Asymptotic Analysis, Code Modeling

Due date: Friday April 19, 2020 at 11:59 pm

Instructions:

Submit your responses to the "Exercise 1" assignment on Gradescope here: https://www.gradescope.com/courses/97095. Make sure to log in to your Gradescope account using your UW email to access our course.

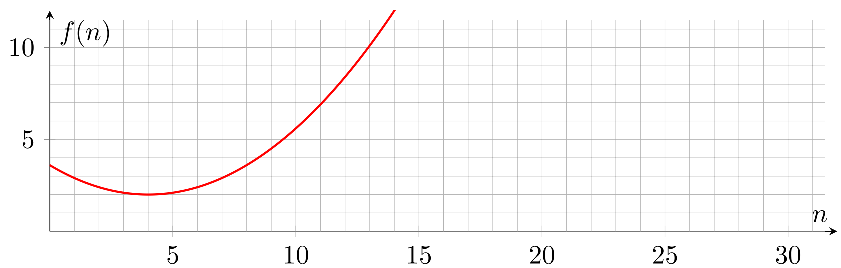
These problems are meant to be done **individually**. If you do want to discuss problems with a partner or group, make sure that you're writing your answers individually later on. Check our course's collaboration policy if you have questions.

1. Asymptotic analysis: Visually

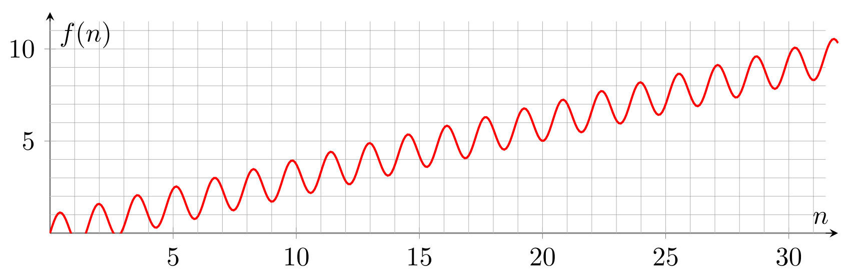
For each of the following plots, provide a tight big-$\mathcal{O}$ bound, a tight big-Ω bound, and a big-Θ bound. You do not need to show your work; just list the bounds. If a particular bound doesn't exist for a given plot, briefly explain why. Assume that the plotted functions continue to follow the same trend shown in the plots as n increases. Each provided bound must either be a constant or a simple polynomial, **from the following possible answers**.

$$n^2, 1, n, log(n), n!, 1/n$$

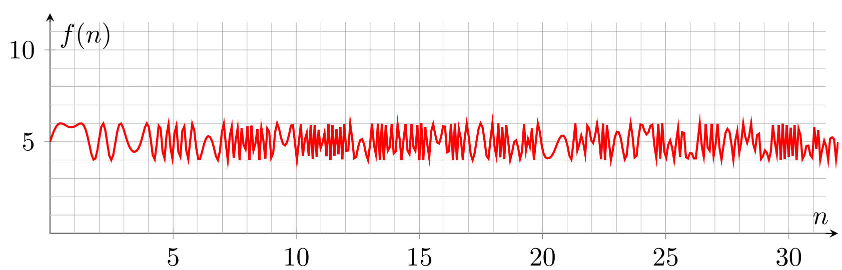
(a)



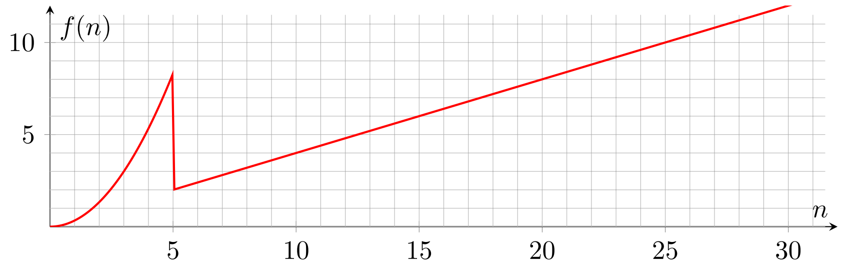
(b)



