A* Search and Design Decisions

CSE 373 Winter 2020

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Announcements

- Midterm is *this Friday*
  - If your student number ends in an odd number, go to KNE 2**1**0
  - If your student ends in an even number, go to KNE 2**2**0
  - Workshops and review session will be focused on your midterm questions – bring your questions and practice midterms!
  - Review session Thursday night: 4:30-6:30 @ ARC 147
- HW6 is released
  - Yes, HW5 and HW6 are both currently released
  - Please prefix your Piazza posts with “HW5: …” or “HW6: …”
- 20sp instructors want current students to TA next quarter!
  - Check Piazza or course webpage for more details

Feedback from Reading Quiz

- If we add diagonals, is it still the Manhattan distance? What is the Euclidean distance?

- I still need a walkthrough of Dijkstra's

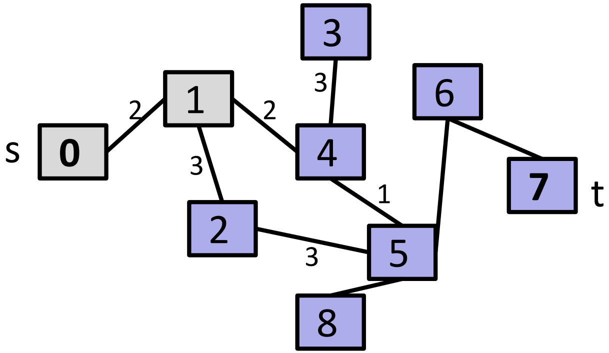
- Does Dijkstra's still work if the grid had different weights?

Lecture Outline

- **Dijkstra's Algorithm, Reviewed**
- A* Search
  - Introducing A*
  - A* Heuristics
- Design Decisions

Dijkstra's Algorithm

- Demo: https://docs.google.com/presentation/d/1_bw2z1ggUkquPdhl7gwdVBoTaoJmaZdpkV6MoAgxlJc/pub?start=false&loop=false&delayms=3000




```
dijkstras(Node s, Graph g) {
  PriorityQueue unvisited;
  unvisited.addAll(g.allNodes(), ∞)
  unvisited.changePriority(s, 0);

  Map<Node, Integer> distances;
  Map<Node, Node> previousNode

  while (! unvisited.isEmpty()) {
    Node n = unvisited.removeMin();
    for (Node i : n.neighbors) {
      if (distances[i]
        < distances[n]
          + g.edgeWeight(n, i)) {
        continue;
      } else {
        distances[i] = distances[n]
          + g.edgeWeight(n, i);
        unvisited.changePriority(i,
          distances[i]);
        previousNode[i] = n;
      }}}}
```

Poll Everywhere

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- Which of the following statements are true?
  - Dijkstra's Algorithm becomes Breadth-first Search if all the edges have the same weight
  - Dijkstra's can find the shortest path from the source to *every node* in the graph
  - At each step of the algorithm, Dijkstra's only considers the path length from the source

A. True / True / True

B. True / True / False

C. False / True / True

D. False / True / False

E. False / False / False

F. I'm not sure …

Dijkstra's Algorithm's Flaws

- Demo: https://qiao.github.io/PathFinding.js/visual/

- If we want a *single shortest path* (instead of *all shortest paths*), Dijkstra's and BFS does unnecessary work
  - The answer is still correct, but we did unnecessary computation

