LEC 08

# CSE 373

# Hash Maps

## BEFORE WE START


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# Announcements

- EX1 (Algo Analysis I) due **TONIGHT 11:59pm PDT**
  - You can use late days on exercises, just like projects!
- P2 (Maps) and EX2 (Algo Analysis II) released today
- Summations Reference published (on course calendar under Wednesday's lecture)
  - Section handout has a cheat-sheet version

# P2: Maps

- Implement everyone's good pal: the Hash Map!
- Like P1, look at multiple data structures under single ADT
  - But this time, we have the algorithmic analysis tools to reason about more complicated situations (especially Case Analysis!)
- 3 Parts:
  - ArrayMap
  - ChainedHashMap
  - Experiments
- Start early! In particular, ChainedHashMap iterator can take a long time!

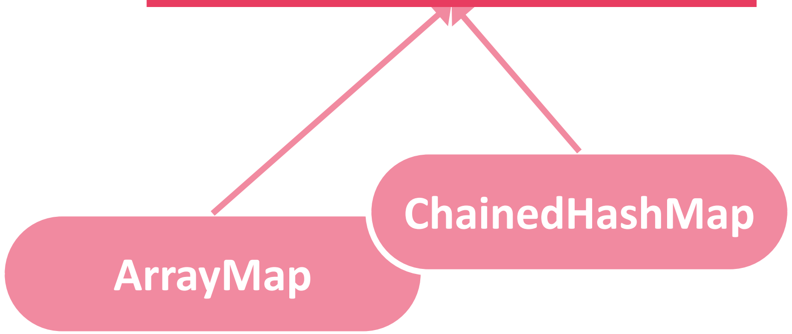
## MAP ADT

**State**

Set of keys, Collection of values
Count of keys

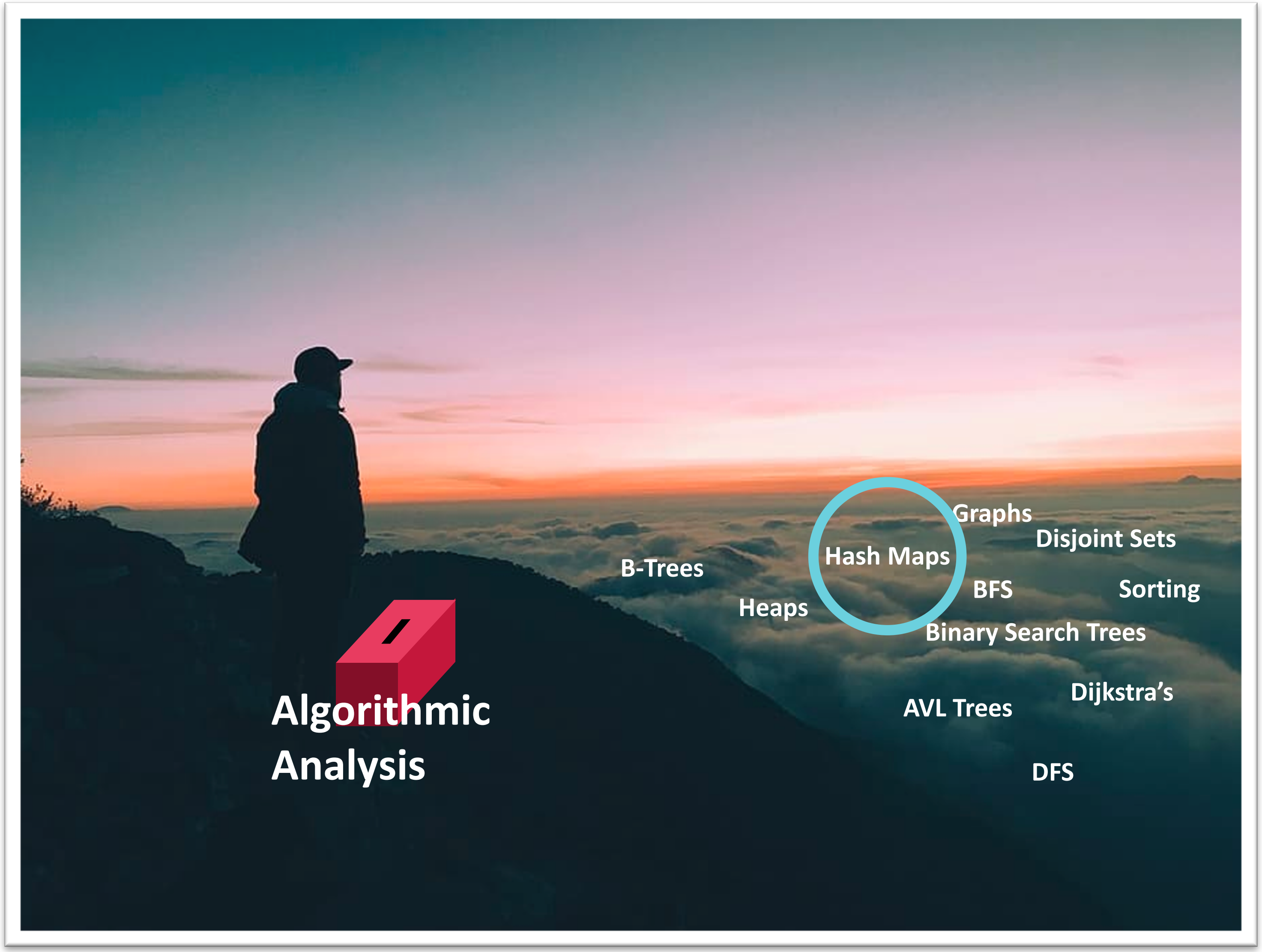
**Behavior**

`put(key, value)` add value to collection, associated with key
`get(key)` return value associated with key
`containsKey(key)` return if key is associated
`remove(key)` remove key and associated value
`size()` return count



# Welcome to the Data Structures Part™

- We're now armed with a toolbox stuffed full of analysis tools
  - Wednesday was the last algorithmic analysis lecture
  - It's time to apply this theory to more practical topics!
- Today, we'll take our first deep dive using those tools on a data structure: Hash Maps!



# Learning Objectives

## After this lecture, you should be able to...

1. Compare the relative pros/cons of various Map implementations, especially given a design like the ones we cover today
2. Trace operations in a Separate Chaining Hash Map on paper (such as insertion, getting an element, resizing)
3. Implement a Separate Chaining Hash Map in code (P2)
4. Differentiate between the “worst” and “in practice” runtimes of a Separate Chaining Hash Map, and describe what assumptions allow us to consider the “in practice” case

# Lecture Outline

MAP ADT

