A* Search and Design Decisions
CSE 373 Winter 2020

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Announcements

- **Midterm is *this Friday***
  - If your student number ends in an odd number, go to KNE 210
  - If your student ends in an even number, go to KNE 220
  - Workshops and review session will be focused on your midterm questions – bring your questions and practice midterms!
  - Review session Thursday night: 4:30-6:30 @ ARC 147

- **HW6 is released**
  - Yes, HW5 and HW6 are both currently released
  - Please prefix your Piazza posts with “HW5: ...” or “HW6: ...”

- **20sp instructors want current students to TA next quarter!**
  - Check Piazza or course webpage for more details
Feedback from Reading Quiz

❖ If we add diagonals, is it still the Manhattan distance? What is the Euclidean distance?

❖ I still need a walkthrough of Dijkstra’s

❖ Does Dijkstra’s still work if the grid had different weights?
Lecture Outline

❖ Dijkstra’s Algorithm, Reviewed

❖ A* Search
  ▪ Introducing A*
  ▪ A* Heuristics

❖ Design Decisions
Dijkstra’s Algorithm

Demo:

https://docs.google.com/presentation/d/1_bw2z1ggUkquPdhl7gwdVBoTaoJmaZdpkV6MoAgxlJc/pub?start=false&loop=false&delayms=3000

dijkstras(Node s, Graph g) {
    PriorityQueue unvisited;
    unvisited.addAll(g.allNodes(), ∞)
    unvisited.changePriority(s, 0);
    Map<Node, Integer> distances;
    Map<Node, Node> previousNode;

    while (!unvisited.isEmpty()) {
        Node n = unvisited.removeMin();
        for (Node i : n.neighbors) {
            if (distances[i] < distances[n] + g.edgeWeight(n, i)) {
                continue;
            } else {
                distances[i] = distances[n] + g.edgeWeight(n, i);
                unvisited.changePriority(i, distances[i]);
                previousNode[i] = n;
            }
        }
    }
}
Which of the following statements are true?

- Dijkstra’s Algorithm becomes Breadth-first Search if all the edges have the same weight
- Dijkstra’s can find the shortest path from the source to every node in the graph
- At each step of the algorithm, Dijkstra’s only considers the path length from the source

A. True / True / True
B. True / True / False
C. False / True / True
D. False / True / False
E. False / False / False
F. I’m not sure ...
Dijkstra’s Algorithm’s Flaws

- Demo: [https://qiao.github.io/PathFinding.js/visual/](https://qiao.github.io/PathFinding.js/visual/)

- If we want a *single shortest path* (instead of *all shortest paths*), Dijkstra’s and BFS does unnecessary work
  - The answer is still correct, but we did unnecessary computation
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Single-Pair Shortest Path Problem
Single-Pair Shortest Path: Dijkstra’s Algorithm
How should we hint to Dijkstra’s that we want it to concentrate its search southward?

BFS -> Dijkstra’s switched the Queue for a Priority Queue

- Can we change our idea of a “priority”?
Introducing A* Search

- **Idea:**
  - Visit vertices in order of \( d(\text{Ravenna Park}, v) + h(v, \text{Japanese Garden}) \), where \( h(v, \text{Japanese Garden}) \) is an *estimate* of the distance from \( v \) to our goal.
  - In other words, prefer a location \( v \) if:
    - We already know the fastest way to reach \( v \)
    - **AND** we suspect that \( v \) might be the fastest way to get to our goal.

