LEC 08

CSE 373

Hash Maps

BEFORE WE START

If the input to a function call on level i is $\left(\frac{n}{3^i}\right)$, and we have this recurrence, what level i is the base case?

$$T(n) = \begin{cases} 4 & \text{if } n \leq 1 \\ T\left(\frac{n}{3}\right) + n & \text{otherwise} \end{cases}$$

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Announcements

- EX1 (Algo Analysis I) due **TONIGHT 11:59pm PDT**
  - You can use late days on exercises, just like projects!
- P2 (Maps) and EX2 (Algo Analysis II) released today
- Don't forget to fill out the P2 Partner Form!
  - Even if you want the default, **please confirm** for us by filling it out!
  - https://courses.cs.washington.edu/courses/cse373/tools/20su/partner/p2/
- Summations Reference published (on course calendar under Wednesday's lecture)

P2: Maps

- Implement everyone's good pal: the Hash Map!
- Like P1, look at multiple data structures under a single ADT
  - But this time, we have the algorithmic analysis tools to reason about more complicated situations (especially Case Analysis!)
- 3 Parts:
  - ArrayMap
  - ChainedHashMap
  - Experiments
- Start early! In particular, the ChainedHashMap iterator can take a long time!

MAP ADT

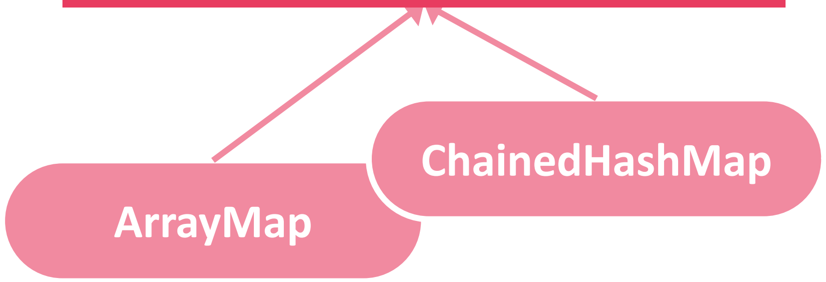
State

Set of keys, Collection of values
Count of keys

Behavior

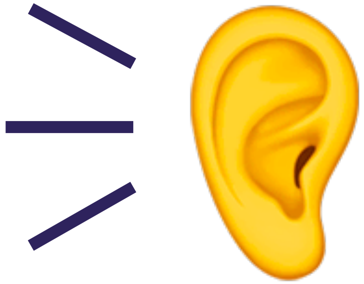
`put(key, value)` add value to collection, associated with key
`get(key)` return value associated with key
`containsKey(key)` return if key is associated
`remove(key)` remove key and associated value
`size()` return count

`clear()` remove all
`iterator()` get an iterator



Fresh from the grapevine, the latest

STUDENT FEEDBACK



- THANK YOU for letting us know how optional review questions could be more helpful for you! Don't stop here: your feedback & ideas are how we make this the best course it can be!

- Post-Lecture Optional Review Questions:
  - ***New*** We'll **publish solutions** at the same time as problems. Use however you prefer!
  - ***New*** **Reflection**: what's one conception you cleared up, or one question you still have?
  - **Extra credit**: No points, but doing lots can round up your GPA 0.1 (**completion only**, not graded on correctness)
  - **No deadline**: Complete anytime during the quarter. Recommendation: before next lecture

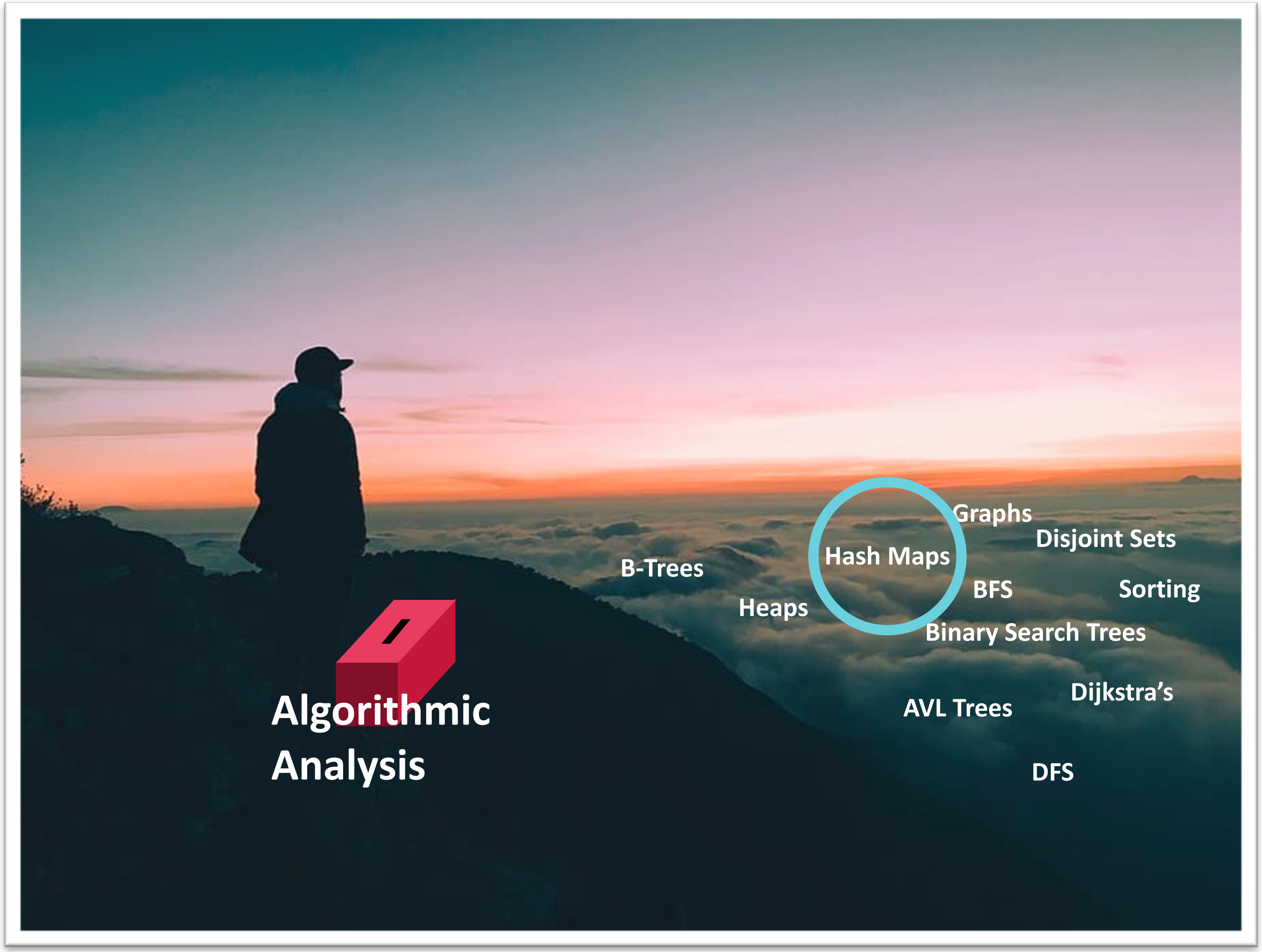
Announcements

- Regarding the fall F-1 online classes visa situation:
  - "The Allen School stands with our international students and is *vehemently opposed* to the planned visa changes that would upend lives and put people at risk during a pandemic. This action goes against our values as a school, a campus community, and a nation, and it should not stand. I want you all to know that school leadership, the University of Washington, and the broader higher education and computing communities are doing everything within our power to try to *prevent these changes* from taking effect." – Magdalena Balazinska (Director, Paul G. Allen School)
- We know this is a stressful time, and you may need flexibility to work on things that aren't this class
  - Effective immediately, **we're giving everyone two extra late days**
    - Apply to P1, EX1, whatever you need. Everyone now has 9 for the quarter.
  - P1 and EX1 late cutoffs are now **5 days after the due date**
  - Next week, we'll offer increased OH coverage and 1:1 meetings availability
- These changes are designed to give flexibility, but we know it's not a one-size-fits-all situation. **Please** reach out if you would benefit from further accommodations – this class should not be a burden as you handle more important things.

