Section 05: Solutions

Section Problems

1. Ternary Heaps

Consider the following sequence of numbers:

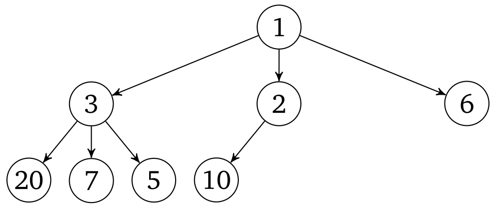
5, 20, 10, 6, 7, 3, 1, 2

(a) Insert these numbers into a min-heap where each node has up to *three* children, instead of two.

(So, instead of inserting into a binary heap, we're inserting into a ternary heap.)

Draw out the tree representation of your completed ternary heap.

Solution:



(b) Draw out the array representation of the above tree. In your array representation, you should start at index 0 (not index 1).

Solution:

1, 3, 2, 6, 20, 7, 5, 10

(c) Given a node at index i, write a formula to find the index of the parent.

Solution:

$$\text{parent}(i) = \left\lfloor \frac{i-1}{3} \right\rfloor$$

(d) Given a node at index i, write a formula to find the j-th child. Assume that $0 \leq j < 3$.

Solution:

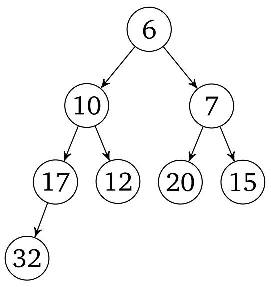
$$\text{child}(i, j) = 3i + j + 1$$

2. Heaps – More Basics

(a) Insert the following sequence of numbers into a *min heap*:

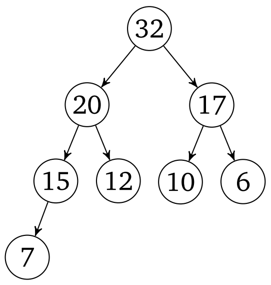
[10, 7, 15, 17, 12, 20, 6, 32]

Solution:



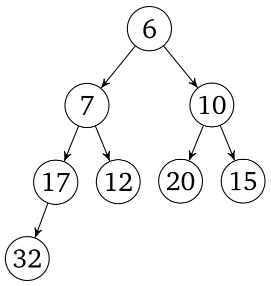
(b) Now, insert the same values into a *max heap*.

Solution:



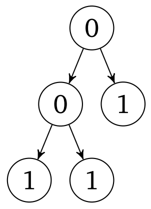
(c) Now, insert the same values into a *min heap*, but use Floyd's `buildHeap` algorithm.

Solution:



(d) Insert 1, 0, 1, 1, 0 into a *min heap*.

Solution:



(e) Call `removeMin` on the min heap stored as the following array: $[2, 5, 7, 8, 10, 9]$ **Solution:**

$[5, 8, 7, 9, 10]$
Credit: https://medium.com/@randerson112358/min-heap-deletion-step-by-step-1e05ff9d3932

3. Sorting and Reversing (with Heaps)

(a) Suppose you have an array representation of a heap. Must the array be sorted? **Solution:**

No, $[1, 2, 5, 4, 3]$ is a valid min-heap, but it isn't sorted.

(b) Suppose you have a sorted array (in increasing order). Must it be the array representation of a valid min-heap? **Solution:**

Yes! Every node appears in the array before its children, so the heap property is satisfied.

(c) You have an array representation of a min-heap. If you reverse the array, does it become an array representation of a max-heap? **Solution:**

No. For example, $[1, 2, 4, 3]$ is a valid min-heap, but if reversed it won't be a valid max-heap, because the maximum element won't appear first.

(d) Describe the most efficient algorithm you can think of to convert the array representation of a min-heap into a max-heap. What is its running time? **Solution:**

You already know an algorithm – just use `buildHeap` (with percolate modified to work for a max-heap instead of a min-heap). The running time is $\mathcal{O}(n)$.

4. Project Prep: Contains

You just finished implementing your heap of `ints` when your boss tells you to add a new method called `contains`. Your solution should not, in general, examine every element in the heap(do it recursively!)

```java
public class DankHeap {
    // NOTE: Data starts at index 0!
    private int[] heapArray;
    private int heapSize;

    // Other heap methods here....

    /**
     * examine whether element k exists in the heap
     * @param int k, the element to find.
     * @return true if found, false otherwise
     */
    public boolean contains(int k) {
        // TODO!
    }
}
```

(a) How efficient do you think you can make this method? **Solution:**

The best you can do in the worst case is $\mathcal{O}(n)$ time. If you start at the top (unlike a binary search tree) the node of priority k could be in either subtree, so you might have to check both. Even if in general we might not need to examine every node, in the worst case, this might lead us to check every node.

