1. **Graph modeling**

Suppose you are working for SpaceY, a private company planning exploration of Mars. You have just built a rover that will be sent to Mars. The rover has two modes.

- **Bulldozer mode**: in bulldozer mode, the rover will permanently clear any obstacle from its path, bulldozer mode uses an extremely large amount of power.
- **Traveling mode**: in traveling mode, the rover does not drain battery, but it cannot move around any obstacles.

In Bulldozer mode, the power used by the rover is proportional to the length of the route it travels.

You have a set of locations on Mars that the rover must visit, and the length of the direct route between each pair. Your goal is to plan which areas to bulldoze so you can reach every location in traveling mode, while using as little power as possible.

(a) Explain how you would model this scenario as a graph. Answer the following questions in bullet points with short sentences, then give an overall description on anything else that is relevant:

(i) What are your vertices and what information is stored in each vertex?

(ii) What are your edges and what information is stored in each edge?

(iii) Is this a weighted graph or an unweighted one? Why? Briefly explain why in 1-2 sentences.

(iv) Is this a directed or undirected graph? Why? Briefly explain why in 1-2 sentences.
(v) Do you permit self-loops (i.e. edges from a vertex to itself)? Parallel edges (i.e. more than one copy of an edge between the same location)? Why? Briefly explain why in 1-2 sentences.

(vi) If there are any other relevant details about your model, describe them here:

(vii) What graph algorithm do you run to find the routes to bulldoze?

(viii) Briefly explain how to convert the output of your algorithm to a plan of how to bulldoze and travel. Your answer should include which routes you are bulldozing, and how/when you will use traveling mode. (2-3 sentences)

(ix) Give a simplified big-$\Theta$ bound for the worst case running time of your algorithm from part (vii) in terms of $k$, the amount of locations on Mars. For clarity, assume that your graph implementation is using an adjacency-list implemented with a HashMap<Vertex, LinkedList<Edge>>.