Quadtrees

CSE 373 Winter 2020

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Announcements

- Homework 4: Heap is released and due *Wednesday*
  - Hint: you will need an additional data structure to improve the runtime for changePriority(). It does not affect the correctness of your PQ at all. Please use a built-in Java collection instead of implementing your own.
  - Hint: If you implemented a unittest that tested the exact thing the autograder described, you could run the autograder's test in the debugger (and also not have to use your tokens).

- Please look at posted QuickCheck; we had a few corrections!

Lecture Outline

- **Heaps, cont.: Floyd's buildHeap**
- Review: Set/Map data structures and logarithmic runtimes
- Multi-dimensional Data
- Uniform and Recursive Partitioning
- Quadtrees

Other Priority Queue Operations

- The two "primary" PQ operations are:
  - removeMax()
  - add()

- However, because PQs are used in so many algorithms there are three common-but-nonstandard operations:
  - merge(): merge two PQs into a single PQ
  - buildHeap(): reorder the elements of an array so that its contents can be interpreted as a valid binary heap
  - changePriority(): change the priority of an item already in the heap

buildHeap: Naïve Implementation

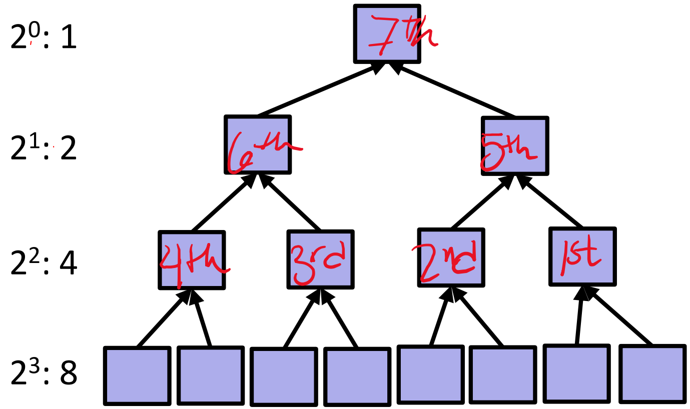
- buildHeap() takes an array of size N and applies the heap-ordering principle to it

- Naïve implementation:
  - Start with an empty array (representing an empty binary heap)
  - Call add() N times
  - Runtime: ?? $\Theta(N \log N)$

- Can we do better?

buildHeap: Clever Implementation

- ~½ of all nodes in a complete binary tree are leaves
  - Remember that $2^0 + 2^1 + \ldots 2^n = 2^{n+1} - 1$
- Clever implementation:
  - Start with full array (representing a binary heap with lots of violations)
  - Call percolateDown() N/2 times starting from the rightmost leaf parent (ie, middle of array)
  - Runtime: ??



This "clever implementation" is called Floyd's Algorithm

Poll Everywhere

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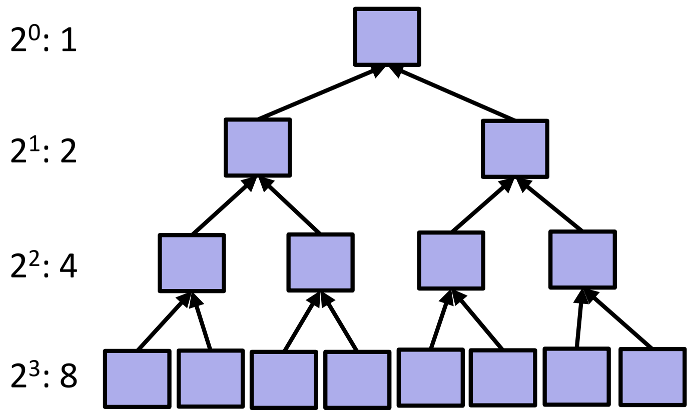
- What is buildHeap()'s runtime?
  - Start with full array (representing a binary heap with lots of violations)
  - Call percolateDown() N/2 times



A. Θ(1)
B. Θ(log N)
C. Θ(N)
D. Θ(N log N)
E. I'm not sure …

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ADT / Data Structure Taxonomy

