Lecture 18: Disjoint Sets
Administrivia

Project 4 Due Wednesday May 20th

Exercise 4 due Friday May 15th

Grades Posted
- Exercise 1
- Project 0
- Project 1
- Project 2
- Project 3
- Midterm 1

Zach & Kasey Grade Office hours
- Today, Monday May 11 9:30-11:30am PDT
- Tuesday May 12 5:45-7:15pm PDT
Roadmap

- Disjoint Sets ADT
- Context, examples
- Different implementations (most of them are just optimizations of the previous):
  1. QuickFind implementation (HashMap based)
  2. QuickUnionTrees
  3. QuickUnionBySizeTrees
  4. QuickUnionBySizeCompressingTrees
  5. ArrayQuickUnionBySizeCompressing
DisjointSets

- are a cool recap of topics / touches on a bunch of different things we’ve seen in this course (trees, arrays, graphs, optimizing runtime, etc.)

- have a lot of details and is fairly complex – it doesn’t seem like a plus at first, but after you learn this / while you’re learning this...you’ve come along way since lists and being able to learn new complex data structures is a great skill to have built)

DisjointSets 😍😍 is Zach’s favorite ADT/data structure that we talk about in this class (if you ever need to bribe Zach just start a conversation w zach about DisjointSets)
Disjoint Sets in mathematics

- “In mathematics, two sets are said to be disjoint sets if they have no element in common.” - Wikipedia

- disjoint = not overlapping

These two sets are disjoint sets

These two sets are not disjoint sets
Disjoint Sets in computer science

In computer science, disjoint sets can refer to this ADT/data structure that keeps track of the multiple “mini” sets that are disjoint (confusing naming, I know)

This overall grey blob thing is the actual disjoint sets, and it’s keeping track of any number of mini-sets, which are all disjoint (the mini sets have no overlapping values).

Note: this might feel really different than ADTs we’ve run into before. The ADTs we’ve seen before (dictionaries, lists, sets, etc.) just store values directly. But the Disjoint Set ADT is particularly interested in letting you group your values into sets and keep track of which particular set your values are in.
DisjointSets ADT methods

Just 3 methods (and makeSet is pretty simple!)

- findSet(value)
- union(valueA, valueB)
- makeSet(value)
**findSet(value)**

`findSet(value)` returns some ID for which particular set the value is in. For Disjoint Sets, we often call this the **representative** (as it’s a value that represents the whole set).

Examples:

- `findSet(Brian)`  
  - 3

- `findSet(Sherdil)`  
  - 2

- `findSet(Velocity)`  
  - 2

- `findSet(Kevin) == findSet(Aileen)`  
  - true
**union(valueA, valueB)**

**union(valueA, valueB)** merges the set that A is in with the set that B is in. (basically add the two sets together into one)

Example: `union(Blarry, Brian)`
**makeSet(value)**

`makeSet(value)` makes a new mini set that just has the value parameter in it.

**Examples:**
- `makeSet(Elena)
- `makeSet(Anish)`
Disjoint Sets ADT Summary

<table>
<thead>
<tr>
<th>Disjoint-Sets ADT</th>
</tr>
</thead>
</table>

**state**

- **Set of Sets**
- **Mini sets are disjoint**: Elements must be unique across mini sets
- No required order
- Each set has id/representative

**behavior**

- **makeSet(value)** – creates a new set within the disjoint set where the only member is the value. Picks id/representative for set
- **findSet(value)** – looks up the set containing the value, returns id/representative of that set
- **union(x, y)** – looks up set containing x and set containing y, combines two sets into one. All of the values of one set are added to the other, and the now empty set goes away.
Roadmap

- Disjoint Sets ADT
- Context, examples
- Different implementations (most of them are just optimizations of the previous)! The data structures we talk about are going to be highly optimized for these interesting ADT methods we just talked about!
  1. QuickFind implementation (HashMap based)
  2. QuickUnionTrees
  3. QuickUnionBySizeTrees
  4. QuickUnionBySizeCompressingTrees
  5. ArrayQuickUnionBySizeCompressing

CSE 373 SP 18 - KASEY CHAMPION
Imagine that Dubs, Kasey, Zach, and Ana Mari are all running for president, and are currently dominating one region of the country in the polls and have secured that these states will vote for these candidates.