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  - **Introducing A***
  - A* Heuristics
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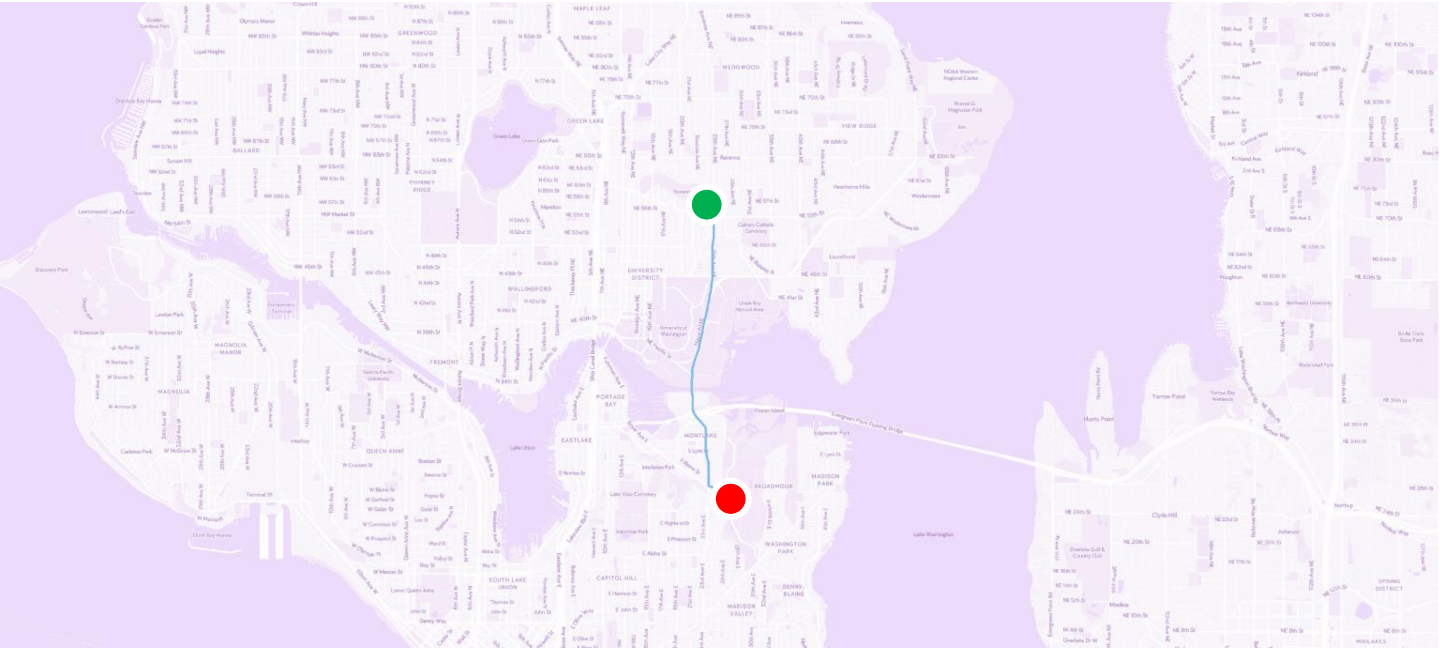
Single-Pair Shortest Path Problem