- Dijkstra’s only considers \( d(\text{Ravenna Park}, v) \)

- Demo: [http://qiao.github.io/PathFinding.js/visual/](http://qiao.github.io/PathFinding.js/visual/)
A* Demo

- Source = 0; Destination = 6
- Use the following estimates:

<table>
<thead>
<tr>
<th>Vertex ID</th>
<th>$h(v, \text{dest})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>$\infty$</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

Demo:

https://docs.google.com/presentation/d/177bRUTdCa60fjExdr9eO04NHm0MRfPtCzvEup1iMccM/edit
Dijkstra’s Algorithm vs A* Search

```java
public class Dijkstra's Algorithm {

    public static PriorityQueue unvisited;

    public static void dijkstras(Node s, Graph g) {
        PriorityQueue unvisited;
        unvisited.addAll(g.allNodes(), ∞);
        unvisited.changePriority(s, 0);
        Map<Node, Integer> distances;
        Map<Node, Node> previousNode;
        while (!unvisited.isEmpty()) {
            Node h = unvisited.removeMin();
            for (Node i : h.neighbors) {
                if (distances[i] < distances[h] + g.edgeWeight(h, i)) {
                    continue;
                } else {
                    distances[i] = distances[h] + g.edgeWeight(h, i);
                    unvisited.changePriority(i, distances[i]);
                    previousNode[i] = h;
                }
            }
        }
    }
}
```

```java
public class A* Search {

    public static PriorityQueue unvisited;

    public static void astar(Node s, Node t, Graph g) {
        PriorityQueue unvisited;
        unvisited.addAll(g.allNodes(), ∞);
        unvisited.changePriority(s, 0);
        Map<Node, Integer> distances;
        Map<Node, Node> previousNode;
        while (!unvisited.isEmpty()) {
            Node h = unvisited.removeMin();
            for (Node i : h.neighbors) {
                if (distances[i] < distances[h] + g.edgeWeight(h, i)) {
                    continue;
                } else {
                    distances[i] = distances[h] + g.edgeWeight(h, i);
                    unvisited.changePriority(i, distances[i] + h(i, t));
                    previousNode[i] = h;
                }
            }
        }
    }
}
```
Lecture Outline

- Dijkstra’s Algorithm, Reviewed

- A* Search
  - Introducing A*
  - A* Heuristics

- Design Decisions
Heuristics

- We call this “estimate function” a **heuristic**
  - Definition: *a solution or choice or judgement that is “good enough” for a purpose, but which could be optimized*
  - In other words: **it doesn’t have to be perfect**

- What is a good heuristic for this map?
Euclidean and Manhattan Distances

- Assume the entire map can be represented as a grid

- Manhattan distance: $\Delta x + \Delta y$

- Euclidean distance: $\sqrt{\Delta x^2 + \Delta y^2}$
Will A* Search return the correct shortest path if $h(v, \text{dest}) = 10$ for every $v$ in the graph?

A. Always  
B. Sometimes  
C. Never  
D. Not enough information  
E. I’m not sure ...
But What If We Have a Lousy Heuristic?

- \( h(v, \text{dest}) = 0 \)
  - That’s just Dijkstra’s

- \( h(v, \text{dest}) = 1,000,000 \)
  - Still just Dijkstra’s

- \( h(\text{Montlake Bridge, dest}) = 1,000,000 \)
  - Inconsistent results!
Good Heuristics are Hard!

- You’ll frequently hear that “A* Search is hard”
  - As we’ve seen, A* Search is an incremental update to Dijkstra’s
  - What’s hard with A* Search is designing a good heuristic

- In this class, we’ll give you a (good) heuristic for HuskyMaps
  - Hint: Manhattan and Euclidean distances are both good heuristics

- If you take an AI class, you’ll learn all about designing heuristics
  - Sneak preview: good heuristics have the following characteristics:
    - $h(v, \text{dest}) \leq \text{true distance from } v \text{ to destination} \quad (\text{“admissible”})$
    - $h(v, \text{dest}) \leq \text{dist}(v, w) + h(w, \text{dest}) \quad (\text{“consistent”})$
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Two Key Skills

❖ In Software Engineering, two important skills to have are:
  ▪ Identifying the requirements (ie, selecting an ADT)
  ▪ Making tradeoffs (ie, selecting the data structure for that ADT)

❖ So let’s review the ADTs’ functionality and the performance characteristics of each data structure
List Functionality

List ADT. A collection storing an ordered sequence of elements.