You could imagine that some common questions about this data is:

- what happens when Zach has to resign and wants his supporters to now join Kasey’s campaign, and Ana Mari does the same for dubs? How do we keep track of the followers all moving over to a different group?
- You can imagine we track the voters \( \leftrightarrow \) nominee data with disjoint sets
Kruskal’s Algorithm Implementation

KruskalMST(Graph G)

initialize each vertex to be an independent component
sort the edges by weight
foreach(edge (u, v) in sorted order){
    if(u and v are in different components){
        update u and v to be in the same component
        add (u,v) to the MST
    }
}

KruskalMST(Graph G)

foreach (V : G.vertices) {
    makeSet(v);
}
sort the edges by weight
foreach(edge (u, v) in sorted order){
    if(findSet(v) is not the same as findSet(u))
        union(u, v)
        add (u, v) to the MST
}


Kruskal’s with disjoint sets on the side example

KruskalMST(Graph G)
  foreach (V : G.vertices) {
    makeSet(v);
  }
  sort the edges by weight
  foreach(edge (u, v) in sorted order) {
    if(findSet(v) is not the same as findSet(u)) {
      union(u, v)
    }
  }
Why are we doing this again? (continued)

Disjoint Sets help us manage groups of distinct values.

This is a common idea in graphs, where we want to keep track of different connected components of a graph.

In Kruskal’s, if each connected-so-far-island of the graph is its own mini set in our disjoint set, we can easily check that we don’t introduce cycles. If we’re considering a new edge, we just check that the two vertices of that edge are in different mini sets by calling findSet.
1 min break for questions / review your notes
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1. QuickFind implementation

Calculate the worst case Big-Theta runtimes for each of the methods (makeSet, findSet, union) for this QuickFind implementation of a DisjointSets.

map of value -> set ID/representative

<table>
<thead>
<tr>
<th>Value</th>
<th>Set ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sheril</td>
<td>1</td>
</tr>
<tr>
<td>Robbie</td>
<td>2</td>
</tr>
<tr>
<td>Sarah</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>method</th>
<th>runtime (assume using HashMap)</th>
</tr>
</thead>
<tbody>
<tr>
<td>makeSet(value)</td>
<td></td>
</tr>
<tr>
<td>findSet(value)</td>
<td></td>
</tr>
<tr>
<td>union(valueA, valueB)</td>
<td></td>
</tr>
</tbody>
</table>
1. QuickFind implementation

Calculate the worst case Big-Theta runtimes for each of the methods (makeSet, findSet, union) for this QuickFind implementation of a DisjointSets.

<table>
<thead>
<tr>
<th>name</th>
<th>setID</th>
<th>method</th>
<th>runtime (with HashMaps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sheril</td>
<td>1</td>
<td>makeSet(value)</td>
<td>O(1)</td>
</tr>
<tr>
<td>Robbie</td>
<td>2</td>
<td>findSet(value)</td>
<td>O(1)</td>
</tr>
<tr>
<td>Sarah</td>
<td>1</td>
<td>union(valueA, valueB)</td>
<td>O(n)</td>
</tr>
</tbody>
</table>
2. QuickUnionTrees implementation

Each mini-set is now represented as a separate tree. If values are somehow connected / in the same tree, they’re in the same mini-set!

(Note: unlike other trees we’ve seen before, the arrows go upwards! We’ll see this is useful so all nodes can access the root)

```
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
</tr>
<tr>
<td>b</td>
<td>2</td>
</tr>
<tr>
<td>c</td>
<td>1</td>
</tr>
<tr>
<td>d</td>
<td>1</td>
</tr>
<tr>
<td>e</td>
<td>2</td>
</tr>
</tbody>
</table>
```
2. QuickUnionTrees implementation:

`findSet(valueA)`

`findSet` has to be different though ...

They all have access to the root node because all the links point up – we can use the root node as our id / representative.

```
findSet(valueA) {
    jump to valueA node
    travel upwards till root
    return ID for set (in this case the node itself)
}
```

`findSet(5) == 1 node`

`findSet(9) == 9 node`

They’re in the same set because they have the same representative!
jumping to nodes:

You can use a Map<\text{T}, \text{Node}> to jump to each node easily (so even though it’s not drawn on the future slides, assume we can just jump to any node)
2. QuickUnionTrees implementation:

union(valueA, valueB)

union(valueA, valueB) -- the method with the problem runtime from before -- should look a lot easier in terms of updating the data structure – all we have to do is change one pointer so they’re connected!