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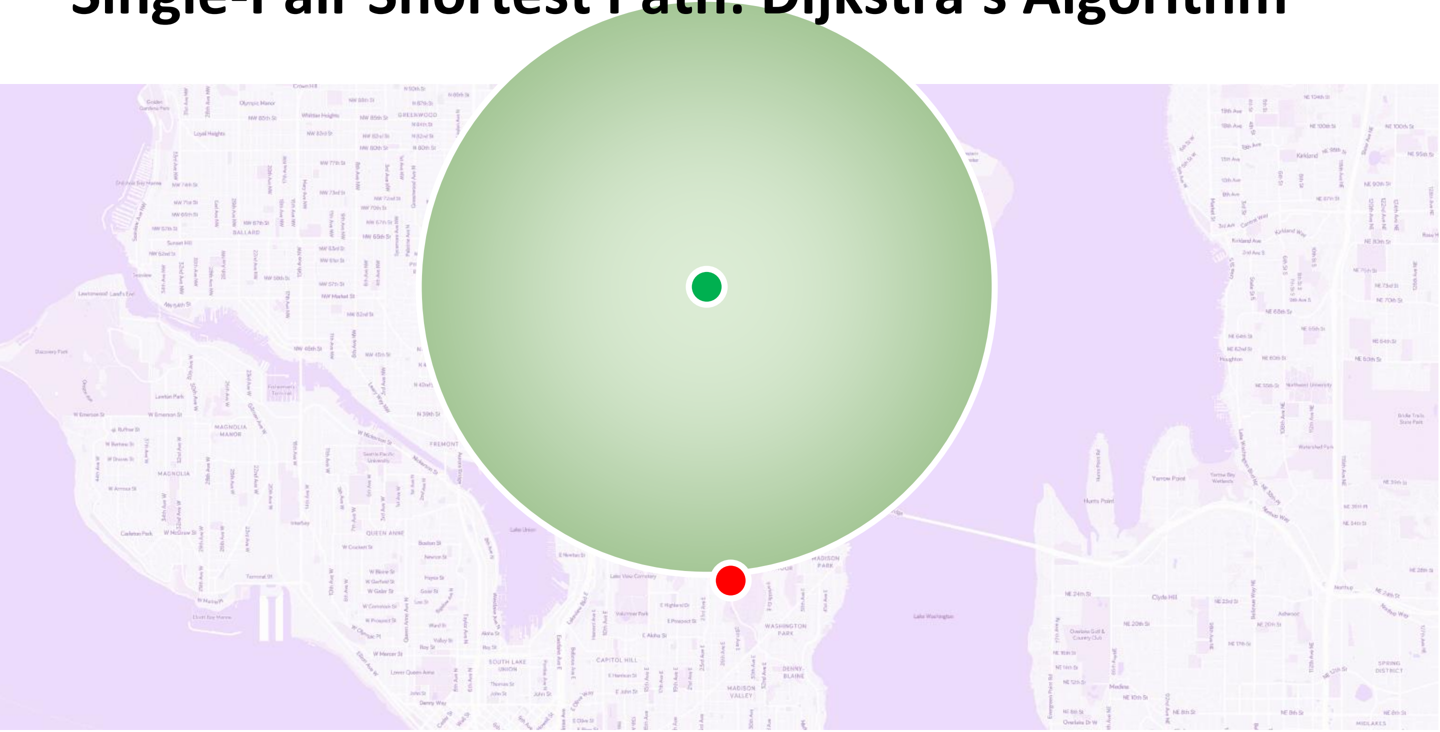
Single-Pair Shortest Path: Dijkstra’s Algorithm



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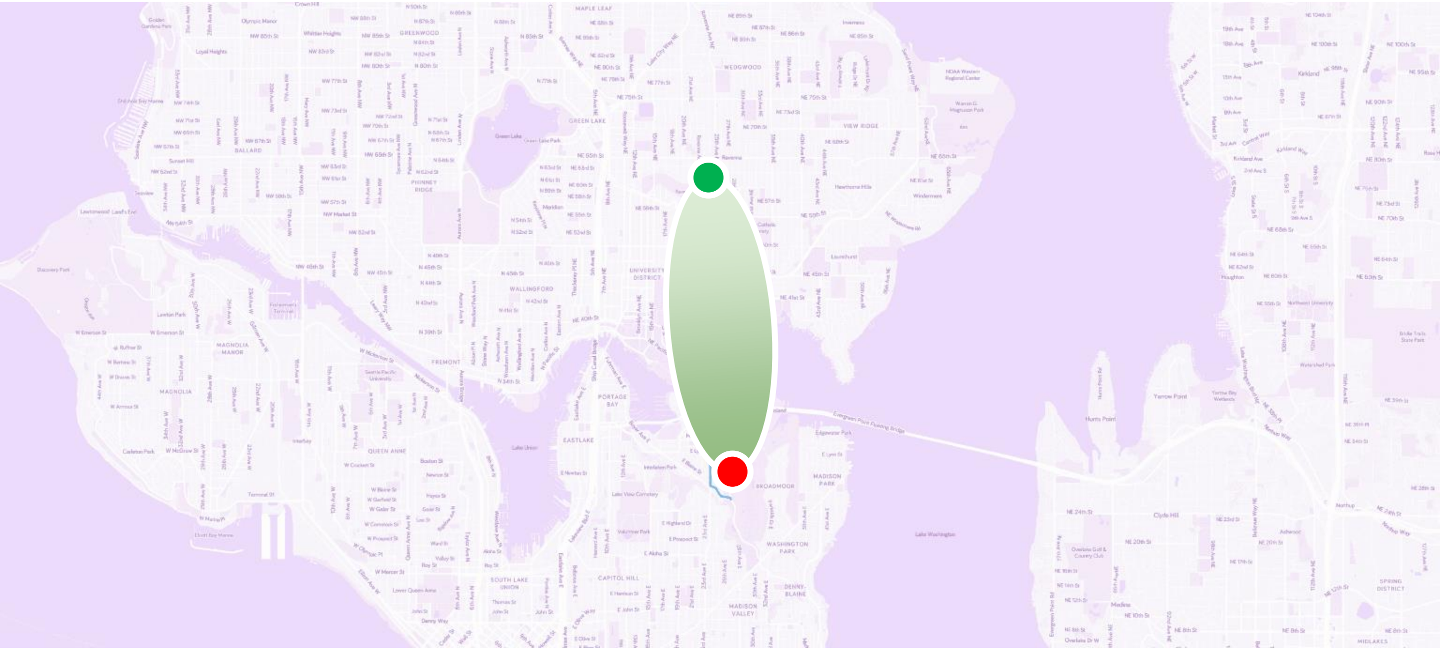
Single-Pair Shortest Path: What We Want