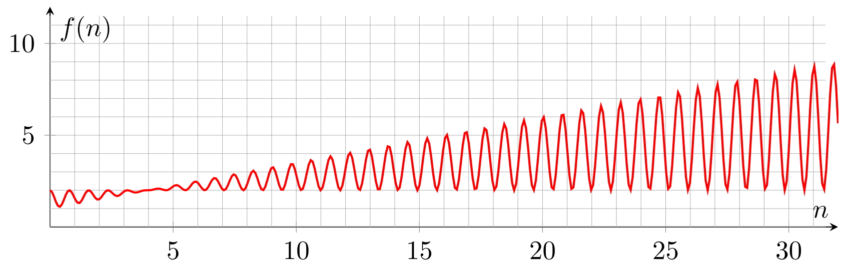
(c)



(d)



(e)



2. Case and Asymptotic Analysis

In this problem we will analyze code in the `printSmilies` method below. The code is not particularly efficient (i.e. you should not use this code snippet as a model for how to use data structures).

```
1  public void printSmilies(ArrayList<Integer> input, int target) {
2      LinkedList<Integer> products = new LinkedList<>();
3      for (int i = 0; i < input.size(); i++) {
4          for (int j = 0; j < input.size(); j++) {
5              products.add(input.get(i) * input.get(j));
6          }
7      }
8
9      LinkedList<Integer> occurrencesOfToFind = new LinkedList<>();
10
11     while (products.size() != 0) {
12         int next = products.remove(0); // remove the value at index 0
13         if (next == target) {
14             occurrencesOfToFind.add(next);
15         }
16     }
17
18     for (int i = 0; i < occurrencesOfToFind.size(); i++) {
19         for (int j = 0; j < occurrencesOfToFind.size(); j++) {
20             System.out.println(":D");
21         }
22     }
23 }
```

Answer the following questions about the runtime of the `printSmilies` method. In this problem, `input` is defined to have n elements. For each of the problems asking for asymptotic bounds, your answers should be in terms of n. All big-$\mathcal{O}$, big-Ω, and big-Θ bounds should be simplified. You can assume `System.out.println` calls will always run in constant time.

Remember that "best/worst-case" refer to the inputs that yield the fastest or slowest possible runtime functions, respectively.

(a) Give the simplified big-Θ bound for runtime of the first loop on lines 3-7. (Note: the runtime bound is the same across all cases). Your answer should be in terms of n, the size of the input.

(b) Note that runtime for last loop is based on the size of `occurrencesOfToFind` (note: this is not the same variable as n, the input size to this method). To figure out that last loop's runtime, let's break it down by analyzing the previous loop on lines 11-16:

- What sort of state of the parameters `input` and `target` will trigger the worst case where `occurrencesOfToFind` is as big as possible? You should only need a maximum of 1 sentence for your response.

- Give a simplified big-Θ bound for the size of `occurrencesOfToFind` in this worst case. Your answer should be in terms of n, the size of the input.

- What sort of state of the parameters `input` and `target` will trigger the best case where `occurrencesOfToFind` is as small as possible? You should only need a maximum of 1 sentence for your response.

- Give a simplified big-Θ bound for the size of `occurrencesOfToFind` in this best case. Your answer should be in terms of n, the size of the input.

(c) Consider the worst case runtime situation discussed in the previous question for the loop on lines 18-22 for the following problems.

- Give a tight big-$\mathcal{O}$ for the runtime of lines 18-22. Your answer should be in terms of n, the size of the input.

- Give a tight big-Ω for the runtime of lines 18-22. Your answer should be in terms of n, the size of the input.

(d) Consider the best case runtime situation discussed in the previous question for the loop on lines 18-22 for the following problems.

- Give a tight big-$\mathcal{O}$ for the runtime of lines 18-22. Your answer should be in terms of n, the size of the input.

- Give a tight big-Ω for the runtime of lines 18-22. Your answer should be in terms of n, the size of the input.