Set and Map ADTs: Binary Search Trees

CSE 373 Winter 2020

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About how long did Homework 2 take?

A. 0-2 Hours
B. 2-4 Hours
C. 4-6 Hours
D. 5-10 Hours
E. 10-14 Hours
F. 14+ Hours
G. I haven’t finished yet / I don’t want to say

Announcements

- Homework 3: Autocomplete is released
  - We’ve started to implement a rate-limiting / token-saving policy to encourage you to write your own tests and to start early.
  - Thresholds are “reasonable”
  - Hint: If you implemented a unittest that tested the exact thing the autograder described, you could run the autograder’s test in the debugger (and also not have to use your tokens).
  - Hint: MatchResult takes an *inclusive* start but an *exclusive* end index

- HW2 feedback survey
  - Similar to HW1; help us improve our homeworks

- Extra DITs added Monday morning
  - 11-12:30, CSE 4th floor breakout

Questions from Reading Quiz

- Do map values need to be unique as well?
- Is the Java TreeMap a BST?
- What if the map keys aren’t numbers?

Lecture Outline

- **Binary Search and Binary Range Search**
- ADTs: Sets and Maps
- Binary Search Trees as Sets and Maps
- BST Operations:
    - Find/Contains
    - Add
    - Remove

Case Analysis != Asymptotic Analysis

- Case analysis deals with a specific input or a specific class of inputs

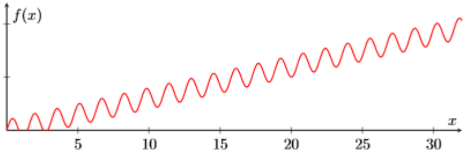
- Asymptotic analysis deals with “the shape of the curve near infinity”

- Demos (each case has their own O, Θ, and Ω bounds):
  - Best: https://www.desmos.com/calculator/uovi22xfwq
  - Worst: https://www.desmos.com/calculator/v3u5hviyqe
  - Overall: https://www.desmos.com/calculator/huqfxcwu05

"Shapes Near Infinity"

has Θ bound for overall case

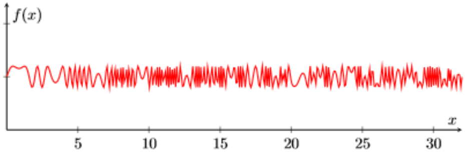
(b)



Solution:

$\mathcal{O}(n)$, $\Omega(n)$, and $\Theta(n)$.

(c)

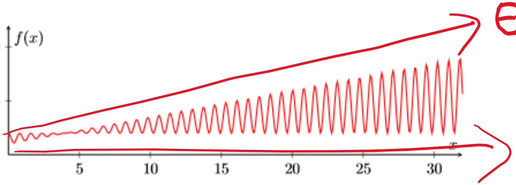


Solution:

$\mathcal{O}(1)$, $\Omega(1)$, and $\Theta(1)$.

no Θ-bound for overall case

(e)



Θ-bound for worst case

Θ-bound for best case

Solution:

$\mathcal{O}(n)$ and $\Omega(1)$. This function does not have a big-Θ, because the tightest upper and lower bounds are not the same.

Binary Search Runtime

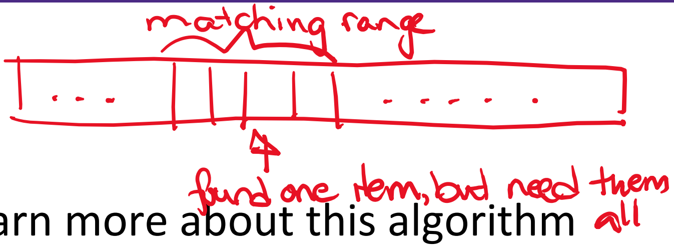
```java
public static boolean binarySearch(int[] sorted, int findMe) {
  if (sorted.length == 0)
    return false;

  int mid = sorted.length / 2;
  if (findMe < sorted[mid])
    int[] subrange = Arrays.copyOfRange(sorted, 0, mid);
    return binarySearch(subrange, findMe);
  else if (x > sorted[mid])
    int[] subrange = Arrays.copyOfRange(sorted, mid, sorted.length);
    return binarySearch(subrange, findMe);
  else
    return true;
}
```

early exit causes $\Theta(1)$ best case runtime

Case	Big-O	Big-Theta	Big-Omega
Best	$O(1)$	$\Theta(1)$	$\Omega(1)$
Worst	$O(\log_2 N)$	$\Theta(\log_2 N)$	$\Omega(\log_2 N)$
Overall	$O(\log_2 N)$	DNE	$\Omega(1)$

