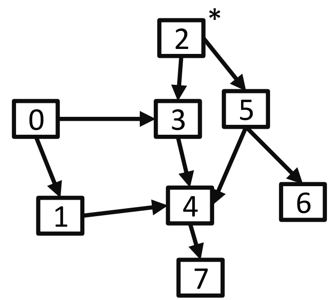
Post-order: [7, 4, 1, 3, 0]



Post-order: [7, 4, 1, 3, 0, 6]



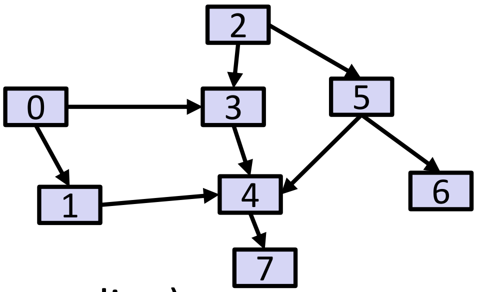
Post-order: [7, 4, 1, 3, 0, 6, 5]



Post-order: [7, 4, 1, 3, 0, 6, 5, 2]

How Did We Find a Topological Sort?

- Does this graph have a topological ordering? If so find one

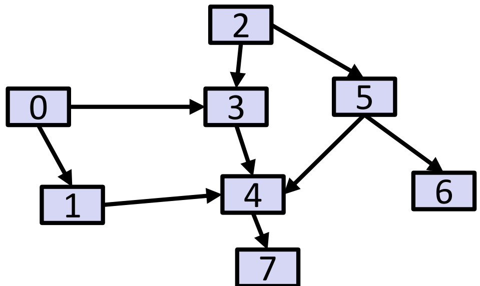


- Algorithm (from reading):
  - Perform DFS post-order traversal(s) from every vertex with in-degree 0, *remembering marked vertices between traversals*

- Topological Sort is given by reverse of the DFS post-order list
  - Post-order list: [7, 4, 1, 3, 0, 6, 5, 2]
  - Reversed: [2, 5, 6, 0, 3, 1, 4, 7]

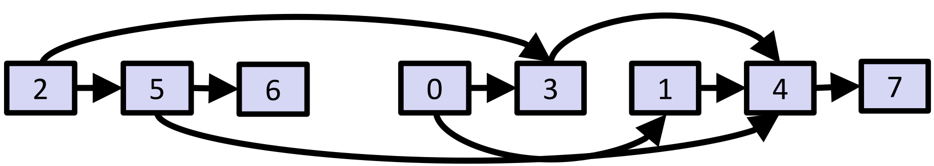
How Did We Find a Topological Sort?

- Does this graph have a topological ordering? If so find one



- Topological Sort is given by reverse of the DFS post-order list
  - Post-order list: [7, 4, 1, 3, 0, 6, 5, 2]
  - Reversed: [2, 5, 6, 0, 3, 1, 4, 7]

- Our ”topological sort”:



Lecture Outline

- Topological Sorting
- **General Technique: Reductions**
  - Reducing Shortest Path in a negative DAG to TopoSort
  - Reducing Seam Carving to A* Search

Decomposition vs Reduction

- **Decomposition**: Taking a complex task and breaking it into smaller parts
  - This is the heart of computer science and programming
  - We might use "canned solutions" for solving these smaller parts

- **Reduction**: Using an algorithm for Problem Q to solve a different Problem P
  - We do not modify the algorithm for Problem Q; we only modify the inputs/outputs
  - "Problem P reduces to Q"
  - Hopefully problem Q is easier than problem P!

Decomposition

- **Decomposition**: Taking a complex task and breaking it into smaller parts

- Examples:
  - Decomposing HuskyMaps to:
    - Autocomplete (search bar prefix queries)
    - Heap (priority queue for route-finding)
    - A* search (route-finding)
    - k-d tree (nearest neighbors to find start and goal vertices)
    - Rasterer (map tile display)
  - Recognizing that “finding the next trie node” meant using a Map ADT

Reduction

- **Reduction**: Using an algorithm for Problem Q to solve a different Problem P

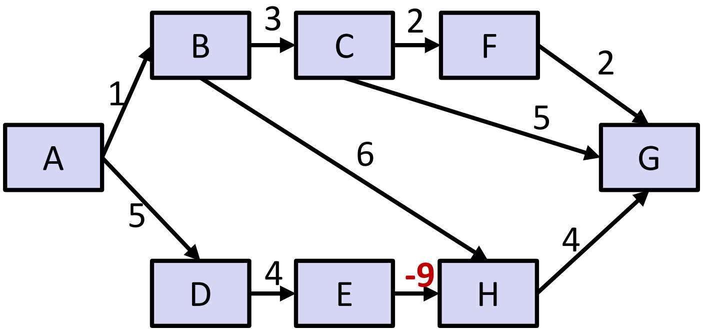
- Examples and Non-examples:
  - A* does not reduce to Dijkstra's
    - We *modified* Dijkstra's by adding a heuristic
  - "Climbing a hill" reduces to:
    - "Riding a ski lift"
    - "Being shot out of a cannon"
    - "Riding a bike"
  - Shortest path in a negative DAG reduces to Topological Sort
  - … does Topological Sort reduce to DFS? Yes!