*Review*

**1**

**ArrayMap**

As seen on **Project 2**

**2**

**DirectAccessMap**

**FASTER**: Jump directly to element, only int keys

**3**

**SimpleHashMap**

**MORE FLEXIBLE**: Hash function supports any type of key

**4**

**SeparateChaining HashMap**

**YOUR BEST FRIEND**: Addresses limitations with hash collisions, but still fast!

As seen on **Project 2**

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# *Review* The Map ADT

- **Map**: an ADT representing a set of distinct keys and a collection of values, where each key is associated with one value.
  - Also known as a **dictionary**
  - If a key is already associated with something, calling put(key, value) replaces the old value
- Used all over the place
  - It's hard to work on a big project without needing one sooner or later
  - CSE 143 introduced:
    - `Map<String, Integer> map1 = new HashMap<>();`
    - `Map<String, String> map2 =  new TreeMap<>();`

**MAP ADT**

State

Set of keys, Collection of values
Count of keys

Behavior

`put(key, value)` add value to collection, associated with key
`get(key)` return value associated with key
`containsKey(key)` return if key is associated
`remove(key)` remove key and associated value
`size()` return count

*`clear()` remove all*
*`iterator()` get an iterator*

# *Review* Implementing a Map with an Array

## MAP ADT

**State**

Set of keys, Collection of values
Count of keys

**Behavior**

put(key, value) add value to collection, associated with key
get(key) return value associated with key
containsKey(key) return if key is associated
remove(key) remove key and associated value
size() return count

## ArrayMap<K, V>

**State**

```
Pair<K, V>[] data
```

**Behavior**

put find key, overwrite value if there. Otherwise create new pair, add to next available spot, grow array if necessary
get scan all pairs looking for given key, return associated item if found
containsKey scan all pairs, return if key is found
remove scan all pairs, replace pair to be removed with last pair in collection
size return count of items in dictionary