Binary *Range* Search



- You are highly encouraged to learn more about this algorithm with a websearch or by talking with a friend
  - Remember: Don’t copy-n-paste other people’s (or your) code

- Basic idea is that you’re looking for the *first* and *last* elements of a range
  - Which means, unlike binary search, there’s no early exit when you’ve found a matching item

Case	Big-O	Big-Theta	Big-Omega
Best	$O(\log_2 N)$	$\Theta(\log_2 N)$	$\Omega(\log_2 N)$
Worst	$O(\log_2 N)$	$\Theta(\log_2 N)$	$\Omega(\log_2 N)$
Overall			

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- Binary Search vs Binary Range Search
- **ADTs: Sets and Maps**
- Binary Search Trees as Sets and Maps
- BST Operations:
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ADTs So Far

List ADT. A collection storing an ordered sequence of elements.

- Each element is accessible by a zero-based index.
- A list has a size defined as the number of elements in the list.
- Elements can be added to the front, back, *or any index in the list*.
- Optionally, elements can be removed from the front, back, *or any index in the list*.

- Data structures that implemented the List ADT include LinkedList and ArrayList
- When we restrict List's functionality, we end up with the 3 other ADTs we've seen so far

ADTs So Far

Deque ADT. A collection storing an ordered sequence of elements.

- Each element is accessible by a zero-based index.
- A deque has a size defined as the number of elements in the deque.
- Elements can be added to the front or back.
- Optionally, elements can be removed from the front or back.

Stack ADT. A collection storing an ordered sequence of elements.

- A stack has a size defined as the number of elements in the stack.
- Elements can only be added and removed from the top (“LIFO”)

Queue ADT. A collection storing an ordered sequence of elements.

- A queue has a size defined as the number of elements in the queue.
- Elements can only be added to one end and removed from the other (“FIFO”)

- Data structures that implemented these ADTs are LinkedList and ArrayList variants

Set ADT

Set ADT. A collection of values.

- A set has a size defined as the number of elements in the set.
- You can add and remove values.
- Each value is accessible via a “get” or “contains” operation.

- Naïve implementation: a list of items
  - add(v):
  - contains(v):
  - remove(v):

```
class Item<Value> {
  Value v;
}

LinkedList<Item> set;
```

Map ADT

Map ADT. A collection of keys, each associated with a value.

- A map has a size defined as the number of elements in the map.
- You can add and remove (key, value) pairs.
- Each value is accessible by its key via a "get" or "contains" operation.

- Also known as "**Dictionary ADT**"
- Naïve implementation: a set of (key, value) pairs
  - add(k, v): $\Theta(1)$
  - find(k):
  - contains(k): $\Theta(N)$
  - remove(k):

```
class KVPair<Key, Value> {
  Key k;
  Value v;
}

LinkedList<KVPair> map;
```

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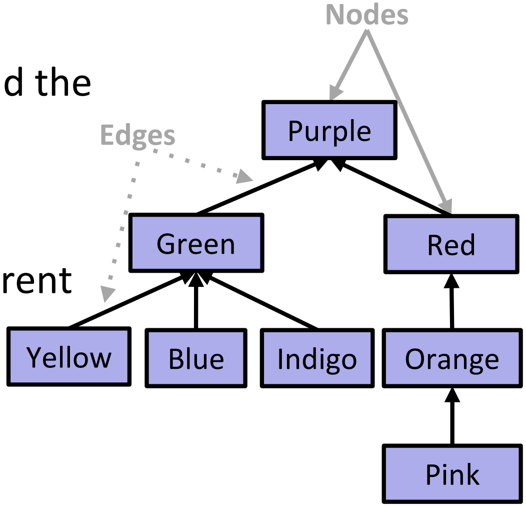
Tree Data Structure

