Do not turn the page until 15:30.

Caveat

- While this practice exam is representative of the format and the topics that might be covered, it hasn’t been tested to ensure that it is exactly 50 minutes’ worth of work. We think this looks like a 70-75 minute exam, but erred on the side of giving you more practice material 😊

Advice

- Read the questions carefully before starting. Skip questions that are taking a long time.
- Read all questions first and start where you feel the most confident.
- Take a deep breath and relax. We believe in you!
Question 1: Listomania!
What ADT would be best used in the following situations:

(a) You need to move apartments and have rented a moving truck. Once something has been loaded into the truck, it can't be removed until everything that was loaded after it has been removed.

(b) The TAs want to keep track of who needs help at office hours and they want to help people in the order they arrive.

(c) The UW is building a new registration system and wants to be able to tell if a student is already registered for a particular class, to prevent them from registering for it twice.

(d) You are an ER doctor, and you are writing a queueing app that allows you to always take in the patient with the most severe symptoms first. Assume that any two illnesses are comparable.

(e) When grading the midterm, the TAs want to be able to record which student got which score.
Question 2: If a Tree Falls in a Forest

a) Draw the entire resulting tree after each of the following insertions into a Left-Leaning Red-Black tree. The first one has been done for you. Indicate red edges in the tree with a dashed line or by labeling them clearly with “RED” next to them.

<table>
<thead>
<tr>
<th>Insert 1</th>
<th>Insert 5</th>
<th>Insert 3</th>
<th>Insert 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Tree Diagram" /></td>
<td><img src="image2" alt="Tree Diagram" /></td>
<td><img src="image3" alt="Tree Diagram" /></td>
<td><img src="image4" alt="Tree Diagram" /></td>
</tr>
</tbody>
</table>

b) Consider the following 2-3 tree. In the box below, draw the final tree after performing the following insertions in order: 8, 20, 9.

<table>
<thead>
<tr>
<th>Before insertion</th>
<th>After inserting 8, 20, 9</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5" alt="Tree Diagram" /></td>
<td><img src="image6" alt="Tree Diagram" /></td>
</tr>
</tbody>
</table>

c) Suppose we want to store the following elements in a regular binary search tree (i.e. with no balancing properties). Give an order we could insert the elements to create a worst-case tree, and an order to create a best-case tree.

<table>
<thead>
<tr>
<th>Elements</th>
<th>8, 12, 6, 9, 5, 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best-Case Insertion Order</td>
<td></td>
</tr>
<tr>
<td>Worst-Case Insertion Order</td>
<td></td>
</tr>
</tbody>
</table>
Question 3: Big Oh-No!

(A) Give the simplified worst case runtime in $\Theta(\cdot)$ notation, as a function of $n$.

```java
public void q1(int n) {
    // Assume TreeMap<K, V> uses an automatically-balanced search tree.
    Map<Integer, Integer> map = new TreeMap<>();
    for (int i = n; i > -n; i--) {
        map.put(i, n);
        for (int j = 0; j < i; j++) {
            System.out.println(j);
        }
    }
}
```

$\Theta( )$

(B) Give the simplified worst case runtime in $\Theta(\cdot)$ notation, as a function of $n$.

```java
public int f(int n) {
    if (n < 1) {
        return 0;
    }
    for (int i = 0; i < n; i++) {
        System.out.println("hiya");
    }
    int first = 2 * f(n/3);
    int second = f(n/3);
    int third = f(n/3);
    return first + second + third;
}
```

$\Theta( )$
The following two sub-questions pertain to this code snippet:

```java
public int q3(int n) {
    int tuition = 0;
    if (n < 100000000) {
        for (int i = 0; i < n * n * n; i++) {
            tuition++;
        }
        return tuition;
    }
    if (n % 2 == 0) {
        for (int i = 0; i < n; i++) {
            tuition++;
        }
    } else {
        for (int i = 0; i < n * n; i++) {
            tuition++;
        }
    }
    return tuition;
}
```

a) What set(s) of values will trigger the **best asymptotic runtime** for this function?

b) Which of the following claims about the **overall asymptotic** runtime for this function are true? Fill in the boxes next to all choices that apply.