Welcome to the Data Structures Part™

- We're now armed with a toolbox stuffed full of analysis tools
  - Wednesday was the last algorithmic analysis lecture
  - It's time to apply this theory to more practical topics!

- Today, we'll take our first deep dive using those tools on a data structure: Hash Maps!



Learning Objectives

After this lecture, you should be able to...

1. Compare the relative pros/cons of various Map implementations, especially given a design like the ones we cover today
2. Trace operations in a Separate Chaining Hash Map on paper (such as insertion, getting an element, resizing)
3. Implement a Separate Chaining Hash Map in code (P2)
4. Differentiate between the "worst" and "in practice" runtimes of a Separate Chaining Hash Map, and describe what assumptions allow us to consider the "in practice" case

Lecture Outline

MAP ADT

Review

1

ArrayMap

As seen on **Project 2**

2

DirectAccessMap

FASTER: Jump directly to element, only int keys

3

SimpleHashMap

MORE FLEXIBLE: Hash function supports any type of key

4

SeparateChaining HashMap

YOUR BEST FRIEND: Addresses limitations with hash collisions, but still fast!

As seen on **Project 2**

Lecture Outline

MAP ADT

Review

1 **ArrayMap**

2 **DirectAccessMap**

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4 **SeparateChaining HashMap**

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Review The Map ADT

- **Map**: an ADT representing a set of distinct keys and a collection of values, where each key is associated with one value.
  - Also known as a **dictionary**
  - If a key is already associated with something, calling put(key, value) replaces the old value
- Used all over the place
  - It's hard to work on a big project without needing one sooner or later
  - CSE 143 introduced:
    - `Map<String, Integer> map1 = new HashMap<>();`
    - `Map<String, String> map2 = new TreeMap<>();`

MAP ADT

State

Set of keys, Collection of values
Count of keys

Behavior

`put(key, value)` add value to collection, associated with key
`get(key)` return value associated with key
`containsKey(key)` return if key is associated
`remove(key)` remove key and associated value
`size()` return count

`clear()` remove all
`iterator()` get an iterator

Review Implementing a Map with an Array

MAP ADT

State

Set of keys, Collection of values
Count of keys

Behavior

put(key, value) add value to collection, associated with key
get(key) return value associated with key
containsKey(key) return if key is associated
remove(key) remove key and associated value
size() return count

ArrayMap<K, V>

State

Pair<K, V>[] data

Behavior

put find key, overwrite value if there. Otherwise create new pair, add to next available spot, grow array if necessary
get scan all pairs looking for given key, return associated item if found
containsKey scan all pairs, return if key is found
remove scan all pairs, replace pair to be removed with last pair in collection
size return count of items in dictionary

Big-Oh Analysis – (if key is the last one looked at / not in the dictionary)

put()	O(n) linear
get()	O(n) linear
containsKey()	O(n) linear
remove()	O(n) linear
size()	O(1) constant

Big-Oh Analysis – (if the key is the first one looked at)

put()	O(1) constant
get()	O(1) constant
containsKey()	O(1) constant
remove()	O(1) constant
size()	O(1) constant

put('b', 97)
put('e', 20)

0	1	2	3	4
('a', 1)	('b',97)	('c', 3)	('d', 4)	('e',20)

Review Implementing a Map with Linked Nodes

MAP ADT

State

Set of keys, Collection of values
Count of keys

Behavior

put(key, value) add value to collection, associated with key
get(key) return value associated with key
containsKey(key) return if key is associated
remove(key) remove key and associated value
size() return count

LinkedMap<K, V>

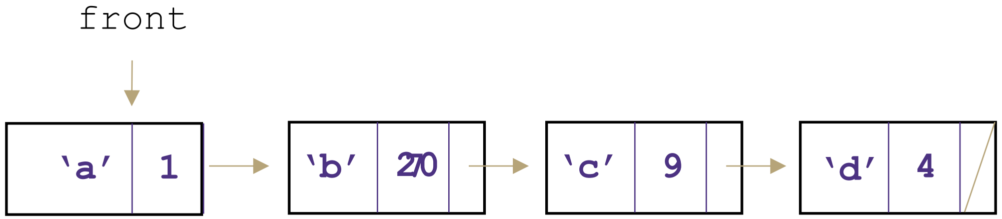
State

front
size

Behavior

put if key is unused, create new with pair, add to front of list, else replace with new value
get scan all pairs looking for given key, return associated item if found
containsKey scan all pairs, return if key is found
remove scan all pairs, skip pair to be removed
size return count of items in dictionary

```
containsKey('c')
get('d')
put('b', 20)
```



Big O Analysis – (if key is the last one looked at / not in the dictionary)

put()	O(n) linear
get()	O(n) linear
containsKey()	O(n) linear
remove()	O(n) linear
size()	O(1) constant

Big O Analysis – (if the key is the first one looked at)

put()	O(1) constant
get()	O(1) constant
containsKey()	O(1) constant
remove()	O(1) constant
size()	O(1) constant

Could we do better?

- `put`, `get`, and `remove` have Θ(n) runtimes. Could we use a Θ(1) operation to improve?
- What about array indexing?
  - data[i] (array access) and data[i] = 2 (array update) are constant runtime!
  - What if we could jump directly to the requested key?
  - We could simplify the problem: **only allow integer keys**

Lecture Outline

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DirectAccessMap

- put, get, and remove have Θ(n) runtimes. Could we use a Θ(1) operation to improve?
- What about array indexing?
  - data[i] (array access) and data[i] = 2 (array update) are constant runtime!
  - What if we could jump directly to the requested key?
  - We could simplify the problem: **only allow integer keys**

```
put(3, "Melissa")

get(3)
```

index	0	1	2	3	4	5	6	7	8	9
data				Melissa						

DirectAccessMap<K, V>

State
- data[]
- size

Behavior
- put put item at given index
- get get item at given index
- containsKey if data[] null at index, return false, return true otherwise
- remove nullify element at index
- size return count of items in dictionary

DirectAccessMap Implementation

```
public void put(int key, V value) {
    this.array[key] = value;
}

public boolean containsKey(int key) {
    return this.array[key] != null;
}

public V get(int key) {
     return this.array[key];
}

public void remove(int key) {
    this.array[key] = null;
}
```

DirectAccessMap<K, V>

State

data[]
size

Behavior

put put item at given index
get get item at given index
containsKey if data[] null at index, return false, return true otherwise
remove nullify element at index
size return count of items in dictionary

Operation	Case	Runtime
put(key,value)	best	$\Theta(1)$
	worst	$\Theta(1)$
get(key)	best	$\Theta(1)$
	worst	$\Theta(1)$
containsKey(key)	best	$\Theta(1)$
	worst	$\Theta(1)$

Poll Everywhere

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Pros and Cons of DirectAccessMap

What's a benefit of using it? What's a drawback?