What should we change? If we change the root of one to point to the other tree, then all the lower nodes in the tree will be updated to be in the same set. It turns out it will be most efficient if we have the root point to the other tree’s root so we can connect all of the values at once and keep a low height (for findSet)

union(valueA, valueB) {
    rootA = findSet(valueA)
    rootB = findSet(valueB)
    set rootA to point to rootB
}
2. QuickUnionTrees implementation:

union(valueA, valueB)

Note: we need to change one of the roots to point to the tree for correctness. If we did union(2, 11) for example and just set the 2 node to point directly to 11... what’s wrong about this picture?

badUnion1(valueA, valueB) {
    rootA = findSet(valueA)
    rootB = findSet(valueB)
    set rootA to point to rootB
}

original union(valueA, valueB) {
    rootA = findSet(valueA)
    rootB = findSet(valueB)
    set rootA to point to rootB
}
2. QuickUnionTrees implementation:

```plaintext
union(valueA, valueB)
```

What about if we did the other way around, what happens? It’s a little bit inefficient for future calls! Try badUnion2(11, 9) – what does this do to future findSets?

```plaintext
original union(valueA, valueB) {
  rootA = findSet(valueA)
  rootB = findSet(valueB)
  set rootA to point to rootB
}

badUnion2(valueA, valueB) {
  rootA = findSet(valueA)
  rootB = findSet(valueB)
  set rootA to point to valueB
}
```
union(valueA, valueB) why modify the roots

summary

Recap:

What should we change? If we change the root of one to point to the other tree, then all the lower nodes in the tree will be updated to be in the same set. It turns out it will be most efficient if we have the root point to the other tree’s root so we can connect all of the values at once and keep a low height (for findSet).

See the previous slides for the visual examples!
Questions break

QuickFind implementation
QuickUnionTrees
- union
- find
Roadmap

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Let’s try to construct a worst-case scenario 😊

makeSet(a)

makeSet(b)

makeSet(c)

makeSet(d)

makeSet(e)

union(valueA, valueB) {
    rootA = findSet(valueA)
    rootB = findSet(valueB)
    set rootA to point to rootB
}

Take 1 min to figure out how: what is a worst case scenario for QuickUnionTrees’s findSet/union runtime? What type of union()s do we need to call to produce this?
Let’s try to construct a worst-case scenario 😊

makeSet(a)  
makeSet(b)  
makeSet(c)  
makeSet(d)  
makeSet(e)

union(valueA, valueB) {
    rootA = findSet(valueA)  
    rootB = findSet(valueB)  
    set rootA to point to rootB
}

union(e, d)  
union(d, c)  
union(c, b)  
union(b, a)  
findSet(e)

Take 1 min to figure out how: what is a worst case scenario for QuickUnionTrees’s findSet/union runtime?   What type of union()s do we need to call to produce this?
3. QuickUnionBySizeTrees

**Problem:** Trees can be unbalanced (and look linked-list-like) so our findSet runtime can be linear runtime in the worst case (if it’s linked-list like and we findSet a node towards the bottom of the linked list)

**Solution:** When union’ing, choose the parent to be the bigger tree

- have the root of each mini-set tree store that tree’s size
- **When union’ing make the tree with larger size the root** (If it’s a tie, pick one arbitrarily)
- increase the size of the new mini-set as appropriate
3. QuickUnionBySizeTrees

Solution: When union’ing, choose the parent to be the bigger tree
- have the root of each mini-set tree store that tree’s size
- When union’ing make the tree with larger size the root (If it’s a tie, pick one arbitrarily)
- increase the size of the new mini-set as appropriate

possible without union by size

with union by size
3. QuickUnionBySizeTrees worst case heights

Consider the worst case where the tree height grows as fast as possible for the number of nodes it has

<table>
<thead>
<tr>
<th># nodes</th>
<th>height</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

a
3. QuickUnionBySizeTrees worst case heights

Consider the worst case where the tree height grows as fast as possible for the number of nodes it has.

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<tbody>
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<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

```
 a
  
 b
```
3. QuickUnionBySizeTrees worst case heights

Consider the worst case where the tree height grows as fast as possible for the number of nodes it has.

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<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>
3. QuickUnionBySizeTrees worst case heights

Consider the worst case where the tree height grows as fast as possible for the number of nodes it has.

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<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

![Diagram showing tree structures and heights]
Consider the worst case where the tree height grows as fast as possible for the number of nodes it has.