ADT

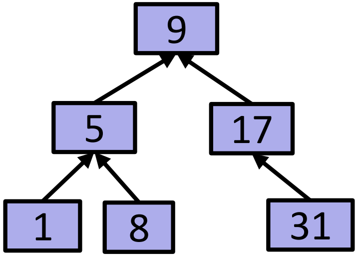
Maps and Sets

Data Structures that Implement

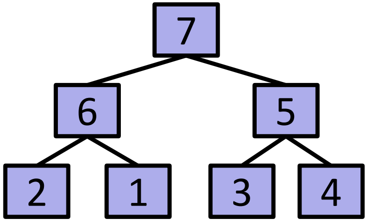
- Search Trees ("left is less-than, right is greater-than")
  - Binary Search Trees (branching factor == 2)
    - Plain BST (unbalanced) start here!
      - Balanced BSTs: LLRB (other examples: "Classic" Red-Black, AVL, Splay, etc)
  - B-Trees (have a branching factor >2; balanced)
    - 2-3 Trees
    - 2-3-4 Trees
- Hash Tables (will cover later!)

Why Does Balance Matter?

- Balanced trees help us avoid considering all of the data all of the time
  - **Binary Search Tree**: Discarding approximately half of the remaining data at each recursive step leads to a logarithmic runtime



  - **Binary Heap:** Recursively percolating up one level approximately halves the number of potential positions to consider, again leading to a logarithmic runtime



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Autocomplete as a 1-Dimensional Range Search

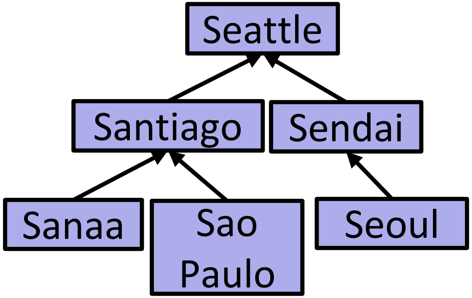
- Location names can be sorted 1-dimensionally (lexicographically aka dictionary ordering)

- Since the data is sorted, we could run two binary searches on the array
  - Range Search Runtime: ?? O(logN) (twice)
  - Insert Runtime: ?? O(N)

Sanaa	Santiago	Sao Paulo	Seattle	Sendai	Seoul

Autocomplete as a 1-Dimensional Range Search

- Or we could do a range search in a balanced BST
  - Range Search Runtime: ?? O(log N)
  - Insert Runtime: ?? O(log N)



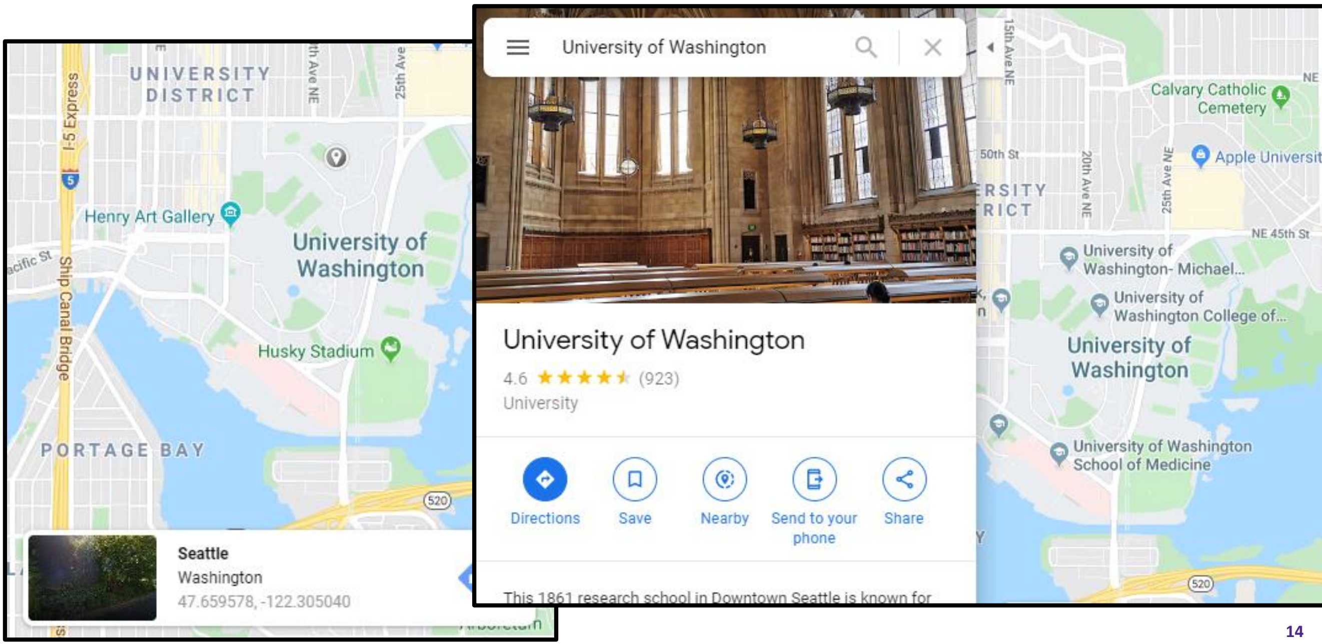

```
void printRange(Node root, Key lo, Key hi) {
  if (root == null) return;

  if (lo < root->key)
    printRange(root->left, lo, hi);
  if (lo <= root->key && root->key >= hi)
    print(root->key);
  if (root->key > hi)
    printRange(root->right, lo, hi);
}
```