Pros and Cons of DirectAccessMap

Top

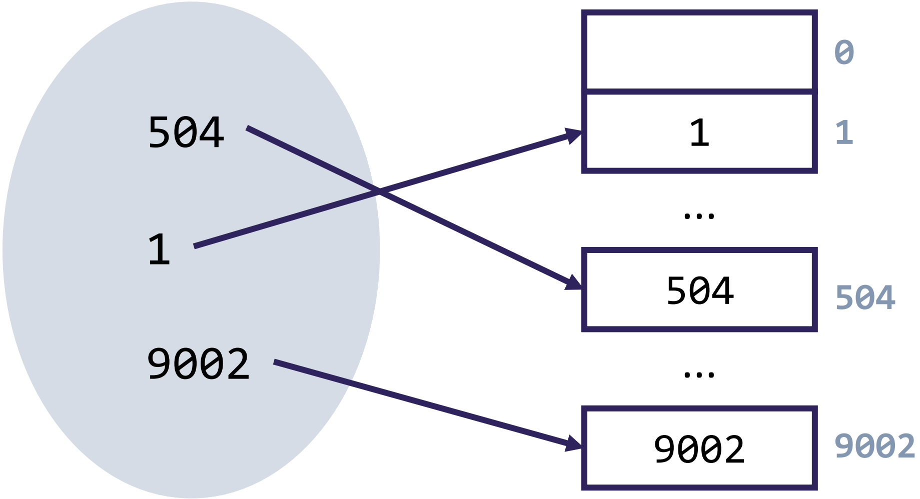
Pros and Cons of DirectAccessMap

- Super Fast!
  - Everything is $\Theta(1)$
- Wasted Space
  - Say we want to store 0 and 999999999. This implementation would waste all the space inbetween ☹
- Only Integer Keys
  - Would be nice to store any type of data ☹
  - But note what's so useful here: being able to go quickly from key to array index

Can We Store Any Integer?

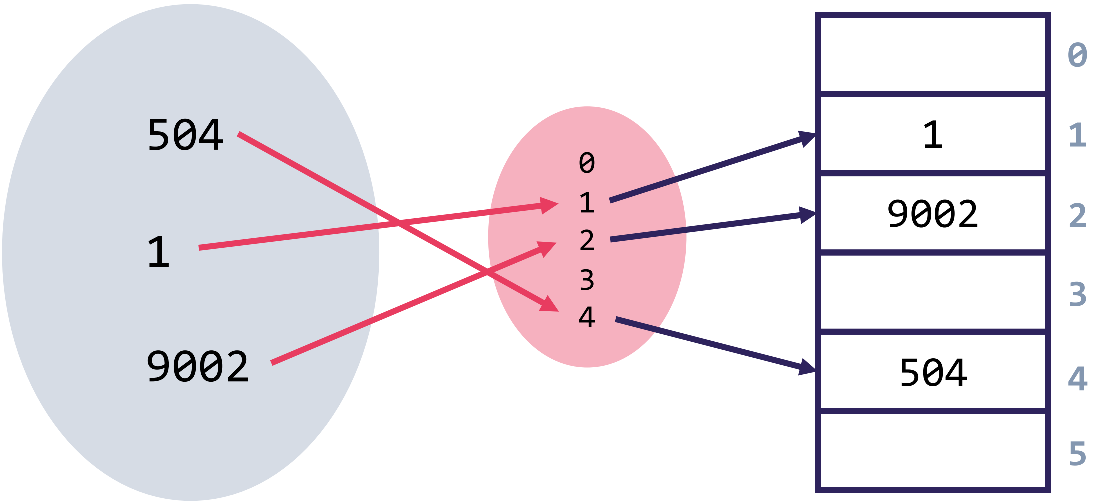
IDEA 1

- Create a GIANT array with every possible integer as an index
- Problems:
  - Can we allocate an array big enough?
  - Super wasteful



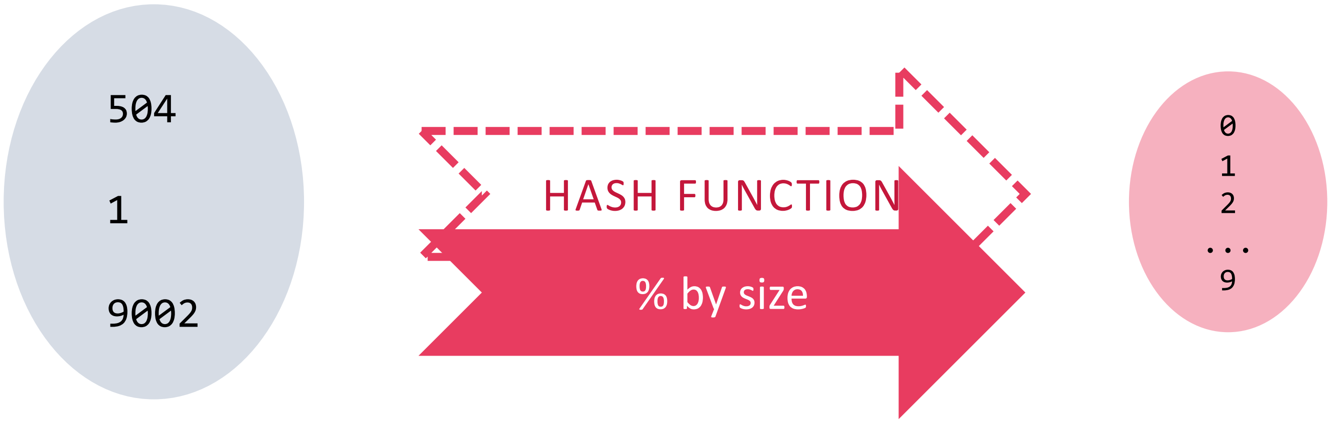
IDEA 2

- Create a smaller array, with a translation from integer keys into available indices
- Problems:
  - How can we construct a translation?



Hash Functions

- **Hash Function**: any function that can be used to map data of an arbitrary size to fixed-size values.
  - We want to translate from the set of all integers to the set of valid indexes in our array



9002 % 10 = 2 (so store it in index 2 of the array)

- One simple approach: take the key and % (mod) it by size of the array

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Mod: Remainder

- The `%` operator computes the remainder from integer division.