Lecture Outline

- Topological Sorting
- General Technique: Reductions
  - **Reducing Shortest Path in a negative DAG to TopoSort**
  - Reducing Seam Carving to A* Search

Shortest Path in a Negative DAG

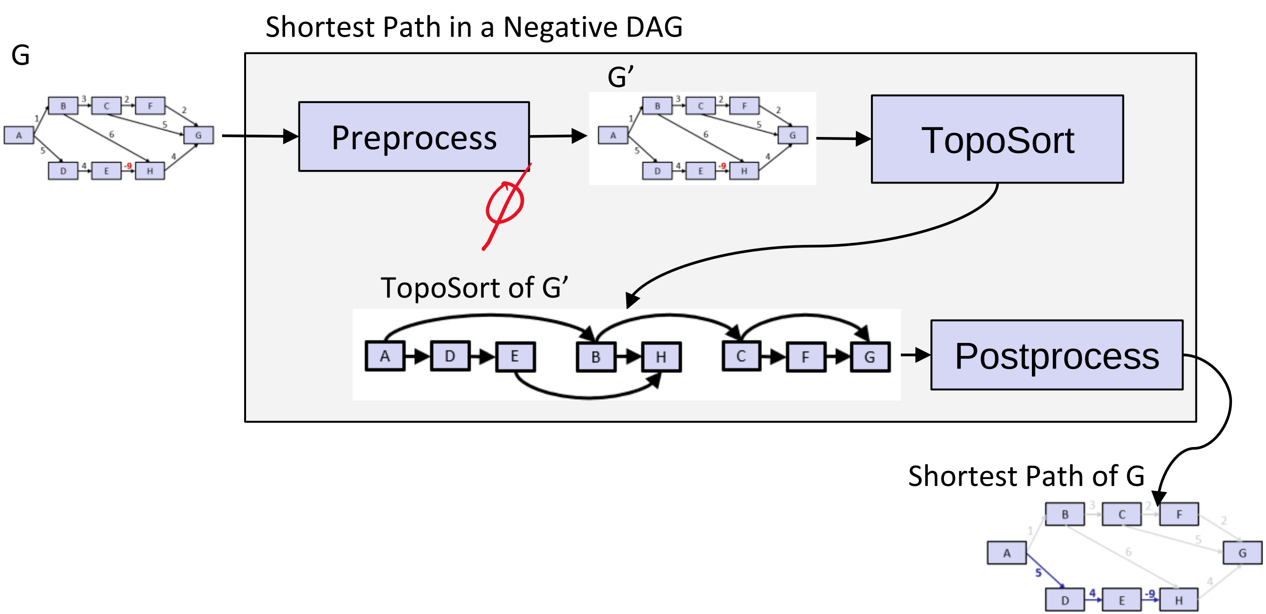
- Recall this graph from our reading:



- We can reduce this problem to TopoSort!

Reductions in 3 Easy Steps!

1. Transform the input so that it can be solved by the standard algorithm
2. Run the standard algorithm as-is on the transformed input
3. Transform the output of the algorithm to solve the original problem



Lecture Outline

- Topological Sorting

- General Technique: Reductions
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  - **Reducing Seam Carving to A* Search**

Content-Aware Image Resizing

Seam carving: A distortion-free technique for resizing an image by removing "unimportant seams"

Original Photo

Horizontally-Scaled
(castle and person are distorted)

Seam-Carved
(castle and person are undistorted; "unimportant" sky removed instead)

Seam carving for content-aware image resizing (Avidan, Shamir/ACM); Broadway Tower (Newton2, Yummifruitbat/Wikimedia)