## Big-Oh Analysis – (if key is the last one looked at / not in the dictionary)

| | |
|---|---|
| put() | O(n) linear |
| get() | O(n) linear |
| containsKey() | O(n) linear |
| remove() | O(n) linear |
| size() | O(1) constant |

## Big-Oh Analysis – (if the key is the first one looked at)

| | |
|---|---|
| put() | O(1) constant |
| get() | O(1) constant |
| containsKey() | O(1) constant |
| remove() | O(1) constant |
| size() | O(1) constant |

```
put('b', 97)
put('e', 20)
```

| 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| ('a', 1) | ('b',97) | ('c', 3) | ('d', 4) | ('e',20) |

# *Review* Implementing a Map with Linked Nodes

## MAP ADT

State

Set of keys, Collection of values
Count of keys

Behavior

put(key, value) add value to collection, associated with key
get(key) return value associated with key
containsKey(key) return if key is associated
remove(key) remove key and associated value
size() return count

## LinkedMap<K, V>

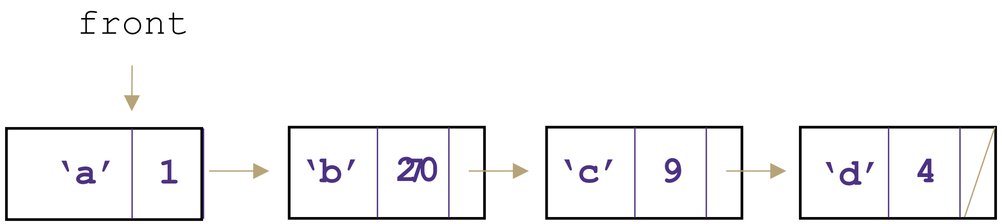
State

front
size

Behavior

put if key is unused, create new with pair, add to front of list, else replace with new value
get scan all pairs looking for given key, return associated item if found
containsKey scan all pairs, return if key is found
remove scan all pairs, skip pair to be removed
size return count of items in dictionary

```
containsKey('c')
get('d')
put('b', 20)
```




## Big O Analysis – (if key is the last one looked at / not in the dictionary)

| | |
|---|---|
| put() | O(n) linear |
| get() | O(n) linear |
| containsKey() | O(n) linear |
| remove() | O(n) linear |
| size() | O(1) constant |

## Big O Analysis – (if the key is the first one looked at)

| | |
|---|---|
| put() | O(1) constant |
| get() | O(1) constant |
| containsKey() | O(1) constant |
| remove() | O(1) constant |
| size() | O(1) constant |

# Could we do better?

- `put`, `get`, and `remove` have Θ(n) runtimes. Could we use a Θ(1) operation to improve?
- What about array indexing?
  - data[i] (array access) and data[i] = 2 (array update) are constant runtime!
  - What if we could jump directly to the requested key?
  - We could simplify the problem: **only allow integer keys**

# Lecture Outline

MAP ADT

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1

ArrayMap

2

DirectAccessMap

**FASTER**: Jump directly to element, only int keys

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SimpleHashMap

**MORE FLEXIBLE**: Hash function supports any type of key

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SeparateChaining HashMap

**YOUR BEST FRIEND**: Addresses limitations with hash collisions, but still fast!

# DirectAccessMap

- put, get, and remove have Θ(n) runtimes. Could we use a Θ(1) operation to improve?
- What about array indexing?
  - data[i] (array access) and data[i] = 2 (array update) are constant runtime!
  - What if we could jump directly to the requested key?
  - We could simplify the problem: **only allow integer keys**

```
put(3, "Alex")

get(3)
```

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| data | | | | Alex | | | | | | |

## DirectAccessMap<K, V>

**State**

data[]
size

**Behavior**

put put item at given index
get get item at given index
containsKey if data[] null at index, return false, return true otherwise
remove nullify element at index
size return count of items in dictionary

# DirectAccessMap Implementation

```
public void put(int key, V value) {
    this.array[key] = value;
}

public boolean containsKey(int key) {
    return this.array[key] != null;
}

public V get(int key) {
     return this.array[key];
}

public void remove(int key) {
    this.array[key] = null;
}
```

**DirectAccessMap<K, V>**

State
- data[]
- size

Behavior
- put put item at given index
- get get item at given index
- containsKey if data[] null at index, return false, return true otherwise
- remove nullify element at index
- size return count of items in dictionary

| Operation | Case | Runtime |
| --- | --- | --- |
| put(key,value) | best | $\Theta(1)$ |
| | worst | $\Theta(1)$ |
| get(key) | best | $\Theta(1)$ |
| | worst | $\Theta(1)$ |
| containsKey(key) | best | $\Theta(1)$ |
| | worst | $\Theta(1)$ |

# Pros and Cons of DirectAccessMap

What’s a benefit of using it? What’s a drawback?