`14 % 4` is `2` **`218 % 5` is `3`**

```
      3
4 ) 14
    12
     2
```

```
       43
5 ) 218
    20
     18
     15
      3
```

Equivalently, to find `a % b` (for `a,b > 0`):

```
while(a > b-1)
      a -= b;
return a;
```

- Applications of `%` operator:
  - Obtain last digit of a number: `230857 % 10` is `7`
  - See whether a number is odd: `7 % 2` is `1`, `42 % 2` is `0`
  - Limit integers to specific range: `8 % 12` is `8`, `18 % 12` is `6`

Limit keys to indices within array

For more review/practice, check out https://www.khanacademy.org/computing/computer-science/cryptography/modarithmetic/a/what-is-modular-arithmetic

SimpleHashMap: "% by size" as Hash Function

```
put(0, "I")        0 % 10 = 0
put(8, "Maps")     8 % 10 = 8
put(11, "<3")     11 % 10 = 1
put(23, "Hash")   23 % 10 = 3
```

IMPLEMENTATION

```
public void put(int key, int value) {
    data[hashToValidIndex(key)] = value;
}

public V get(int key) {
    return data[hashToValidIndex(key)];
}

public int hashToValidIndex(int k) {
   return k % this.data.length;
}
```

index	0	1	2	3	4	5	6	7	8	9
data	I	<3		Hash					Maps	

SimpleHashMap: Collisions?!

```
put(0, “I”)        0 % 10 = 0
put(8, “Maps”)     8 % 10 = 8
put(11, “<3”)     11 % 10 = 1
put(23, “Hash”)   23 % 10 = 3
put(20, “We”)     20 % 10 = 0
```

IMPLEMENTATION

```
public void put(int key, int value) {
    data[hashToValidIndex(key)] = value;
}

public V get(int key) {
    return data[hashToValidIndex(key)];
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public int hashToValidIndex(int k) {
   return k % this.data.length;
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Handling Collisions

- Two common strategies to handle collisions:

1. Separate Chaining

”Chain” together multiple values stored in a single bucket

2. Open Addressing

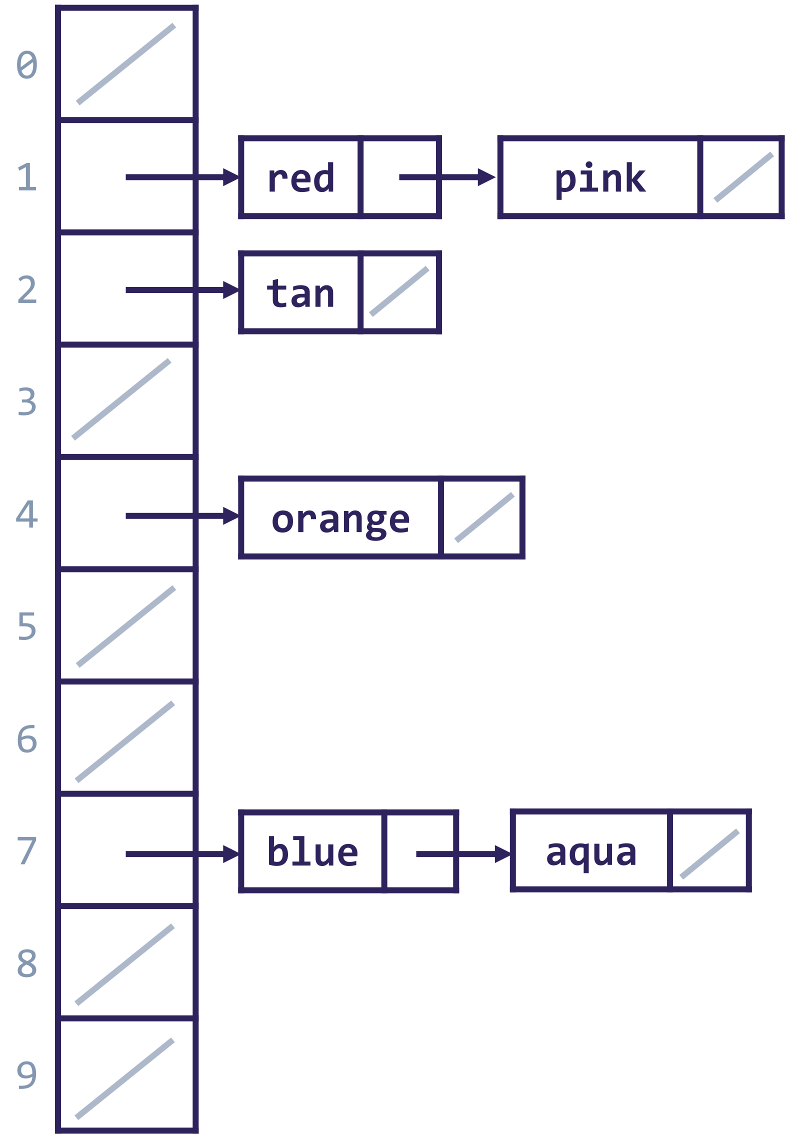
If a bucket is taken, find a new bucket using some strategy:
Linear Probing
Quadratic Probing
Double Hashing

We’ll focus on separate chaining this quarter, much more common in practice

Bonus topic beyond the scope of the class

Separate Chaining

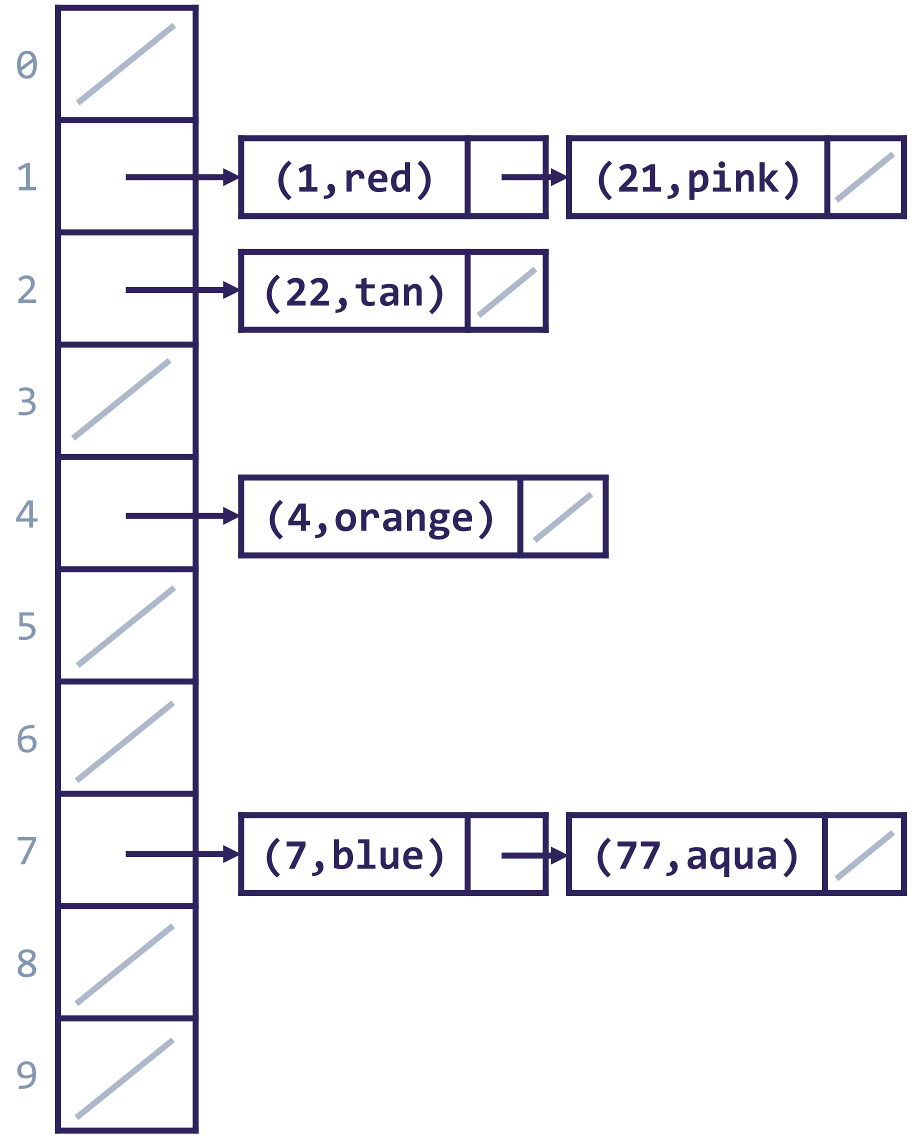
- If two values want to live in the same index, let's just let them be roommates!
- Each index is a "bucket"
  - Linked Nodes are a common implementation for these bucket "chains"
- When item x hashes to index h:
  - If bucket at h is empty, create new list with x
  - Else, add x to the list



Separate Chaining

- If two values want to live in the same index, let's just let them be roommates!
- Each index is a "bucket"
  - Linked Nodes are a common implementation for these bucket "chains"
- When item x hashes to index h:
  - If bucket at h is empty, create new list with x
  - Else, add x to the list

- But if multiple keys can hash to the same index, need to store the key too!



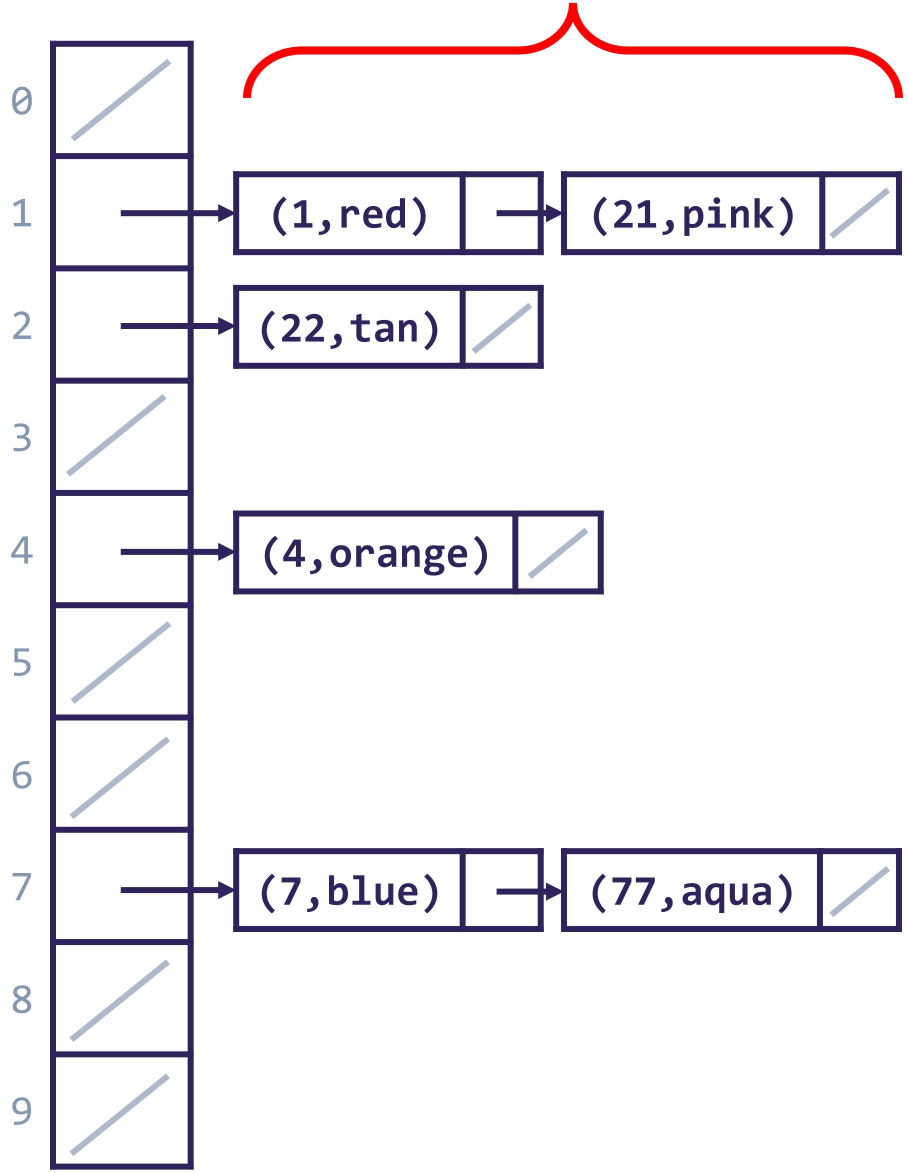
Separate Chaining

- Implementation of get/put/containsKey very similar

PSEUDOCODE