single pass algorithm

Geo-locating a Click on a 2D Map

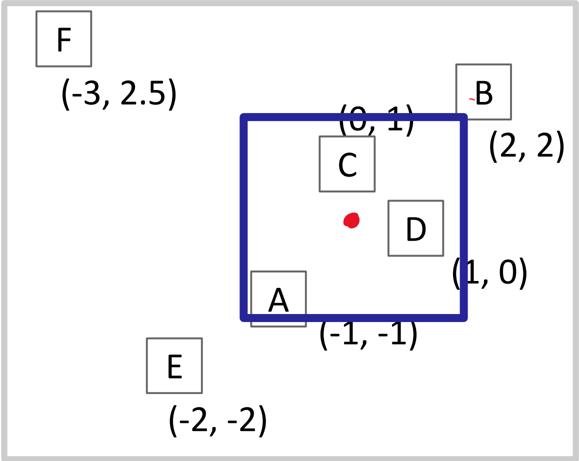
- Why do some map clicks resolve to a lat/lng?
- And other clicks resolve to a point-of-interest?



2-d Range Search: Naïve Implementation

- Check every point for containment in the click target (ie, consider all of the data)

- **Range Search**
  - Scan through all the keys and collect matching results
  - Runtime: ?? O(N)



- **Nearest Neighbour**
  - Range Search, hope for an non-empty result, iterate through results and choose nearest
  - Runtime: ?? O(N)

- **Insert**
  - Put key anywhere
  - Runtime: ?? O(1)

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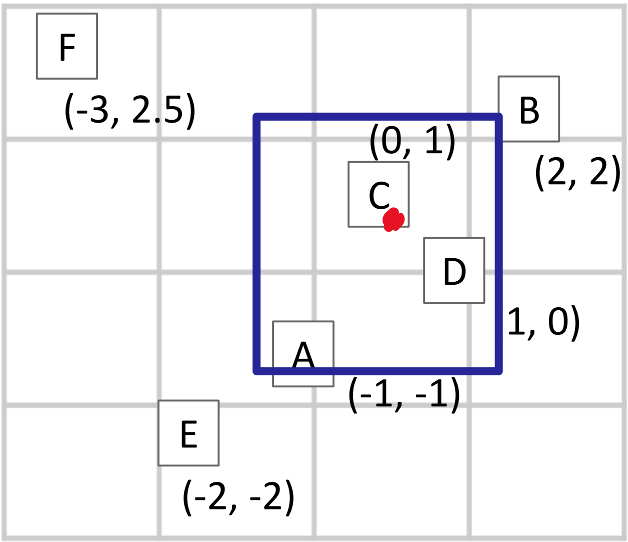
- **Uniform and Recursive Partitioning**
- Quad-Trees

Uniform Partitioning

- Divide space into non-overlapping **subspaces**
  - Known as "spatial partitioning problem"
- **Uniform partitioning strategy**
  - Partition space into uniform rectangular buckets ("bins")
  - Ex: 4x4 grid of such buckets.