- A **Tree** is a collection of nodes; each node has <= 1 parent and >= 0 children
  - **Root node**: the "top" of the tree and the only node with no parent
  - **Leaf node**: a node with no children
  - **Edge**: the connection between a parent and child
  - There is exactly one path between any pair of nodes

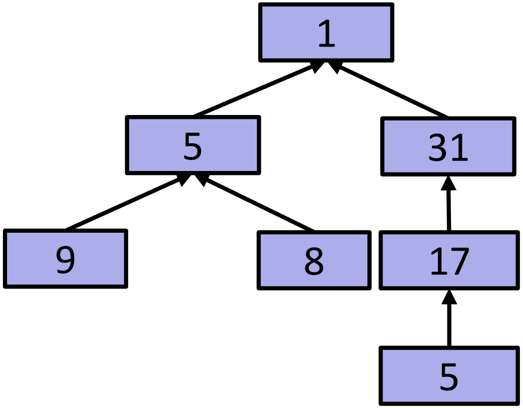


- **Subtree**: a node and all of its descendants
  - Trees are defined recursively!

```
class Node<Value> {
  Value v;
  List<Node> children;
}
```

Binary Tree Data Structure

- A **Binary Tree** is a tree where each node has 0 <= children <= 2



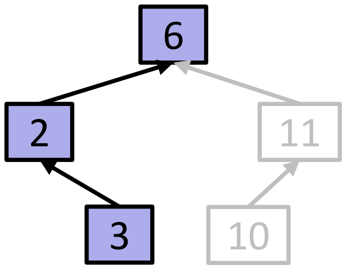
```
class BinaryNode<Value> {
  Value v;
  BinaryNode left;
  BinaryNode right;
}
```

Review: Binary Search

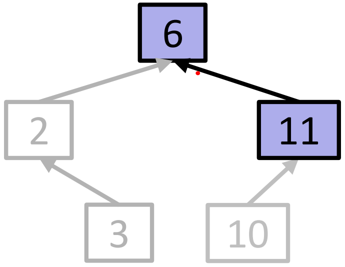
- Remember Binary Search's "function call tree"?

2	3	6	10	11

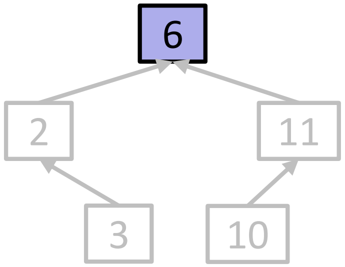
binarySearch(3)



binarySearch(11)

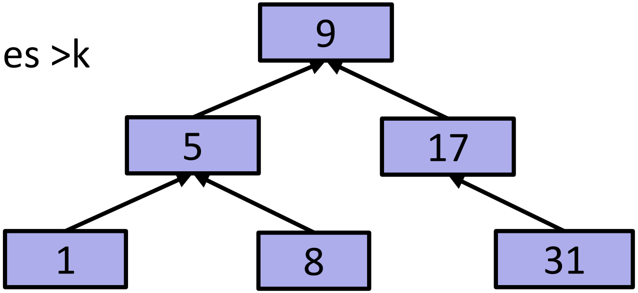


binarySearch(6)



Binary *Search* Trees

- A **Binary Search Tree** is a binary tree with the following invariant: for every node with value k in the BST:
  - The left subtree only contains values <k
  - The right subtree only contains values >k



```
class BSTNode<Value> {
  Value v;
  BSTNode left;
  BSTNode right;
}
```

