Master Theorem

For recurrences in this form, where a, b, c, e are constants:

$$T(n) = \begin{cases} d & \text{if } n \leq \text{some constant} \\ aT(n/b) + e \cdot n^c & \text{otherwise} \end{cases}$$

$$T(n) \text{ is } \begin{cases} \Theta(n^c) & \text{if } \log_b(a) < c \\ \Theta(n^c \log n) & \text{if } \log_b(a) = c \\ \Theta\left(n^{\log_b(a)}\right) & \text{if } \log_b(a) > c \end{cases}$$

Useful summation identities

Splitting a sum

$$\sum_{i=a}^{b} (x + y) = \sum_{i=a}^{b} x + \sum_{i=a}^{b} y$$

Adjusting summation bounds

$$\sum_{i=a}^{b} f(x) = \sum_{i=0}^{b} f(x) - \sum_{i=0}^{a-1} f(x)$$

Factoring out a constant

$$\sum_{i=a}^{b} cf(i) = c \sum_{i=a}^{b} f(i)$$

Summation of a constant

$$\sum_{i=0}^{n-1} c = \underbrace{c + c + \ldots + c}_{n \text{ times}} = cn$$

Note: this rule is a special case of the rule on the left

Gauss's identity

$$\sum_{i=0}^{n-1} i = 0 + 1 + 2 + \ldots + n - 1 = \frac{n(n-1)}{2}$$

Finite geometric series

$$\sum_{i=0}^{n-1} x^i = 1 + x + x^2 + \ldots + x^{n-1} = \frac{x^n - 1}{x - 1}$$

Infinite geometric series

$$\sum_{i=0}^{\infty} x^i = 1 + x + x^2 + \ldots = \frac{1}{1 - x}$$

Note: applicable only when $-1 < x < 1$

Useful Log Rules

Power of a log identity

$a^{\log_b c} = c^{\log_b a}$

Product rule

$\log_c(a * b) = \log_c a + \log_c b$

Quotient rule

$\log_c(a/b) = \log_c a - \log_c b$

Power rule

$\log_c(a^b) = b * \log_c a$

Change of base formula

$\log_b a = (\log_c a)/(\log_c b)$