Poll Everywhere

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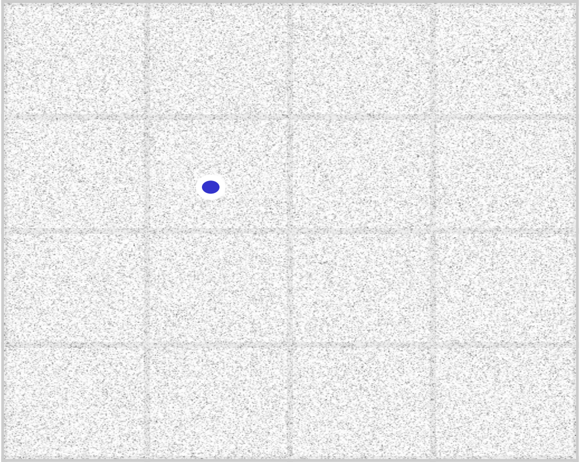
What is the runtime to find the nearest neighbour to our blue point, assuming N points are evenly spread out across a 16-bin uniform partition?



A. $\Theta(1)$

B. $\Theta(\log N)$

C. $\Theta(N)$

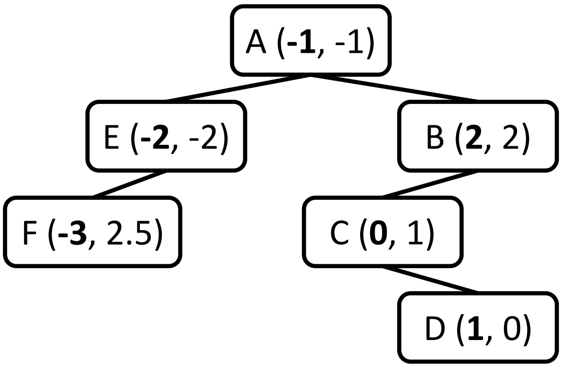
D. $\Theta(N^2)$

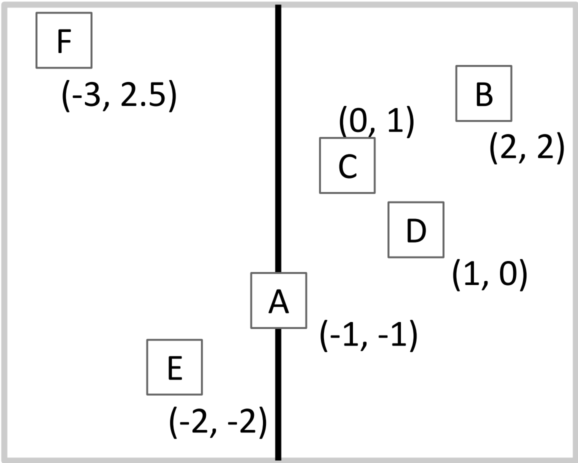
E. I'm not sure …

$N/16 \in \Theta(N)$, so we are, essentially, still considering all of our data. We can do better!

Recursive Partitioning: An x-coordinate BST?

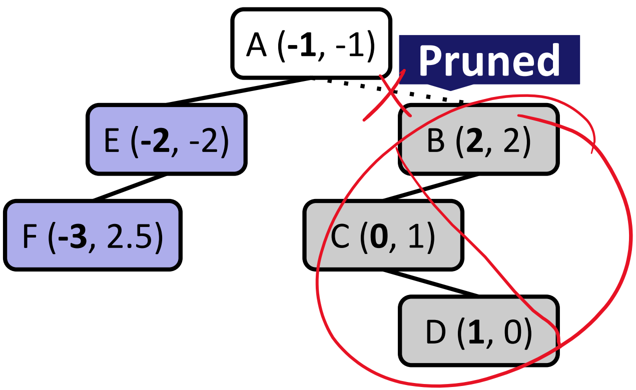
- Suppose we put points into a BST map ordered by x-coordinate.