Reminder: the BST ordering applies recursively to the entire subtree

Poll Everywhere

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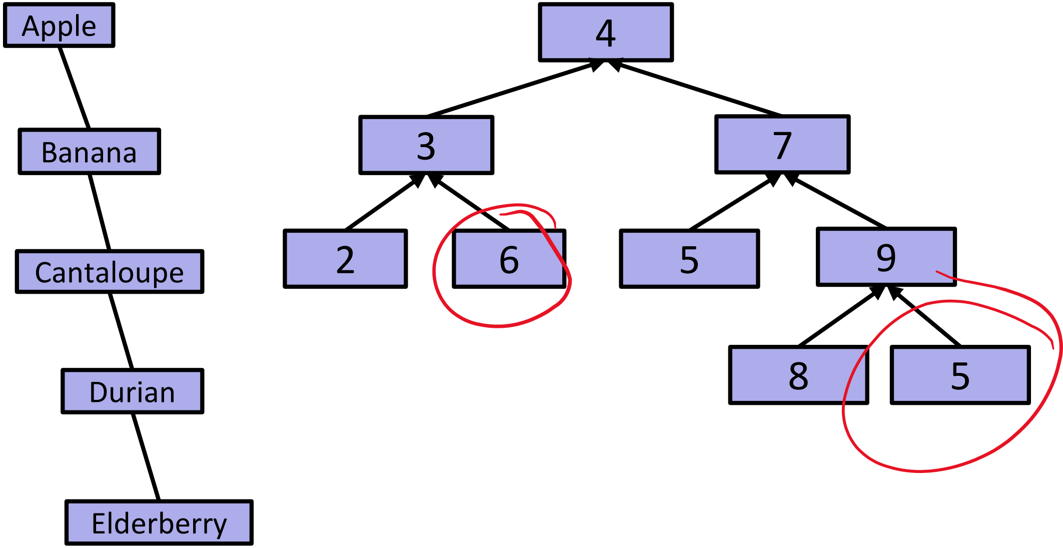
- Are these Binary Search Trees?

A. Yes / Yes

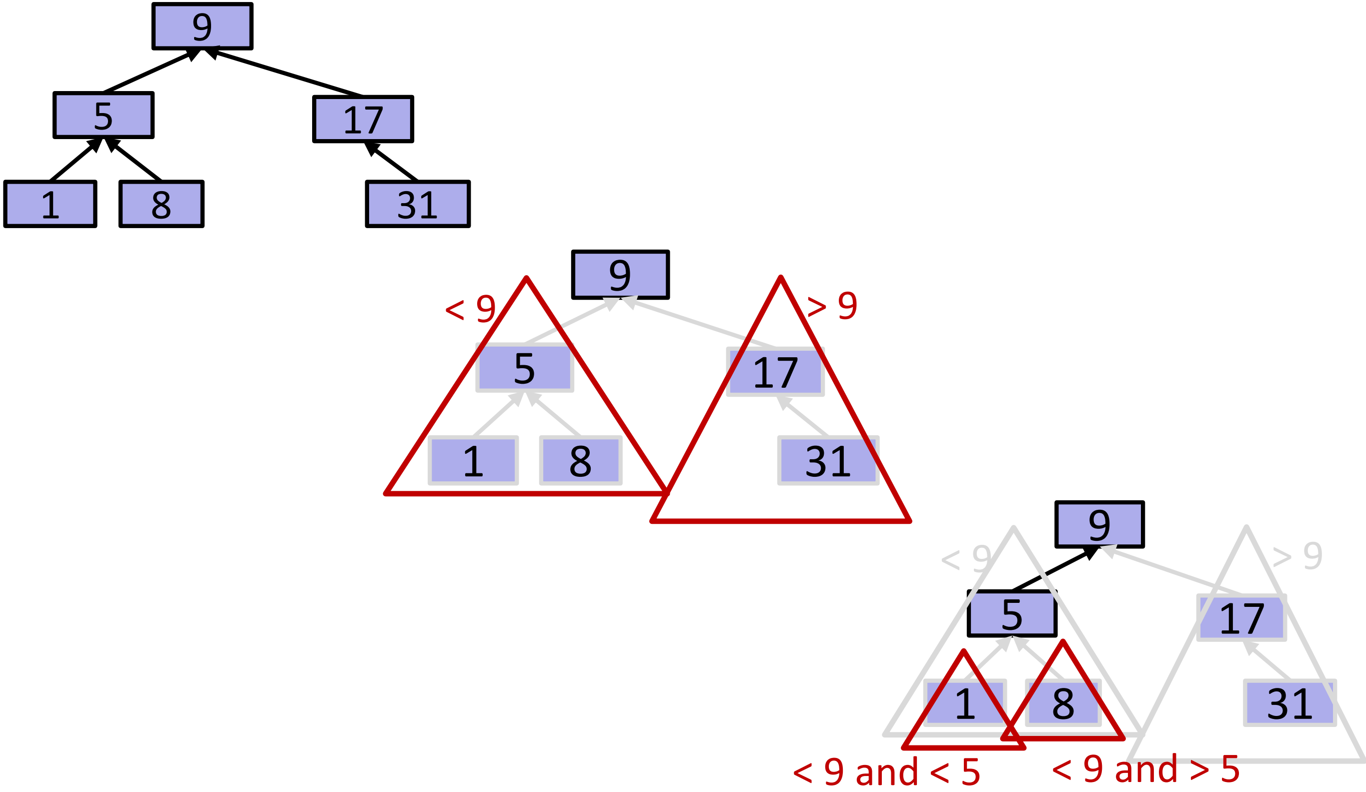
[✓ B. Yes / No]