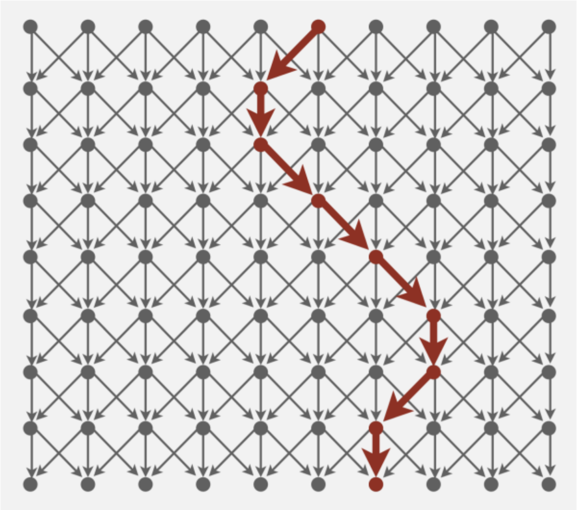
Demo: https://www.youtube.com/watch?v=vIFCV2spKtg

Seam Carving Reduces to A* Search

1. *Transform the input so that it can be solved by the standard algorithm*
   - Formulate the image as a graph
     - **Vertices**: pixel in the image
     - **Edges**: connects a pixel to its 3 downward neighbors
     - **Edge Weights**: the "energy" (visual difference) between adjacent pixels

2. *Run the standard algorithm as-is on the transformed input*
   - Run A* Search to find the shortest path (sum of weights) from top row to bottom row

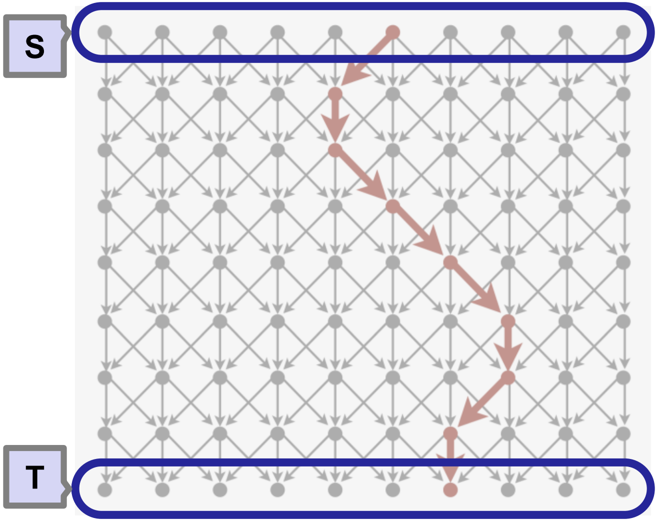
3. *Transform the output of the algorithm to solve the original problem*
   - Interpret the path as a removable "seam" of unimportant pixels



Shortest Paths (Robert Sedgewick, Kevin Wayne/Princeton)

Formal Problem Statement

- Using AStarSolver, find the seam from any top vertex to any bottom vertex
- Given a digraph with positive edge weights and two distinguished subsets of vertices S and T, find a shortest path from any vertex in S to any vertex in T
- Your algorithm should run in time proportional to E log V in the worst case

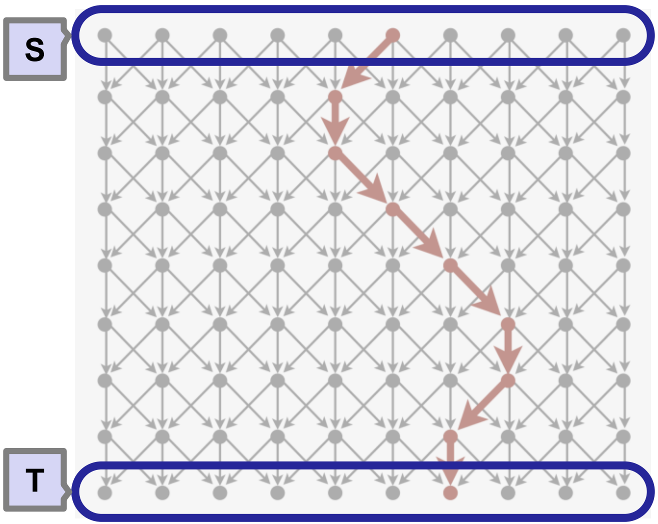


Shortest Paths (Robert Sedgewick, Kevin Wayne/Princeton)

An Incomplete Reduction

- A* Search starts with a single vertex S and ends with a single vertex T
  - This problem specifies *sets of vertices* for the start and end

- **Your turn**: brainstorm how to transform this graph into something A* Search knows how to operate on



Shortest Paths (Robert Sedgewick, Kevin Wayne/Princeton)

tl;dr

- TopoSort is widely applicable; many problems can be *decomposed* into a dependency graph

- But problems can also be *reduced* to TopoSort!

- Reductions are a powerful tool in theoretical computer science
  - You won't have to do a reduction on the final, but you should be aware of the general principles