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- How should we hint to Dijkstra's that we want it to concentrate its search southward?
- BFS -> Dijkstra's switched the Queue for a Priority Queue
  - Can we change our idea of a "priority"?

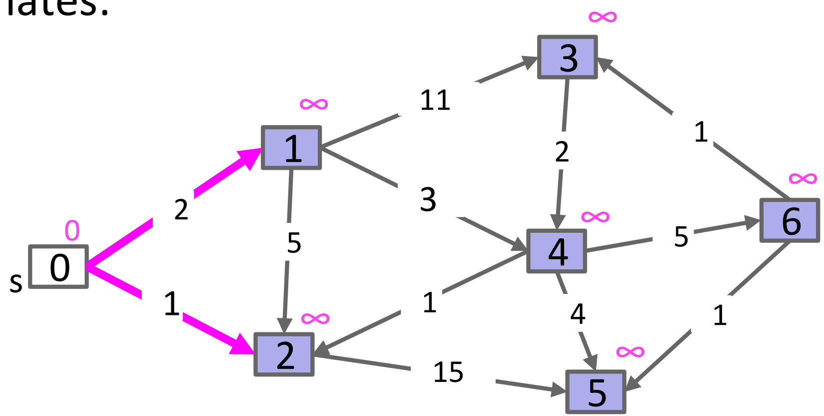
Introducing A* Search

- Idea:
  - Visit vertices in order of d(Ravenna Park, v) + h(v, Japanese Garden), where h(v, Japanese Garden) is an *estimate* of the distance from v to our goal
  - In other words, prefer a location v if:
    - We already know the fastest way to reach v
    - **AND** we suspect that v might be the fastest way to get to our goal
- Dijkstra's only considers d(Ravenna Park, v)
- Demo: http://qiao.github.io/PathFinding.js/visual/

A* Demo

- Source = 0; Destination = 6
- Use the following estimates:

Vertex ID	h(v, dest)
0	1
1	3
2	15
3	2
4	5
5	∞
6	0



- Demo: https://docs.google.com/presentation/d/177bRUTdCa60fjExdr9eO04NHm0MRfPtCzvEup1iMccM/edit