<table>
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<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

```
3. QuickUnionBySizeTrees worst case heights

Consider the worst case where the tree height grows as fast as possible for the number of nodes it has.

```
```
3. QuickUnionBySizeTrees worst case heights

Consider the worst case where the tree height grows as fast as possible for the number of nodes it has.

Worst case tree height is $\Theta(\log N)$

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
</tr>
</tbody>
</table>
3. QuickUnionBySizeTrees bad situations are still bounded by the worst case heights

union(a, e) – which one becomes the parent when doing union-by-size?

a will point to e because a’s tree size is 4, but e’s tree size is 6. The height increases by one even though it didn’t need to! If we had e point to a the height (the max distance) would have stayed the same.

Why not use the height of the tree?

QuickUnionByHeightTrees runtime is asymptotically the same: \( \Theta(\log(N)) \)

It’s easier to track weights than heights

main point of this slide: QuickUnionBySizeTrees produces a suboptimal structures, such as this one, in specific cases. But for the most part it works out as you increase the number of nodes towards infinity.
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Modifying Data Structures To Preserve Invariants

Thus far, the modifications we’ve studied are designed to preserve invariants (aka “repair the data structure”)

- **Tree rotations**: preserve AVL height invariant so we guarantee log(n) height and log(n) runtime for worst case if we need to traverse to the bottom of the tree
- **Heap percolations**: preserve heap sorted invariants so we can find Min/Max still in constant time

Notably, the modifications don’t improve runtime between identical method calls

Path compression is entirely different: we are modifying the tree structure to improve future performance
This is the worst-case topology if we use WeightedQuickUnion

**Idea:** When we do findSet(p), move all visited nodes under the root

- Additional cost is insignificant (same order of growth)
This is the worst-case topology if we use WeightedQuickUnion

Idea: When we do findSet(p), move all visited nodes under the root

- Doesn’t meaningfully change runtime for this invocation of findSet(p), but subsequent findSet(p)s (and subsequent findSet(o)s and findSet(m)s and ...) will be faster
Run path compression on every findSet()!
- Including the findSet()s that are invoked as part of a union()

Understanding the performance of M>1 operations requires amortized analysis

We won’t go into it here, but we’ve sort of seen this before
- It’s how we can actually say that appending to an array is “O(1) on average” if we double whenever we resize. You can google it more if you’re curious!
4. QuickUnionBySizeCompressingTrees
Subtleties of Path Compression

Path compression is an optimization written into the `findSet` code.

It does not appear directly in the `union` code.
- It’s not worth it; you’d have to rewrite the entire `findSet` code inside `union` to make it happen.

But `union` does make two `findSet` calls,
- So path compression will happen when you do a `union` call, just indirectly.
Questions break

QuickunionBySizeTrees
QuickUnionBySizeCompressingTrees
4. QuickUnionBySizeCompressingTrees runtimes

<table>
<thead>
<tr>
<th></th>
<th>makeSet</th>
<th>findSet</th>
<th>Union</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worst-Case</td>
<td>$\Theta(1)$</td>
<td>$\Theta(\log n)$</td>
<td>$\Theta(\log n)$</td>
</tr>
<tr>
<td>Best-Case</td>
<td>$\Theta(1)$</td>
<td>$\Theta(1)$</td>
<td>$\Theta(1)$</td>
</tr>
<tr>
<td>In-Practice</td>
<td>$\Theta(1)$</td>
<td>$O(\log^* n)$</td>
<td>$O(\log^* n)$</td>
</tr>
</tbody>
</table>

Hey why are some of those $O()$ not $\Theta()$?
And...wait what’s that * above the log?
\[ \log^*(n) \]

\( \log^*(n) \) is the “iterated logarithm”

It answers the question “how many times do I have to take the log of this to get a number at most 1?”

E.g. \( \log^*(16) = 3 \)

\[ \log(16) = 4 \quad \log(4) = 2 \quad \log(2) = 1. \]

\( \log^* \) grows ridiculously slowly.

\[ \log^*(10^{80}) = 5. \]

\( 10^{80} \) is the number of atoms in the observable universe. For all practical purposes these operations are constant time.

But they aren’t \( O(1) \).
$\log^* n$ isn’t tight – that’s why those $\Theta()$ bounds became $O()$ bounds.

There is a tight bound. It’s a function that grows even slower than $\log^* n$

- Google “inverse Ackerman function”
4. QuickUnionBySizeCompressingTrees methods recap

findSet(value):

1. jump to the node of value and traverse up to get to the root (representative)
2. after finding the representative do **path compression** (point every node from the path you visited to the root directly)
3. return the root (representative) of the set value is in

union(valueA, valueB):

1. call findSet(valueA) and findSet(valueB) to get access to the root (representative) of both
2. merge by setting one root to point to the other root (one root becomes the parent of the other root). Have the smaller sized tree’s root point to the bigger tree’s root
   - if treeA’s rank == treeB’s size, It doesn’t matter which is the parent so choose arbitrarily
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Array implementation motivation

Instead of nodes, let’s use an array implementation!

Just like heaps, the trees and node objects will exist in our mind, but not in our programs.

It won’t be asymptotically faster, but check out all these benefits:
- this will be more memory compact
- get better caching benefits because we’ll be using arrays
- simplify the implementation
What are we going to put in the array and what is it going to mean?

One of the most common things we do with Disjoint Sets is: go to a node and traverse upwards to the root (go to your parent, then go to your parent’s parent, then go to your parent’s parent’s parent, etc.).

A couple of ideas:

• represent each node as a position in our array

• at each node’s position, store the index of the parent node. This will let us jump to the parent node position in the array, and then we can look up our parent’s parent node position, etc.
  • if we’re storing indices, this mean this is an array of ints
at each node’s position, store the index of the parent node

<table>
<thead>
<tr>
<th>index</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>?</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
</tr>
</tbody>
</table>
at each node’s position, store the index of the parent node

<table>
<thead>
<tr>
<th>index</th>
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<tbody>
<tr>
<td>0</td>
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</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>
Exercise (1 min) at each node’s position, store the index of the parent node

<table>
<thead>
<tr>
<th>index</th>
<th>z</th>
<th>y</th>
<th>t</th>
<th>x</th>
<th>w</th>
<th>v</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