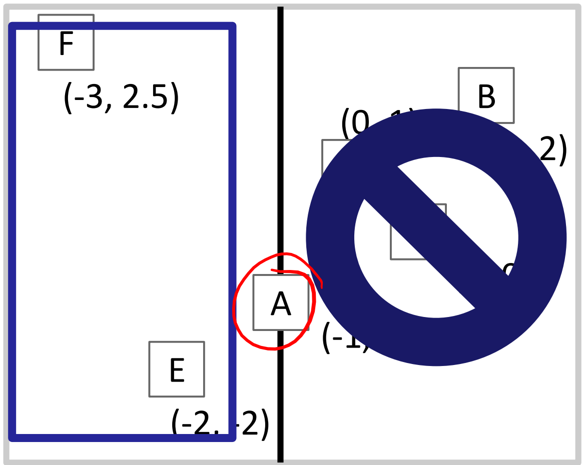




Recursive Partitioning: An x-coordinate BST?

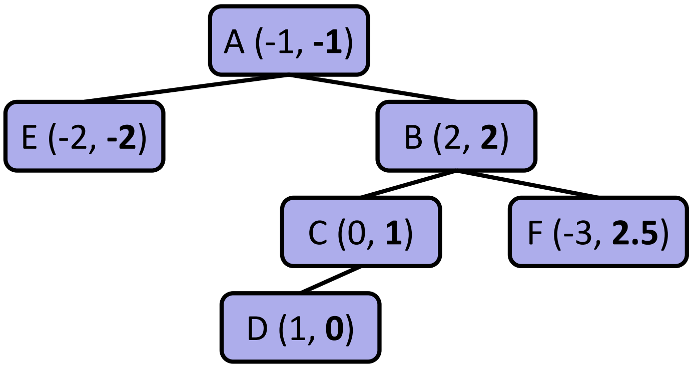
- Range Searching becomes: "What are all the points with x-coordinate less than -1.5?"

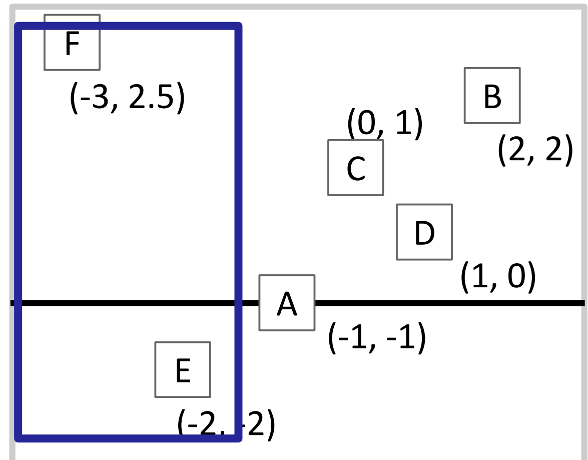




Recursive Partitioning: A y-coordinate BST?

But in a y-coordinate BST, we can't prune anything!





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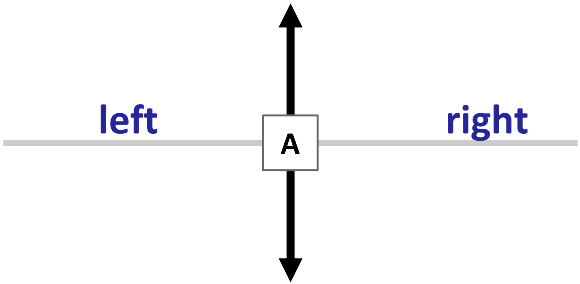
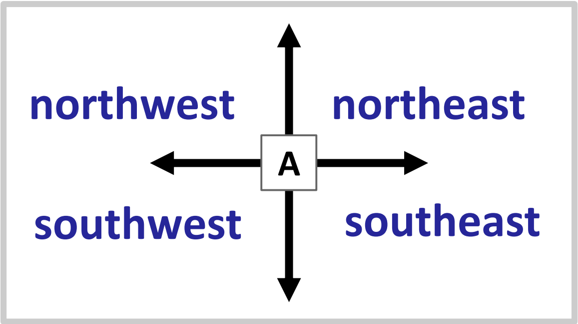
Recursive Partitioning: Quadtree

- **1-dimensional data**
  - Keys are ordered on a line
  - Recursive decision: left or right



Binary Search Tree

- **2-dimensional data**
  - Keys are located on a plane
  - Recursive decision: northwest, northeast, southwest, southeast.