- Each element is accessible by a zero-based index.
- A list has a size defined as the number of elements in the list.
- Elements can be added to the front, back, or any index in the list.
- Optionally, elements can be removed from the front, back, or any index in the list.

Possible Implementations:
- ArrayList
- LinkedList
# List Performance Tradeoffs

<table>
<thead>
<tr>
<th></th>
<th>ArrayList</th>
<th>LinkedList</th>
</tr>
</thead>
<tbody>
<tr>
<td>addFront</td>
<td>linear</td>
<td>constant</td>
</tr>
<tr>
<td>removeFront</td>
<td>linear</td>
<td>constant</td>
</tr>
<tr>
<td>addBack</td>
<td>constant*</td>
<td>linear</td>
</tr>
<tr>
<td>removeBack</td>
<td>constant</td>
<td>linear</td>
</tr>
<tr>
<td>get(idx)</td>
<td>const</td>
<td>linear</td>
</tr>
<tr>
<td>put(idx)</td>
<td>linear</td>
<td>linear</td>
</tr>
</tbody>
</table>

* constant for most invocations
Stack and Queue Functionality

**Stack ADT.** A collection storing an ordered sequence of elements.
- A stack has a size defined as the number of elements in the stack.
- Elements can only be added and removed from the top (“LIFO”)

- Possible Implementations:
  - ArrayStack, LinkedStack

**Queue ADT.** A collection storing an ordered sequence of elements.
- A queue has a size defined as the number of elements in the queue.
- Elements can only be added to one end and removed from the other (“FIFO”)

- Possible Implementations:
  - ArrayQueue, LinkedQueue
## Stack and Queue Performance Tradeoffs

### Stack (LIFO):

<table>
<thead>
<tr>
<th></th>
<th>ArrayStack</th>
<th>LinkedStack</th>
</tr>
</thead>
<tbody>
<tr>
<td>push</td>
<td>constant*</td>
<td>constant</td>
</tr>
<tr>
<td>pop</td>
<td>constant</td>
<td>constant</td>
</tr>
</tbody>
</table>

* constant for most invocations

### Queue (FIFO):

<table>
<thead>
<tr>
<th></th>
<th>Array Queue (v2)</th>
<th>LinkedQueue (v2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>enqueue</td>
<td>constant*</td>
<td>constant</td>
</tr>
<tr>
<td>dequeue</td>
<td>constant</td>
<td>constant</td>
</tr>
</tbody>
</table>

* constant for most invocations
Deque Functionality

**Deque ADT.** A collection storing an ordered sequence of elements.

- Each element is accessible by a zero-based index.
- A deque has a size defined as the number of elements in the deque.
- Elements can be added to the front or back.
- Optionally, elements can be removed from the front or back.

- Possible Implementations:
  - ArrayDeque, LinkedDeque
# Deque Performance Tradeoffs

<table>
<thead>
<tr>
<th></th>
<th>CircularArrayDeque</th>
<th>LinkedDeque</th>
</tr>
</thead>
<tbody>
<tr>
<td>addFirst</td>
<td>constant*</td>
<td>constant</td>
</tr>
<tr>
<td>removeFirst</td>
<td>constant</td>
<td>constant</td>
</tr>
<tr>
<td>addLast</td>
<td>constant*</td>
<td>constant</td>
</tr>
<tr>
<td>removeLast</td>
<td>constant</td>
<td>constant</td>
</tr>
</tbody>
</table>

* constant for most invocations
Set and Map Functionality

**Set ADT.** A collection of values.
- A set has a size defined as the number of elements in the set.
- You can add and remove values.
- Each value is accessible via a “get” or “contains” operation.

**Map ADT.** A collection of keys, each associated with a value.
- A map has a size defined as the number of elements in the map.
- You can add and remove (key, value) pairs.
- Each value is accessible by its key via a “get” or “contains” operation.

