Quadtrees
CSE 373 Winter 2020

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Announcements

❖ Homework 4: Heap is released and due Wednesday
  ▪ Hint: you will need an additional data structure to improve the runtime for changePriority(). It does not affect the correctness of your PQ at all. Please use a built-in Java collection instead of implementing your own.
  ▪ Hint: If you implemented a unittest that tested the exact thing the autograder described, you could run the autograder’s test in the debugger (and also not have to use your tokens).

❖ Please look at posted QuickCheck; we had a few corrections!
Lecture Outline

❖ Heaps, cont.: Floyd’s buildHeap

❖ Review: Set/Map data structures and logarithmic runtimes

❖ Multi-dimensional Data

❖ Uniform and Recursive Partitioning

❖ Quadtrees
Other Priority Queue Operations

❖ The two “primary” PQ operations are:
  ▪ removeMax()
  ▪ add()

❖ However, because PQs are used in so many algorithms there are three common-but-nonstandard operations:
  ▪ merge(): merge two PQs into a single PQ
  ▪ buildHeap(): reorder the elements of an array so that its contents can be interpreted as a valid binary heap
  ▪ changePriority(): change the priority of an item already in the heap
buildHeap: Naïve Implementation

- buildHeap() takes an array of size N and applies the heap-ordering principle to it

- Naïve implementation:
  - Start with an empty array (representing an empty binary heap)
  - Call add() N times
  - Runtime: \( \Theta(N \log N) \)

- Can we do better?
~½ of all nodes in a complete binary tree are leaves

- Remember that $2^0 + 2^1 + \ldots + 2^n = 2^{n+1} - 1$

Clever implementation:

- Start with full array (representing a binary heap with lots of violations)
- Call `percolateDown()` N/2 times starting from the rightmost leaf parent (i.e., middle of array)
- Runtime: ??

This “clever implementation” is called Floyd’s Algorithm
What is buildHeap()’s runtime?

- Start with full array (representing a binary heap with lots of violations)
- Call percolateDown() N/2 times

A. $\Theta(1)$
B. $\Theta(\log N)$
C. $\Theta(N)$
D. $\Theta(N \log N)$
E. I’m not sure ...
Lecture Outline

❖ Heaps, cont.: Floyd’s buildHeap

❖ **Review: Set/Map data structures and logarithmic runtimes**

❖ Multi-dimensional Data

❖ Uniform and Recursive Partitioning

❖ Quad-Trees
ADT / Data Structure Taxonomy

Maps and Sets

- Search Trees (“left is less-than, right is greater-than”)
  - Binary Search Trees (branching factor == 2)
    - Plain BST (unbalanced)
    - Balanced BSTs: LLRB (other examples: “Classic” Red-Black, AVL, Splay, etc)
  - B-Trees (have a branching factor >2; balanced)
    - 2-3 Trees
    - 2-3-4 Trees

- Hash Tables (will cover later!)
Why Does Balance Matter?

- Balanced trees help us avoid considering all of the data all of the time

- **Binary Search Tree**: Discarding approximately half of the remaining data at each recursive step leads to a logarithmic runtime

- **Binary Heap**: Recursively percolating up one level approximately halves the number of potential positions to consider, again leading to a logarithmic runtime
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- Uniform and Recursive Partitioning
- Quad-Trees
Autocomplete as a 1-Dimensional Range Search

- Location names can be sorted 1-dimensionally (lexicographically aka dictionary ordering)

- Since the data is sorted, we could run two binary searches on the array
  - Range Search Runtime: $O(\log N)$ (twice)
  - Insert Runtime: $O(N)$

<table>
<thead>
<tr>
<th>Sanaa</th>
<th>Santiago</th>
<th>Sao Paulo</th>
<th>Seattle</th>
<th>Sendai</th>
<th>Seoul</th>
</tr>
</thead>
</table>

Autocomplete as a 1-Dimensional Range Search

- Or we could do a range search in a balanced BST
  - Range Search Runtime: $O(\log N)$
  - Insert Runtime: $O(\log N)$

```c
void printRange(Node root, Key lo, Key hi) {
    if (root == null) return;

    if (lo < root->key)
        printRange(root->left, lo, hi);
    if (lo <= root->key && root->key >= hi)
        print(root->key);
    if (root->key > hi)
        printRange(root->right, lo, hi);
}
```
Geo-locating a Click on a 2D Map

❖ Why do some map clicks resolve to a lat/lng?
❖ And other clicks resolve to a point-of-interest?
2-d Range Search: Naïve Implementation

- Check every point for containment in the click target (i.e., consider all of the data)

- **Range Search**
  - Scan through all the keys and collect matching results
  - Runtime: $O(N)$

- **Nearest Neighbour**
  - Range Search, hope for a non-empty result, iterate through results and choose nearest
  - Runtime: $O(N)$

- **Insert**
  - Put key anywhere
  - Runtime: $O(1)$
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Uniform Partitioning

- Divide space into non-overlapping **subspaces**
  - Known as “spatial partitioning problem”

- **Uniform partitioning strategy**
  - Partition space into uniform rectangular buckets (“bins”)
  - Ex: 4x4 grid of such buckets.

![Diagram showing uniform partitioning]
What is the runtime to find the nearest neighbour to our blue point, assuming N points are evenly spread out across a 16-bin uniform partition?

A. $\Theta(1)$  
B. $\Theta(\log N)$  
C. $\Theta(N)$  
D. $\Theta(N^2)$  
E. I’m not sure ...
Recursive Partitioning: An x-coordinate BST?

- Suppose we put points into a BST map ordered by x-coordinate.
Recursive Partitioning: An x-coordinate BST?

- Range Searching becomes: “What are all the points with x-coordinate less than -1.5?”

- Pruned
Recursive Partitioning: A y-coordinate BST?

But in a y-coordinate BST, we can’t prune anything!
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Recursive Partitioning: Quadtree

- **1-dimensional data**
  - Keys are ordered on a line
  - Recursive decision: left or right

- **2-dimensional data**
  - Keys are located on a plane
  - Recursive decision: northwest, northeast, southwest, southeast.

Binary Search Tree

Quadtree
Using Quadtrees to Recursively Partition

- Quadtrees produce recursive, hierarchical partitionings
  - Each point owns 4 subspaces

Uniform Partitioning

Recursive Partition (quadtrees)
Quadtree: Insert

Demo: https://docs.google.com/presentation/d/1vqAJkvUxSh-Eq4iJZevjpY29nagNTjx-4N3HpDi0UQ/present?ueb=true&slide=id.g11ecaef56_0_0
Quadtree: Range Search

We can prune unnecessary subspaces!

Demo:
https://docs.google.com/presentation/d/1ZVvh_Q15Lh2D1_NnzZ4PR_aDsLBwvAU9JYQAwISuXSM/present?ueb=true&slide=id.g52a9824549_0_129
3-dimensional Data and Beyond

- Oct-trees are generalization of quadtrees for 3D data
- Quadtree Applications: https://www.ics.uci.edu/~eppstein/gina/quadtree.html
tl;dr

❖ A Priority Queue’s core functionality is removeMax and add
  ▪ changePriority can be $\Theta(\log N)$ if you use an auxiliary data structure
  ▪ buildHeap can be $\Theta(N)$ if you percolate carefully

❖ Recursively subdividing input:
  ▪ allows you to find one piece data without examining all of it
  ▪ often yields logarithmic runtime

❖ **Quadtrees** allow you to recursively partition 2-dimensional data using a single 4-way question

<table>
<thead>
<tr>
<th>Range Search</th>
<th>Nearest Neighbour</th>
<th>Add</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta(\log N)$</td>
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