```
public boolean get(int key) {
    int bucketIndex = key % data.length;
    loop through each pair in data[bucketIndex]
        if pair.key == key
            return pair.value
    return null if we get here
}
```

Let's analyze the runtime. First, are there different possible states for this HashMap to make the code faster or slower, assuming n key/value pairs are already stored?



Separate Chaining Worst Case

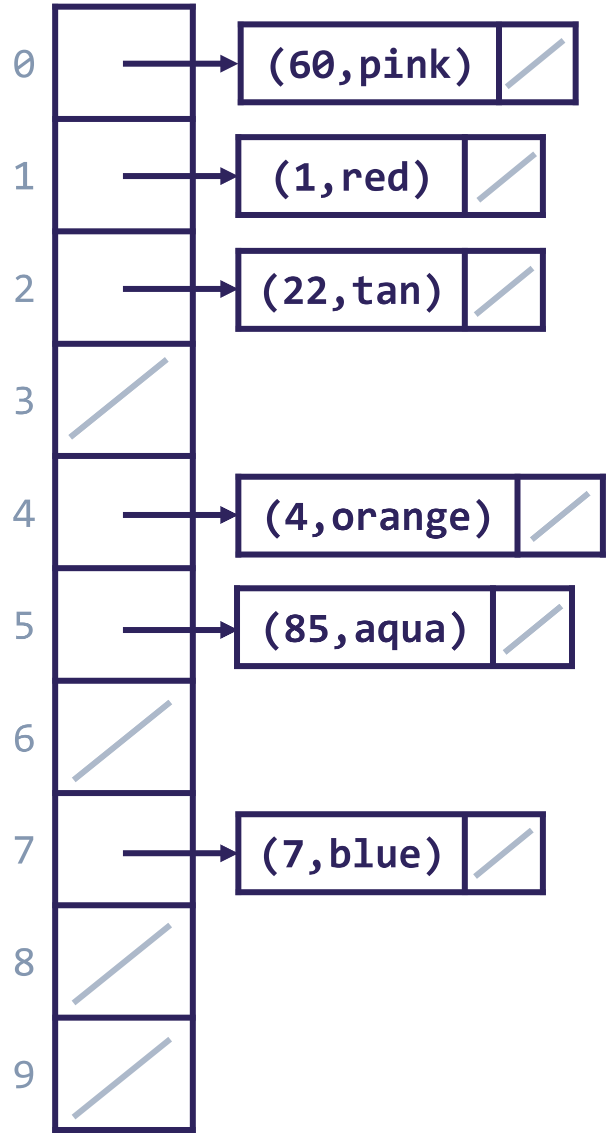
0
1 → (1,red) → (21,pink) → (11,tan) → (91,orange) → (41,aqua) → (51,blue)
2
3
4
5
6
7
8
9

- It’s possible that everything hashes to the same bucket by chance!
  - get would take Θ(n) time ☹
- Consider `get(51)`
  - Use hash function (% 10) to get index (5)
  - Check every element in bucket for key 51
- We’ve lost that Θ(1) runtime

PSEUDOCODE