```
0 1 2 3 4 5 6
```
Exercise (1 min)

at each node’s position, store the index of the parent node

<table>
<thead>
<tr>
<th>index</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>3</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Diagram: (Diagram not described in natural text)
Exercise (1 min)
at each node’s position, store the index of the parent node

<table>
<thead>
<tr>
<th>index</th>
<th>z</th>
<th>y</th>
<th>t</th>
<th>x</th>
<th>w</th>
<th>v</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>3</td>
<td>3</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Diagram of tree with nodes z, y, t, x, w, v, u.
Exercise (1 min) at each node’s position, store the index of the parent node

<table>
<thead>
<tr>
<th>index</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>?</td>
</tr>
<tr>
<td>4</td>
<td>?</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
</tr>
</tbody>
</table>
Exercise (1 min)
at each node’s position, store the index of the parent node

<table>
<thead>
<tr>
<th>Index</th>
<th>z</th>
<th>y</th>
<th>t</th>
<th>x</th>
<th>w</th>
<th>v</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Value</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>?</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Exercise (1 min)
at each node’s position, store the index of the parent node

<table>
<thead>
<tr>
<th>index</th>
<th>z</th>
<th>y</th>
<th>t</th>
<th>x</th>
<th>w</th>
<th>v</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

```
index: 0 1 2 3 4 5 6
value: 3 3 4 5 6 - -
```
How would findSet work for array implementation?

<table>
<thead>
<tr>
<th>index</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

element: findSet(y)
- look up the index of y in our array (index 1)
- keep traversing till we get to the root / no more parent indices available
- path compression (set everything to point to the index of the root - in this case set everything on the path to 5)
- return the index of the root (in this case return 5). Instead of the actual node itself, we now have access to an index which is a simpler, but still unique ID
How would `findSet` work for array implementation?
(Looking up the index for a given value)

<table>
<thead>
<tr>
<th>index</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
</tr>
</tbody>
</table>

In `findSet` we have to figure out where to start traversing upwards from ... so what index do we use and how do we keep track of the values indices? (In the above example) basically, how would we map each letter to a position?

Whenever you add new values into your disjoint set, keep track of what index you stored it at with a dictionary of value to index! This is similar to the thing as what we did in our ArrayHeap.

This is a big idea!
How would `findSet` work for array implementation? (What do we store at the root position so we know when to stop?)

We just mentioned for `findSet` that we need to traverse starting from a node (like `y`) to its parent and then its parent’s parent until we get to a root. What type of int could we put there as a sign that we’ve reached the root?
How would findSet work for array implementation? (What do we store at the root position so we know when to stop?)

<table>
<thead>
<tr>
<th>index</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>-4</td>
</tr>
<tr>
<td>6</td>
<td>-3</td>
</tr>
</tbody>
</table>

We just mentioned for findSet that we need to traverse starting from a node (like y) to its parent and then its parent’s parent until we get to a root. What type of int could we put there as a sign that we’ve reached the root?

A negative number! (since valid array indices are only 0 and positive numbers)

We’re going to actually be extra clever and store a strictly negative version of the size for our root nodes.
How would `findSet` work for array implementation?
(after ironing out details)

<table>
<thead>
<tr>
<th>index</th>
<th>z</th>
<th>y</th>
<th>t</th>
<th>x</th>
<th>w</th>
<th>v</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>-4</td>
<td>-3</td>
</tr>
</tbody>
</table>

example: `findSet(y)`
- look up the index of y in our array with index dictionary (index 1)
- keep traversing till we get to the root, signified by negative numbers