Dijkstra's Algorithm vs A* Search

```
dijkstras(Node s, Graph g) {
  PriorityQueue unvisited;
  unvisited.addAll(g.allNodes(), ∞)
  unvisited.changePriority(s, 0);

  Map<Node, Integer> distances;
  Map<Node, Node> previousNode

  while (! unvisited.isEmpty()) {
    Node n = unvisited.removeMin();
    for (Node i : n.neighbors) {
      if (distances[i]
        < distances[n]
          + g.edgeWeight(n, i)) {
        continue;
      } else {
        distances[i] = distances[n]
          + g.edgeWeight(n, i);
        unvisited.changePriority(i,
          distances[i]);
        previousNode[i] = n;
      }}}}
```

```
astar(Node s, Node t, Graph g) {
  PriorityQueue unvisited;
  unvisited.addAll(g.allNodes(), ∞)
  unvisited.changePriority(s, 0);

  Map<Node, Integer> distances;
  Map<Node, Node> previousNode

  while (! unvisited.isEmpty()) {
    Node n = unvisited.removeMin();
    for (Node i : n.neighbors) {
      if (distances[i]
        < distances[n]
          + g.edgeWeight(n, i)) {
        continue;
      } else {
        distances[i] = distances[n]
          + g.edgeWeight(n, i);
        unvisited.changePriority(i,
          distances[i] + h(i, t));
        previousNode[i] = n;
      }}}}
```

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  - **A* Heuristics**
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Heuristics

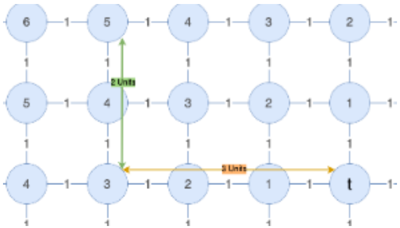
- We call this "estimate function" a **heuristic**
  - Definition: *a solution or choice or judgement that is "good enough" for a purpose, but which could be optimized*
  - In other words: **it doesn't have to be perfect**

- What is a good heuristic for this map?

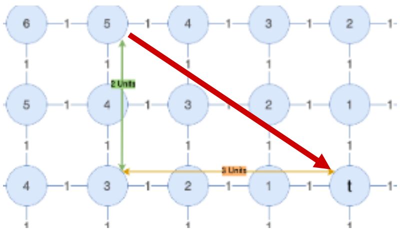
Euclidean and Manhattan Distances

- Assume the entire map can be represented as a grid

- Manhattan distance: Δx + Δy



- Euclidean distance: sqrt($\Delta x^2 + \Delta y^2$)



Poll Everywhere

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- Will A* Search return the correct shortest path if h(v, dest) = 10 for every v in the graph?

A. Always
B. Sometimes
C. Never
D. Not enough information
E. I'm not sure …

But What If We Have a Lousy Heuristic?

- h(v, dest) = 0
  - That's just Dijkstra's

- h(v, dest) = 1,000,000
  - Still just Dijkstra's

- h(Montlake Bridge, dest) = 1,000,000
  - Inconsistent results!

Good Heuristics are Hard!

- You'll frequently hear that "A* Search is hard"
  - As we've seen, A* Search is an incremental update to Dijkstra's
  - What's hard with A* Search is *designing a good heuristic*

- In this class, we'll give you a (good) heuristic for HuskyMaps
  - Hint: Manhattan and Euclidean distances are both good heuristics

- If you take an AI class, you'll learn all about designing heuristics
  - Sneak preview: good heuristics have the following characteristics:
    - h(v, dest) <= true distance from v to destination *("admissible")*
    - h(v, dest) <= dist(v, w) + h(w, dest) *("consistent")*

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Two Key Skills

- In Software Engineering, two important skills to have are:
  - Identifying the requirements (ie, selecting an ADT)
  - Making tradeoffs (ie, selecting the data structure for that ADT)

- So let's review the ADTs' functionality and the performance characteristics of each data structure

List Functionality

List ADT. A collection storing an ordered sequence of elements.

- Each element is accessible by a zero-based index.
- A list has a size defined as the number of elements in the list.
- Elements can be added to the front, back, *or any index in the list*.
- Optionally, elements can be removed from the front, back, *or any index in the list*.

- Possible Implementations:
  - ArrayList
  - LinkedList

List Performance Tradeoffs

	ArrayList	LinkedList
addFront	linear	constant
removeFront	linear	constant
addBack	constant*	linear
removeBack	constant	linear
get(idx)	const	linear
put(idx)	linear	linear

* constant for most invocations

Stack and Queue Functionality

Stack ADT. A collection storing an ordered sequence of elements.