```
public boolean get(int key) {
    int bucketIndex = key % data.length;
    loop through each pair in data[bucketIndex]
        if pair.key == key
            return pair.value
    return null if we get here
}
```

Separate Chaining Best Case



- However, if everything is spread evenly across the buckets, get takes Θ(1)
- Consider `get(22)`
  - Use hash function (% 10) to get index (2)
  - Check the single element in bucket for key 22 – a constant time operation!
- Key to a successful Hash Map implementation: how can we keep the buckets as close to this distribution as possible?

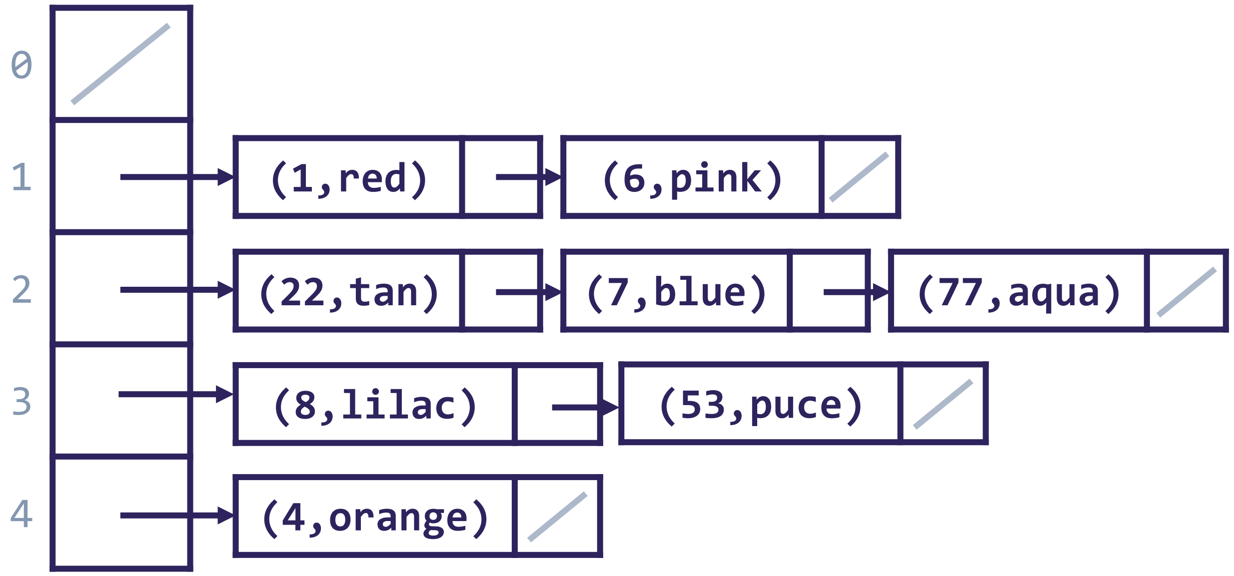
Separate Chaining... In Practice

- A well-implemented separate chaining hash map will stay very close to the best case
  - Most of the time, operations are fast. Rarely, do an expensive operation that restores the map close to best case.
- How to stay close to best case?
  - Good distribution & Resizing!
- We can describe the “in-practice” case as what *almost always* happens:
  - (1) items are fairly evenly distributed
  - (2) assume resizing doesn’t occur
    - This is *similar* to the concept of “amortized”

Operation	Case	Runtime
`put(key,value)`	best	$\Theta(1)$
	In-practice	$\Theta(1)$
	worst	$\Theta(n)$
`get(key)`	In-practice	$\Theta(1)$
	average	$\Theta(1)$
	worst	$\Theta(n)$
`remove(key)`	best	$\Theta(1)$
	In-practice	$\Theta(1)$
	worst	$\Theta(n)$

Resizing

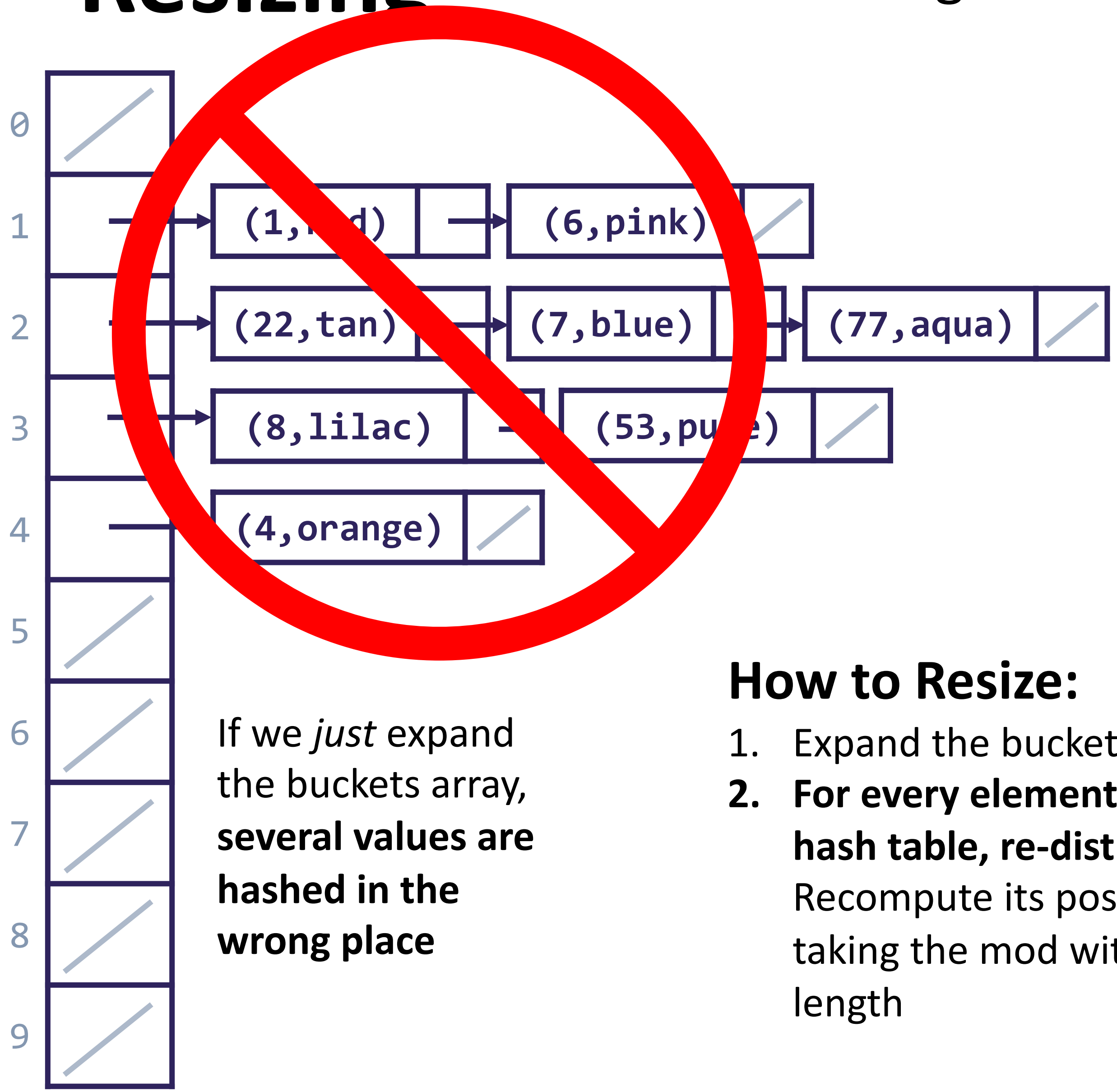
- The runtime to scan each bucket is creeping up
  - If we don't intervene, our in-practice runtime is going to hit Θ(n)
    - number of buckets is a constant, so n / (# buckets) is Θ(n)



Resizing

Don't forget to re-distribute your keys!

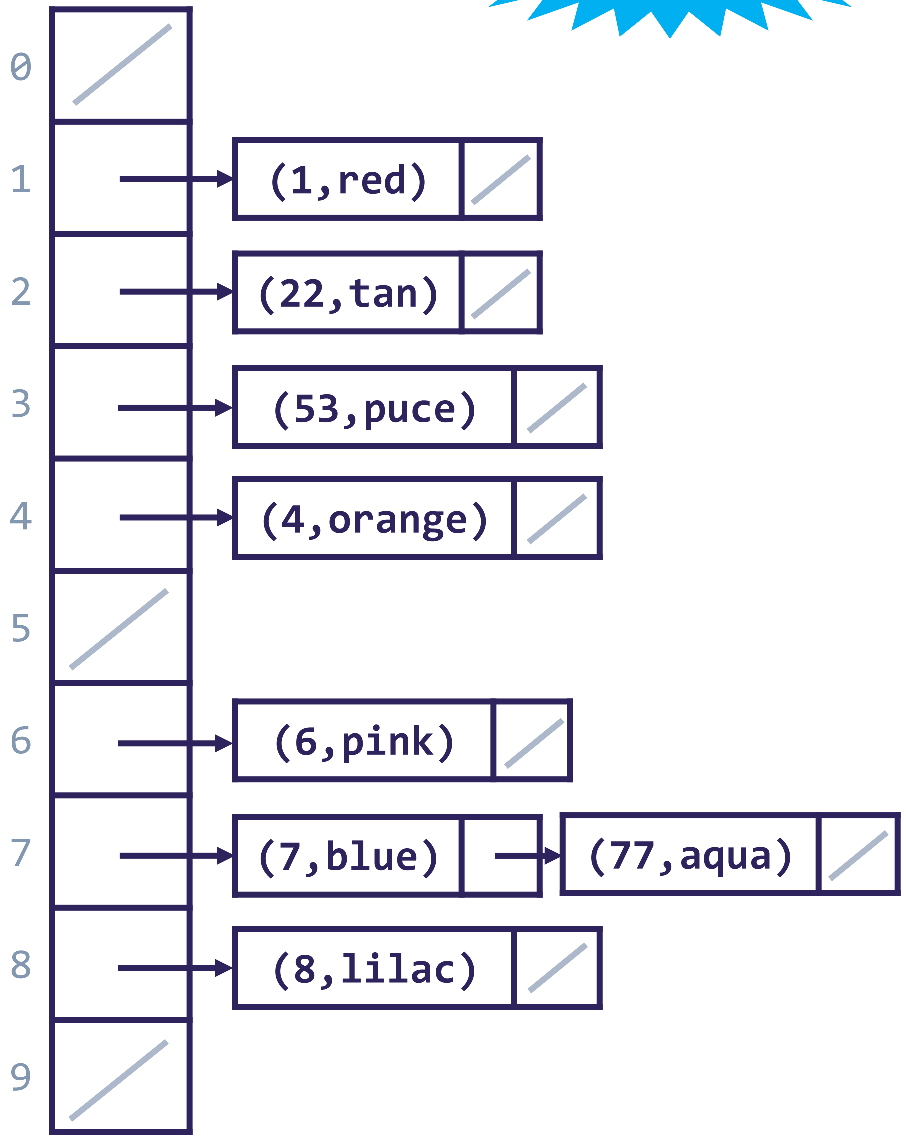
As seen on **Project 2**



If we *just* expand the buckets array, **several values are hashed in the wrong place**