C. No / Yes

D. No / No

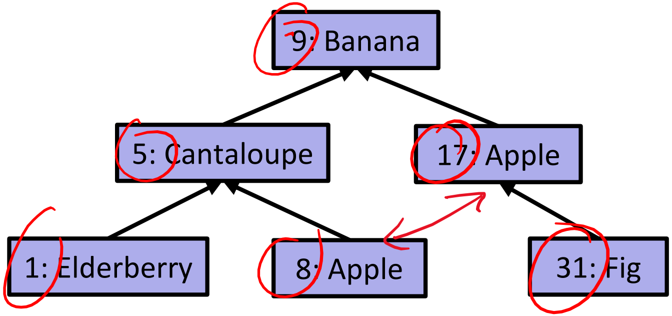


BST Ordering Applies *Recursively*



Binary Search Trees as Maps

- Since BSTs contain keys, they can also contain (key, value) pairs



```
class BSTNode<Key, Value> {
  Key k;
  Value v;
  BSTNode left;
  BSTNode right;
}
```

- Sorted by keys, not values
- Values can be duplicated

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- ADTs: Maps and Sets
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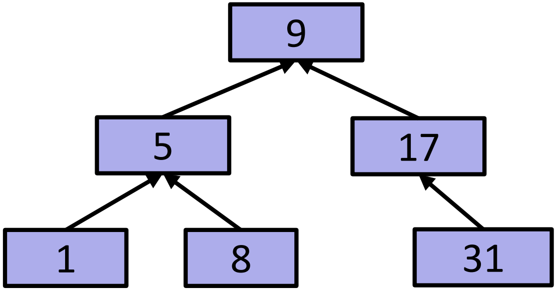
- BST Operations:
  - **Find/Contains**
  - Add
  - Remove

Binary Search Trees: Find/Contains

- Unsurprisingly, this looks a lot like binary search
- Can you implement contains by putting the following statements in the correct order?
  - Hint: remember BST's invariants
- What is find's worst-case runtime?



```
boolean contains(BSTNode n,
                 Key k) {

}
```

A,B,C,D } either will work
A,B,D,C

A
```
if (n == null)
  return false;
```

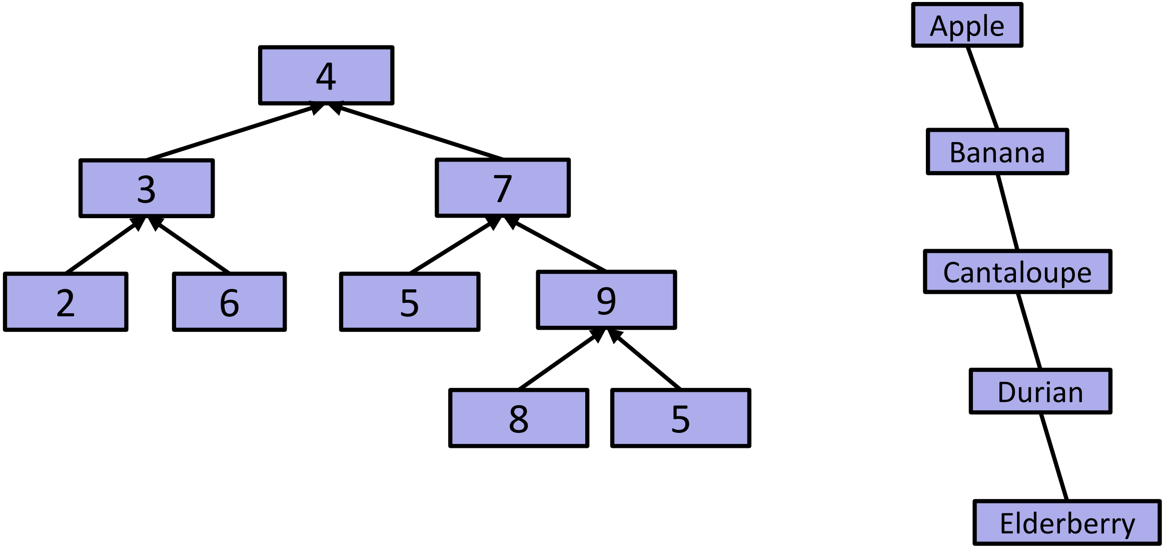
B
```
if (k.equals(n.key))
  return true;
```