- A stack has a size defined as the number of elements in the stack.
- Elements can only be added and removed from the top ("LIFO")

- Possible Implementations:
  - ArrayStack, LinkedStack

Queue ADT. A collection storing an ordered sequence of elements.

- A queue has a size defined as the number of elements in the queue.
- Elements can only be added to one end and removed from the other ("FIFO")

- Possible Implementations:
  - ArrayQueue, LinkedQueue

Stack and Queue Performance Tradeoffs

- Stack (LIFO):

	ArrayStack	LinkedStack
push	constant*	constant
pop	constant	constant

* constant for most invocations

- Queue (FIFO):

	Array Queue (v2)	LinkedQueue (v2)
enqueue	constant*	constant
dequeue	constant	constant

* constant for most invocations

Deque Functionality

Deque ADT. A collection storing an ordered sequence of elements.

- Each element is accessible by a zero-based index.
- A deque has a size defined as the number of elements in the deque.
- Elements can be added to the front or back.
- Optionally, elements can be removed from the front or back.

- Possible Implementations:
  - ArrayDeque, LinkedDeque

Deque Performance Tradeoffs

	CircularArrayDeque	LinkedDeque
addFirst	constant*	constant
removeFirst	constant	constant
addLast	constant*	constant
removeLast	constant	constant

* constant for most invocations

Set and Map Functionality

Set ADT. A collection of values.
- A set has a size defined as the number of elements in the set.
- You can add and remove values.
- Each value is accessible via a "get" or "contains" operation.

Map ADT. A collection of keys, each associated with a value.
- A map has a size defined as the number of elements in the map.
- You can add and remove (key, value) pairs.
- Each value is accessible by its key via a "get" or "contains" operation.

- Possible Implementations:
  - Unbalanced BST
  - LLRB Tree
  - B-Tree (eg, 2-3 Tree)
  - Hash Tables

Set and Map Performance Tradeoffs

	Find	Add	Remove
Resizing Separate Chaining Hash Table *(worst case)*	$Q \in \Theta(N)$	$Q \in \Theta(N)$	$Q \in \Theta(N)$
Resizing Separate Chaining Hash Table *(best/average cases)* +	$\Theta(1)$	$\Theta(1)^*$	$\Theta(1)^*$
LLRB Tree	$h \in \Theta(\log N)$	$h \in \Theta(\log N)$	$h \in \Theta(\log N)$
B-Tree	$h \in \Theta(\log N)$	$h \in \Theta(\log N)$	$h \in \Theta(\log N)$
BST	$h \in \Theta(N)$	$h \in \Theta(N)$	$h \in \Theta(N)$
LinkedList	$\Theta(N)$	$\Theta(N)$	$\Theta(N)$

Priority Queue Functionality

Priority Queue ADT. A collection of values.

- A PQ has a size defined as the number of elements in the set.
- You can add values.
- You cannot access or remove arbitrary values, only the max value.

- Possible Implementations:
  - Balanced BST with "max" pointer
  - Binary Heap
  - (and a ton of others we didn't discuss)

Priority Queue Performance Tradeoffs

	Balanced BST (worst case)	Binary Heap (worst case)
add	O(log N)	O(log N)**
max	O(1)*	O(1)
removeMax	O(log N)	O(log N)

** If we keep a pointer to the largest element in the BST*

*** Average case is constant*

Graph Functionality

Graph ADT. A collection of vertices and the edges connecting them.

- We can query for vertices connected to, or edges leaving from, a vertex v
- Edges are specified as pairs of vertices
  - We can add/remove edges from the graph

- Possible Implementations:
  - Adjacency Matrix
  - Edge Set
  - Adjacency List

Graph Performance Tradeoffs

	getAllEdgesFrom(v)	hasEdge(v, w)	getAllEdges()
Adjacency Matrix	Θ(V)	Θ(1)	$\Theta(V^2)$
Edge Set	Θ(E)	Θ(E)	Θ(E)
Adjacency List	O(V)	Θ(degree(v))	Θ(E + V)

tl;dr

- Dijkstra's is great for *all-pairs shortest path*

- A* is great for *single-pair shortest path*
  - But you need to be careful about picking a good heuristic