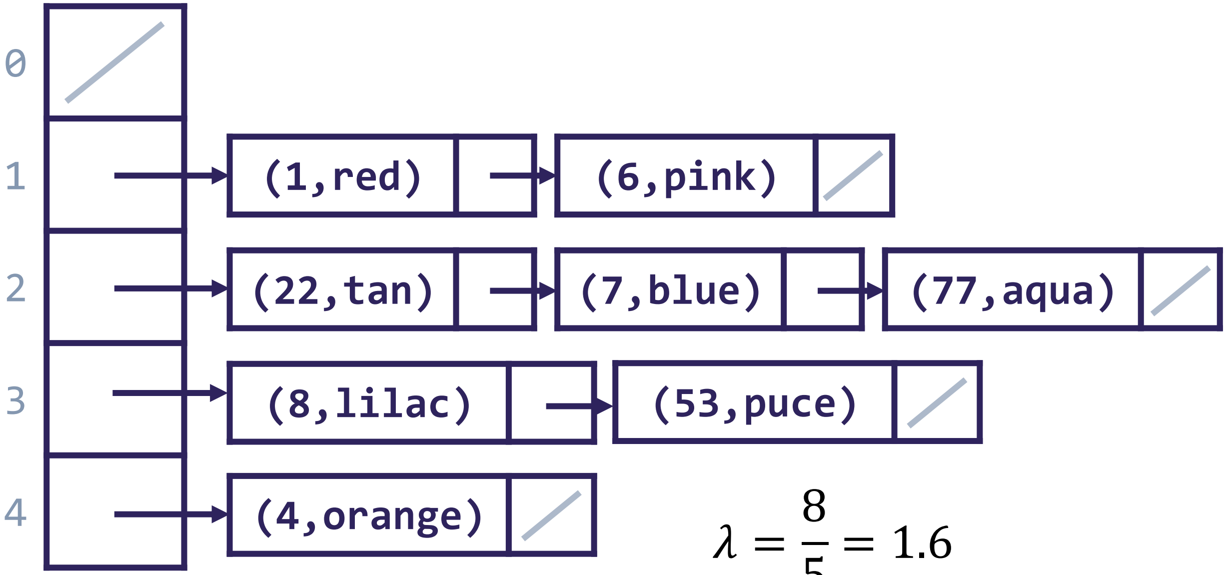
How to Resize:

1. Expand the buckets array
2. **For every element in the old hash table, re-distribute**! Recompute its position by taking the mod with the new length



When to Resize?

- In ArrayList, we were forced to resize when we ran out of room
  - In SeparateChainingHashMap, never *forced* to resize, but we want to make sure the buckets don't get too long for good runtime
- How do we quantify "too full"?
  - Look at the average bucket size: number of elements / number of buckets



$$\lambda = \frac{8}{5} = 1.6$$

LOAD FACTOR λ

n: total number of key/value pairs
c: capacity of the array (# of buckets)

$$\lambda = \frac{n}{c}$$

When to Resize?

- In ArrayList, we were forced to resize when we ran out of room
  - In SeparateChainingHashMap, never *forced* to resize, but we want to make sure the buckets don't get too long for good runtime
- How do we quantify "too full"?
  - Look at the average bucket size: number of elements / number of buckets
- If we resize when λ hits some *constant* value like 1:
  - We expect to see 1 element per bucket: **constant runtime**!
  - If we double the capacity each time, the expensive resize operation becomes less and less frequent

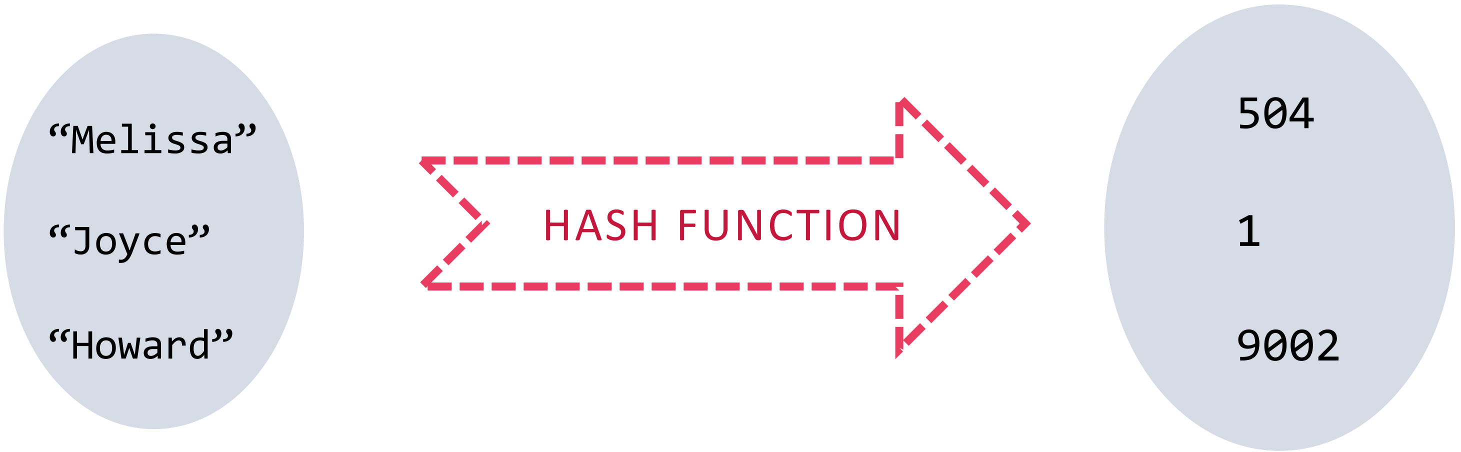
LOAD FACTOR λ

n: total number of key/value pairs
c: capacity of the array (# of buckets)

$$\lambda = \frac{n}{c}$$

Hashing

- What about non-integer data?
  - Remember the definition -- **Hash Function**: any function that can be used to map data of an arbitrary size to fixed-size values.



- Considerations for Hash Functions:
  1. **Deterministic** – same input should generate the same output
  2. **Efficient** – reasonable runtime
  3. **Uniform** – inputs spread "evenly" across output range

Hashing

Implementation 1: Simple aspect of values