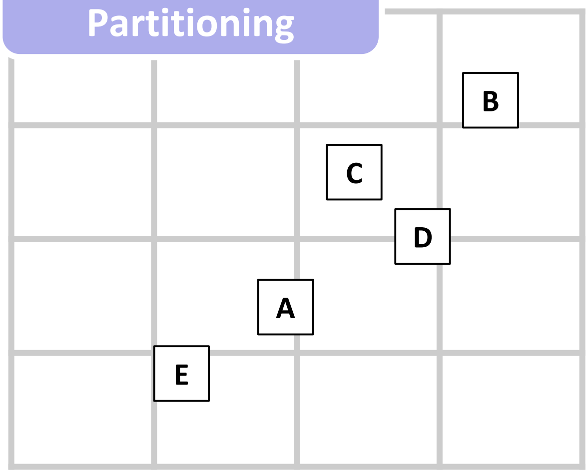


Quadtree

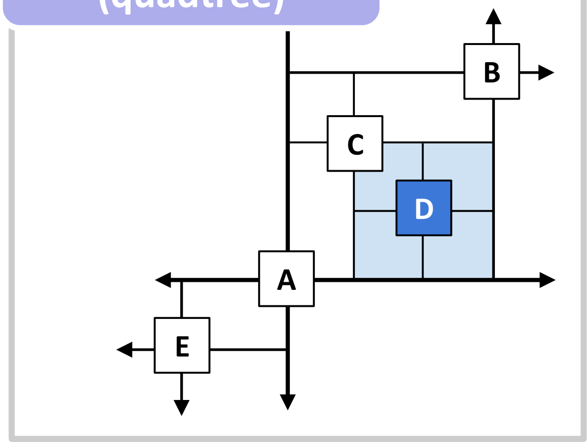
Using Quadtrees to Recursively Partition

- Quadtrees produce recursive, hierarchical partitionings
  - Each point owns 4 subspaces

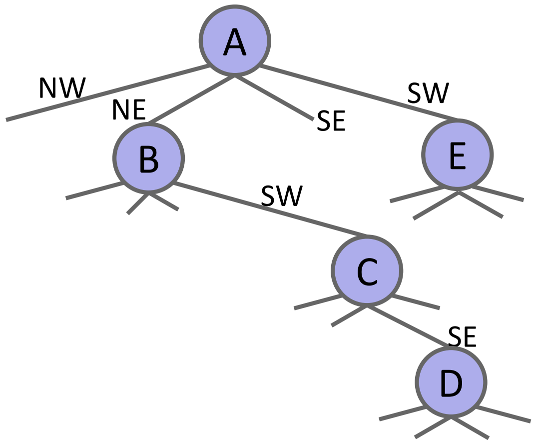
Uniform Partitioning

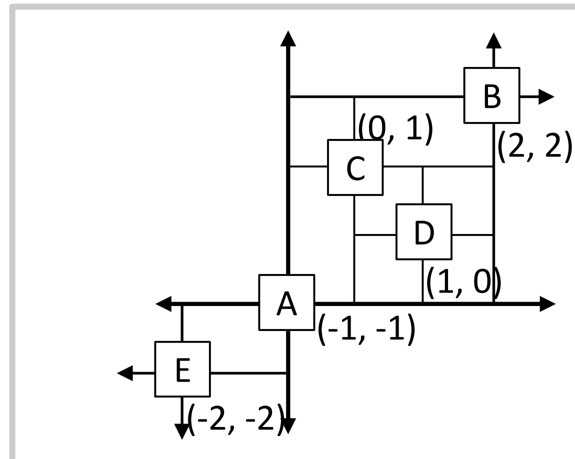


Recursive Partition (quadtree)