(b) Write code for `contains`. Remember that `heapArray` starts at index 0! **Solution:**

```java
private boolean contains(int k){
    if(k < heapArray[0]){ //if k is less than the minimum value
        return false;
    }else if (heapSize == 0){
        return false;
    }
    return containsHelper(k, 0);
}
private boolean containsHelper(int k, int index){
    if(index >= heapSize){ //if the index is larger than the heap's size
        return false;
    }
    if(heapArray[index] == k){
        return true;
    }else if(heapArray[index] < k){
        return containsHelper(k, index * 2 + 1) || containsHelper(k, index * 2 + 2);
    }else{
        return false;
    }
}
```

5. Challenge: Debugging Heaps of Problems

For this problem, we will consider a hypothetical hash table that uses linear probing and implements the `IDictionary` interface. Specifically, we will focus on analyzing and testing one potential implementation of the `remove` method.

(a) Come up with at least 4 different test cases to test this `remove(...)` method. For each test case, describe what the expected outcome is (assuming the method is implemented correctly).

Try and construct test cases that check if the `remove(...)` method is correctly using the key's hash code. (You may assume that you can construct custom key objects that let you customize the behavior of the `equals(...)` and `hashCode()` method.)

Solution:

Some examples of test cases:

- If the dictionary contains null keys, or if we pass in a null key, everything should still work correctly.
- If we try removing a key that doesn't exist, the method should throw an exception.
- If we pass in a key with a large hash value, it should mod and stay within the array.
- If we pass in two different keys that happen to have the same hash value, the method should remove the correct key.
- If we pass in a key where we need to probe in the middle of a cluster, removing that item shouldn't disrupt lookup of anything else in that cluster.

  For example, suppose the table's capacity is 10 and we pass in the integer keys 5, 15, 6, 25, 36 in that order. These keys all collide with each other, forming a primary cluster. If we delete the key 15, we should still successfully be able to look up the values corresponding to the other keys.

(b) Now, consider the following (buggy) implementation of the `remove(...)` method. List all the bugs you can find.

```
public class LinearProbingDictionary<K, V> implements IDictionary<K, V> {
    // Field invariants:
    //
    // 1. Empty, unused slots are null
    // 2. Slots that are actually being used contain an instance of a Pair object

    private Pair<K, V>[] array;

    // ...snip...

    public V remove(K key) {
        int index = key.hashCode();

        while ((this.array[index] != null) && !this.array[index].key.equals(key)) {
            index = (index + 1) % this.array.length;
        }

        if (this.array[index] == null) {
            throw new NoSuchKeyException();
        }
        V returnValue = this.array[index].value;
        this.array[index] = null;
        return returnValue;
    }
}
```

Solution:

The bugs:

- We don't mod the key's hash code at the start
- This implementation doesn't correctly handle null keys
- If the hash table is full, the while loop will never end
- This implementation does not correctly handle the "clustering" test case described up above.

  If we insert 5, 15, 6, 25, and 36 then try deleting 15, future lookups to 6, 25, and 36 will all fail.

Note: The first two bugs are, relatively speaking, trivial ones with easy fixes. The middle bug is not trivial, but we have seen many examples of how to fix this. The last bug is the most critical one and will require some thought to detect and fix.

(c) Briefly describe how you would fix these bug(s).

Solution:

- Mod the key's hash code with the array length at the start.
- Handle null keys in basically the same way we handled them in `ArrayDictionary`
- There should be a size field, with `ensureCapacity()` functionality.
- Ultimately, the problem with the "clustering" bug stems from the fact that breaking a primary cluster into two in any way will inevitably end up disrupting future lookups.

  This means that simply setting the element we want to remove to null is not a viable solution. Here are many different ways we can try and fix this issue, but here are just a few different ideas with some analysis:

  - One potential idea is to "shift" over all the elements on the right over one space to close the gap to try and keep the cluster together. However, this solution also fails.

    Consider an internal array of capacity 10. Now, suppose we try inserting keys with the hash-codes 5, 15, 7. If we remove 15 and shift the "7" over, any future lookups to 7 will end up landing on a null node and fail.
  - Rather then trying to "shift" the entire cluster over, what if we could instead just try and find a single key that could fill the gap. We can search through the cluster and try and find the very last key that, if rehashed, would end up occupying this new slot.

    If no key in the cluster would rehash to the now open slot, we can conclude it's ok to leave it null.

    This would potentially be expensive if the cluster is large, but avoids the issue with the previous solution.
  - Another common solution would be to use lazy deletion. Rather then trying to "fill" the hole, we instead modify each `Pair` object so it contains a third field named `isDeleted`.

    Now, rather then nulling that array entry, we just set that field to true and modify all of our other methods to ignore pairs that have this special flag set. When rehashing, we don't copy over these "ghost" pairs.

    This helps us keep our delete method relatively efficient, since all we need to do is to toggle a flag.

    However, this approach also does complicate the runtime analysis of our other methods (the load factor is no longer as straightforward, for example).