```
public int hashCode(String input) {
   return input.length();
}
```

Pro: super fast
Con: lots of collisions!

Implementation 2: More aspects of value

```
public int hashCode(String input) {
   int output = 0;
   for(char c : input) {
      out += (int)c;
   }
   return output;
}
```

Pro: still really fast
Con: some collisions

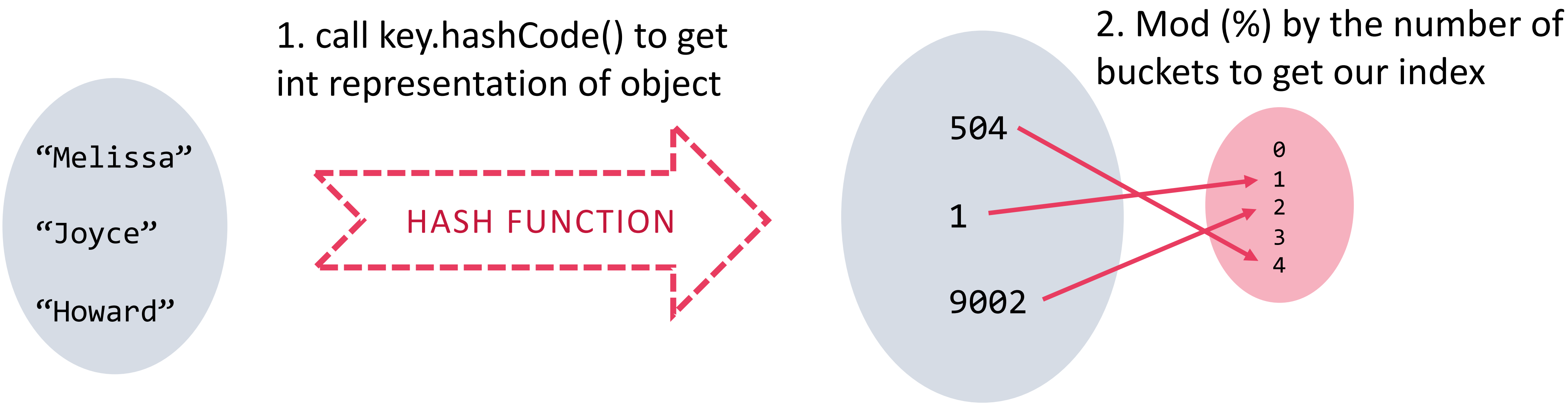
Implementation 3: Multiple aspects of value + math!

```
public int hashCode(String input) {
   int output = 1;
   for (char c : input) {
      int nextPrime = getNextPrime();
      out *= Math.pow(nextPrime, (int)c);
    }
   return Math.pow(nextPrime, input.length());
}
```

Pro: few collisions
Con: slower, gigantic integers

Hashing

- Fortunately, experts have made most of these design decisions for us!
  - All objects in Java have a `.hashCode()` method that does some magic to make a "good" hash for any object type (e.g. String, ArrayList, Scanner)
  - The built-in hashCode() has a good distribution/not a lot of collisions
- More precisely, hashCode() just gets us an int representation: *then* we % by size



Review Iterators

- **Iterator**: a Java interface that dictates how a collection of data should be traversed. Can only move forward and in a single pass.

Iterator Interface

Behavior

`hasNext()` – true if elements remain
`next()` – returns next element

hasNext() – returns true if the iteration has more elements yet to be examined

next() – returns the next element in the iteration and moves the iterator forward to next item

Two ways to *use* an iterator in Java:

```
ArrayList<Integer> list;

Iterator itr = list.iterator();
while (itr.hasNext()) {
   int item = itr.next();
}
```

```
ArrayList<Integer> list;

for (int i : list) {
   int item = i;
}
```

P2 Reminders

- Implementing an iterator for a Hash Map is *complex*!
  - You need to iterate through the elements of a bucket, but when you reach the end of the chain, *have to move to the next bucket*
  - "you're not iterating over some linear data structure, you're playing 2D chess"
    – Howard Xiao
- Start early! P2 available for over 1.5 weeks, but for good reason!
  - Especially the ChainedHashMap iterator
- Remember to read the entire Tips section of the instructions!