Quadtree: Insert

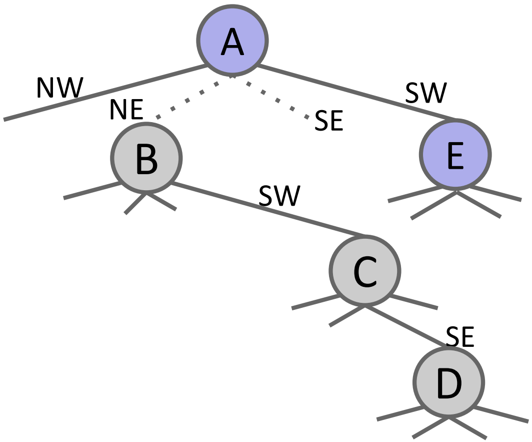


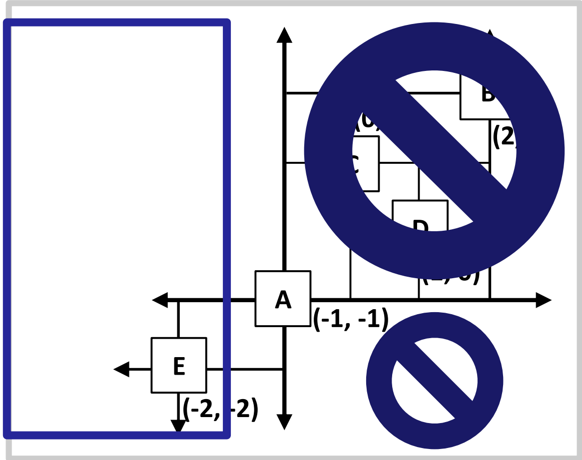


Demo: https://docs.google.com/presentation/d/1vqAJkvUxSh-Eq4iIJZevjpY29nagNTjx-4N3HpDi0UQ/present?ueb=true&slide=id.g11ecaeaf56_0_0

Quadtree: Range Search

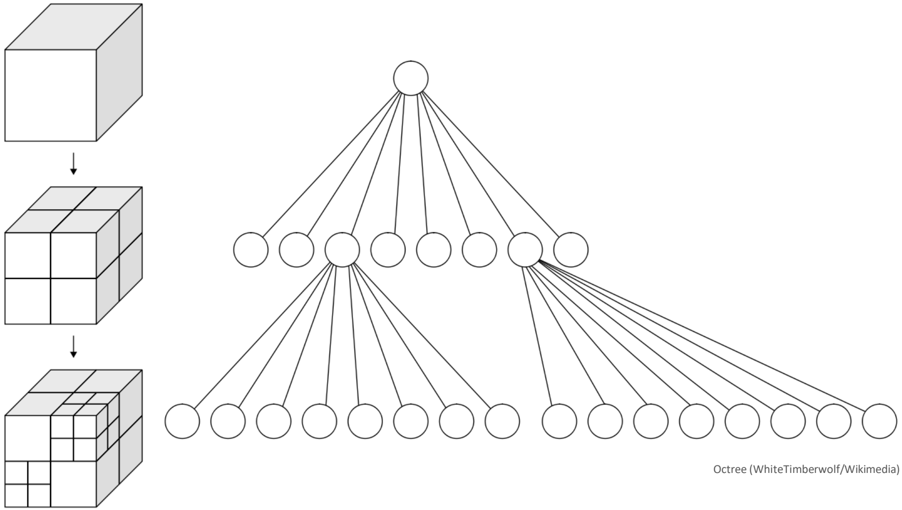
We can prune unnecessary subspaces!





Demo:
https://docs.google.com/presentation/d/1ZVvh_Q15Lh2D1_NnzZ4PR_aDsLBwvAU9JYQAwlSuXSM/present?ueb=true&slide=id.g52a9824549_0_129

3-dimensional Data and Beyond



Octree (WhiteTimberwolf/Wikimedia)

- Oct-trees are generalization of quadtrees for 3D data
- Quadtree Applications: https://www.ics.uci.edu/~eppstein/gina/quadtree.html

tl;dr

- A Priority Queue's core functionality is removeMax and add
  - changePriority can be Θ(log N) if you use an auxiliary data structure
  - buildHeap can be Θ(N) if you percolate carefully

- Recursively subdividing input:
  - allows you to find one piece data without examining all of it
  - often yields logarithmic runtime

- **Quadtrees** allow you to recursively partition 2-dimensional data using a single 4-way question

Range Search	Nearest Neighbour	Add
Θ(log N)	Θ(log N)	Θ(log N)