- path compression (set everything to point to the index of the root - in this case set everything on the path to 5)
- return the **index** of the root (in this case return 5). Instead of the actual node itself, we now have access to an index which is a simpler, but still unique ID
Exercise (1.5 min) – what happens for findSet(s)

- look up the index of value in our array with index dictionary keep traversing till we get to the root, signified by negative numbers
- path compression (set everything to point to the index of the root)
- return the index of the root (in this case return 5). Instead of the actual node itself, we now have access to an index which is a simpler, but still unique ID
Exercise (1.5 min) – what happens for findSet(s)

- look up the index of value in our array with index dictionary keep traversing till we get to the root, signified by negative numbers
- path compression (set everything to point to the index of the root)
- return the index of the root (in this case return 5). Instead of the actual node itself, we now have access to an index which is a simpler, but still unique ID

returns 5
How would union work for array implementation?

<table>
<thead>
<tr>
<th>index</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

note: formula to store in root nodes is negative size

makeSet(u)
makeSet(v)
union(u, v)
How would union work for array implementation?

<table>
<thead>
<tr>
<th>index</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>/</td>
</tr>
<tr>
<td>2</td>
<td>/</td>
</tr>
<tr>
<td>3</td>
<td>/</td>
</tr>
</tbody>
</table>

note: formula to store in root nodes is negative size

\[
\text{makeSet}(u) \\
\text{makeSet}(v) \\
\text{union}(u, v)
\]
How would union work for array implementation?

Note: formula to store in root nodes is negative size.

<table>
<thead>
<tr>
<th>index</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>-1</td>
<td>-1</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

```
makeSet(u)
makeSet(v)
union(u, v)
```
How would union work for array implementation?

index | 0 | 1 | 2 | 3
--- | --- | --- | --- | ---
value | 1 | -1 | / | /

note: formula to store in root nodes is negative size

union – almost the same as before
• update one of the roots to point to the other root (in this case we had node u’s position in the array store index 1, as v is now its parent)

makeSet(u)
makeSet(v)
union(u, v)
How would union work for array implementation?

<table>
<thead>
<tr>
<th>index</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>1</td>
<td>-2</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

- **makeSet(u)**
- **makeSet(v)**
- **union(u, v)**

**union – almost the same as before**

- update one of the roots to point to the other root (in this case we had node u’s position in the array store index 1, as v is now its parent)
- Note: calculate the new size and then multiply it by -1 to turn it into the negative version.

**Note:** formula to store in root nodes is negative size
Exercise maybe

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

already set up all the makeSet calls in the area
- union(a, b)
- union(c, d)
- union(e, f)
- union(a, g)
- union(c, e)
- union(a, c)
Summary of the big ideas

- each node is represented by a position in the int array

- each position stores either:
  - the index of its parent, if not the root node
  - -1 * size if the root node

- keep track of a dictionary of value to index to be able to jump to a node’s position in the array

- apply all the same high level ideas of how the Disjoint Set methods work (findSet and union) for trees, but to the array representation
  - makeSet – store -1 (size of 1) in a new slot in the array
  - findSet(value) – jump to the value’s position in your array, and traverse till you reach a negative number (signifies the root). Do path compression and return the index of the root (the representative of this set).
  - union(valueA, valueB) – call findSet(valueA) and findSet(valueB) to access the sizes and indices of valueA and valueB’s sets. Compare the sizes like in the tree representation. Make sure to update the size when you union the two of them together.