(Hint: Start by finding \(\Omega/\Theta/O\) for individual cases like you did for the QuickCheck. Then, try to find \(\Omega/\Theta/O\) for the overall function.)

<table>
<thead>
<tr>
<th>(\Omega(1))</th>
<th>(\Theta(1))</th>
<th>(O(1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Omega(n))</td>
<td>(\Theta(n))</td>
<td>(O(n))</td>
</tr>
<tr>
<td>(\Omega(n^2))</td>
<td>(\Theta(n^2))</td>
<td>(O(n^2))</td>
</tr>
<tr>
<td>(\Omega(n^3))</td>
<td>(\Theta(n^3))</td>
<td>(O(n^3))</td>
</tr>
</tbody>
</table>
Question 4: Sweet Heaps Are Made of This

a) Consider the following Binary Min Heap:

i) Draw the heap after inserting the value 7.

How many swaps are required during this insertion?

ii) Draw the heap after removing the minimum value. Start with the heap pictured; do not include the value 7 from part (i).

How many swaps are required during this removal?
b) Suppose we wanted to add each of the following methods to our implementation of a Binary Min Heap. For each method, briefly describe how you would implement it in no more than 2 sentences. You do not need to use specific field names as long as your meaning is clear. Then, give a lower bound for the runtime of that method in the worst case, including runtime needed to restore any heap invariants.

<table>
<thead>
<tr>
<th>i) removeSecondToMin() - Removes the second-smallest value in the heap. Does not remove the smallest value.</th>
<th>Implementation (&lt;= 2 sentences)</th>
<th>Lower-Bound Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ω( )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ii) removeArbitrary(int value) - Removes the given value from the heap (if it exists), otherwise does nothing.</th>
<th>Implementation (&lt;= 2 sentences)</th>
<th>Lower-Bound Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ω( )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>iii) checkHeapInvariants(int[] array) - Given an arbitrary array, return true if the array satisfies the heap invariants.</th>
<th>Implementation (&lt;= 2 sentences)</th>
<th>Lower-Bound Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ω( )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>iv) incrementMin() - Adds 1 to the value of the heap’s root node.</th>
<th>Implementation (&lt;= 2 sentences)</th>
<th>Lower-Bound Runtime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Ω( )</td>
</tr>
</tbody>
</table>
Question 5: Sustainability, It’s in Our Nature

Assume we have a hash table that maps characters to integers. The hash table starts with 4 buckets and resizes by doubling the number of buckets if the insert would cause the load factor to exceed 0.75. Collision is resolved with separate chaining, and each individual bucket uses linked lists.

(A) Draw an internal representation of the hash map after inserting key-value pairs:

('r', 1), ('e', 2), ('c', 3), ('y', 4), ('c', 5), ('l', 6), ('e', 7), ('d', 8)

Assume the hash code for 'c' is 2, 'd' is 3, 'e' is 4, 'l' is 11, 'r' is 17, and 'y' is 24. If a key already exists in the table, subsequent inserts with the same key will overwrite its value.

(B) What is the current load factor after all the insertions?
Question 6: Be Boundless (For Runtime / For The World)

Your friend Morgan says that they have come up with the new greatest idea for a data structure: a hash table, but instead of normal chaining (based on linked lists) they will use a Left-Leaning Red-Black tree in each bucket!

They claim that the runtimes should always be at least as good as for regular hash tables, since the asymptotic bounds are as good or better in both the best and worst cases. You want to fact check their claim.

Assuming that the hash table resizes at a reasonable load factor (for example, 1.5):

(A) Give the \textbf{best case} runtime of looking up a key in this data structure, in terms of \(N\), the number of elements in the hash table.

\[ \Theta(\ ) \]

(B) Give the \textbf{worst case} runtime of looking up a key in this data structure, in terms of \(N\), the number of elements in the hash table.

\[ \Theta(\ ) \]

(C) Is Morgan’s claim correct? Briefly justify based on your answers above (in 1-2 sentences).
Question 7: It's the End of the Exam as We Know It (and I Feel Fine)

[Extra Credit] Which do you like better: B-trees, or LLRB trees? Briefly justify by drawing a picture.