- Possible Implementations:
  - Unbalanced BST
  - LLRB Tree
  - B-Tree (eg, 2-3 Tree)
  - Hash Tables
## Set and Map Performance Tradeoffs

<table>
<thead>
<tr>
<th></th>
<th>Find</th>
<th>Add</th>
<th>Remove</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resizing Separate</td>
<td>$Q \in \Theta(N)$</td>
<td>$Q \in \Theta(N)$</td>
<td>$Q \in \Theta(N)$</td>
</tr>
<tr>
<td>Chaining Hash Table</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(worst case)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resizing Separate</td>
<td>$\Theta(1)$</td>
<td>$\Theta(1)^*$</td>
<td>$\Theta(1)^*$</td>
</tr>
<tr>
<td>Chaining Hash Table</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(best/average cases) $^+$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LLRB Tree</td>
<td>$h \in \Theta(\log N)$</td>
<td>$h \in \Theta(\log N)$</td>
<td>$h \in \Theta(\log N)$</td>
</tr>
<tr>
<td>B-Tree</td>
<td>$h \in \Theta(\log N)$</td>
<td>$h \in \Theta(\log N)$</td>
<td>$h \in \Theta(\log N)$</td>
</tr>
<tr>
<td>BST</td>
<td>$h \in \Theta(N)$</td>
<td>$h \in \Theta(N)$</td>
<td>$h \in \Theta(N)$</td>
</tr>
<tr>
<td>LinkedList</td>
<td>$\Theta(N)$</td>
<td>$\Theta(N)$</td>
<td>$\Theta(N)$</td>
</tr>
</tbody>
</table>
Priority Queue Functionality

Priority Queue ADT. A collection of values.
- A PQ has a size defined as the number of elements in the set.
- You can add values.
- You cannot access or remove arbitrary values, only the max value.

Possible Implementations:
- Balanced BST with “max” pointer
- Binary Heap
- (and a ton of others we didn’t discuss)
## Priority Queue Performance Tradeoffs

<table>
<thead>
<tr>
<th>Operation</th>
<th>Balanced BST (worst case)</th>
<th>Binary Heap (worst case)</th>
</tr>
</thead>
<tbody>
<tr>
<td>add</td>
<td>$O(\log N)$</td>
<td>$O(\log N)^{**}$</td>
</tr>
<tr>
<td>max</td>
<td>$O(1)^{*}$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>removeMax</td>
<td>$O(\log N)$</td>
<td>$O(\log N)$</td>
</tr>
</tbody>
</table>

* If we keep a pointer to the largest element in the BST

** Average case is constant
Graph Functionality

**Graph ADT.** A collection of vertices and the edges connecting them.

- We can query for vertices connected to, or edges leaving from, a vertex v
- Edges are specified as pairs of vertices
- We can add/remove edges from the graph

- Possible Implementations:
  - Adjacency Matrix
  - Edge Set
  - Adjacency List
# Graph Performance Tradeoffs

<table>
<thead>
<tr>
<th></th>
<th>getAllEdgesFrom(v)</th>
<th>hasEdge(v, w)</th>
<th>getAllEdges()</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Adjacency Matrix</strong></td>
<td>Θ(V)</td>
<td>Θ(1)</td>
<td>Θ(V^2)</td>
</tr>
<tr>
<td><strong>Edge Set</strong></td>
<td>Θ(E)</td>
<td>Θ(E)</td>
<td>Θ(E)</td>
</tr>
<tr>
<td><strong>Adjacency List</strong></td>
<td>O(V)</td>
<td>Θ(degree(v))</td>
<td>Θ(E + V)</td>
</tr>
</tbody>
</table>
tl;dr

- Dijkstra’s is great for *all-pairs shortest path*

- A* is great for *single-pair shortest path*
  - But you need to be careful about picking a good heuristic