C
```
if (k < n.k) {
  return contains(
         n.left, k);
}
```

D
```
if (k >= n.k) {
  return contains(
         n.right, k);
}
```

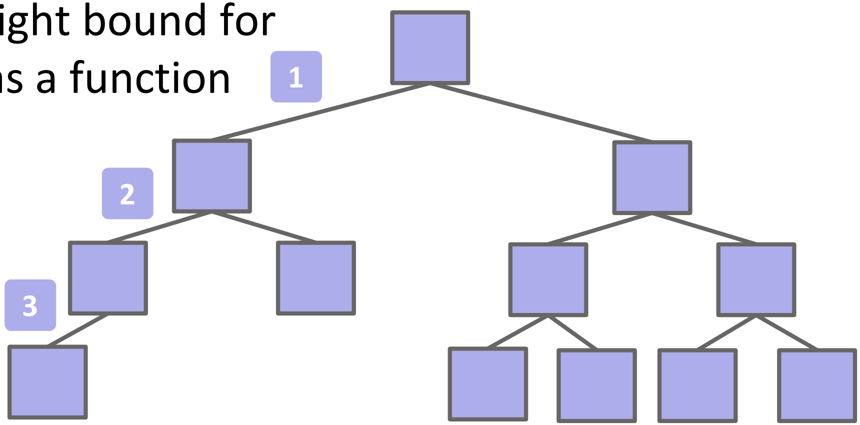
BST Find/Contains's runtime

- What is find's *worst-case* runtime, as a function of n? } don't know yet!
- What is find's worst-case runtime, as a function of *height*? } $\Theta(h)$



BST Height (or depth)

- The **height** of a binary search tree is the number of edges on *the longest path* between the root node and any leaf
  - A **path** is a connected sequence of edges that join parent-child nodes
  - The height of this tree is **3**
- We don't have a tight bound for the tree's height as a function of its size!



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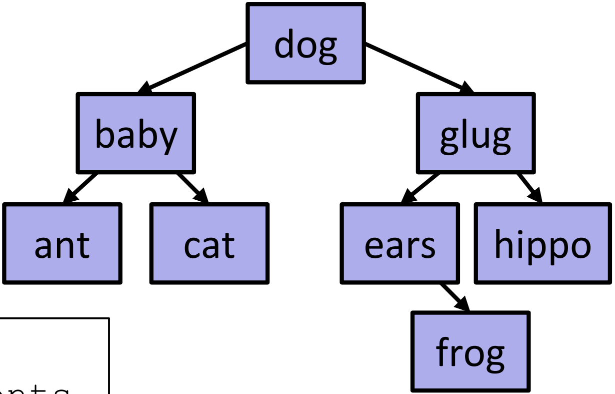
  - **Add**
  - Remove

Binary Search Trees: Add

- Where does the new key belong?

Easiest to add to a leaf

- How do we use BST invariants to ensure the leaf is added correctly?



```
BSTNode add(BST t, Key k) {
  // Implement by putting statements
  // in the correct order
}
```

D, B, C, A } either will work
D, C, B, A

Same runtime as find: Θ(h)

A
```
return t;
```

B
```
if (k < t.key)) {
  t.left
    = add(t.left, k);
}
```

C
```
if (k > t.key) {
  t.right
    = add(t.right, k);
}
```

D
```
if (t == null){
  return
    new BSTNode(k);
}
```

Lecture Outline

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- BST Operations:
  - Find/Contains
  - Add
  - **Remove (to be continued Friday)**