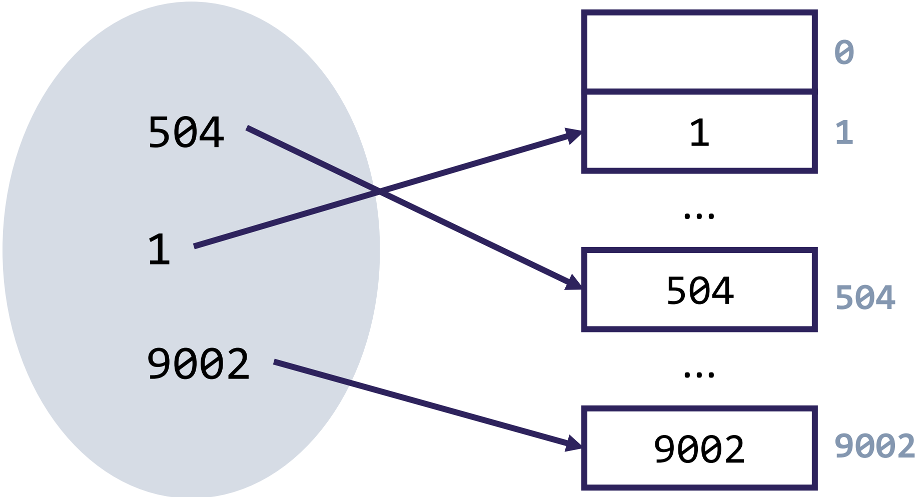
# Pros and Cons of DirectAccessMap

- Super Fast!
  - Everything is $\Theta(1)$
- Wasted Space
  - Say we want to store 0 and 999999999. This implementation would waste all the space inbetween ☹
- Only Integer Keys
  - Would be nice to store any type of data ☹
  - But note what's so useful here: being able to go quickly from key to array index

# Can We Store Any Integer?

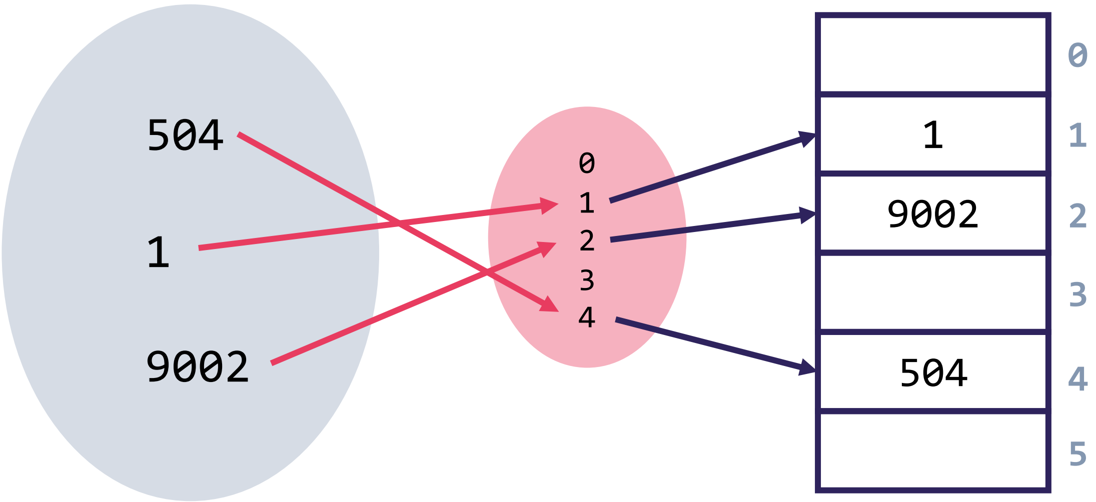
## IDEA 1

- Create a GIANT array with every possible integer as an index
- Problems:
  - Can we allocate an array big enough?
  - Super wasteful



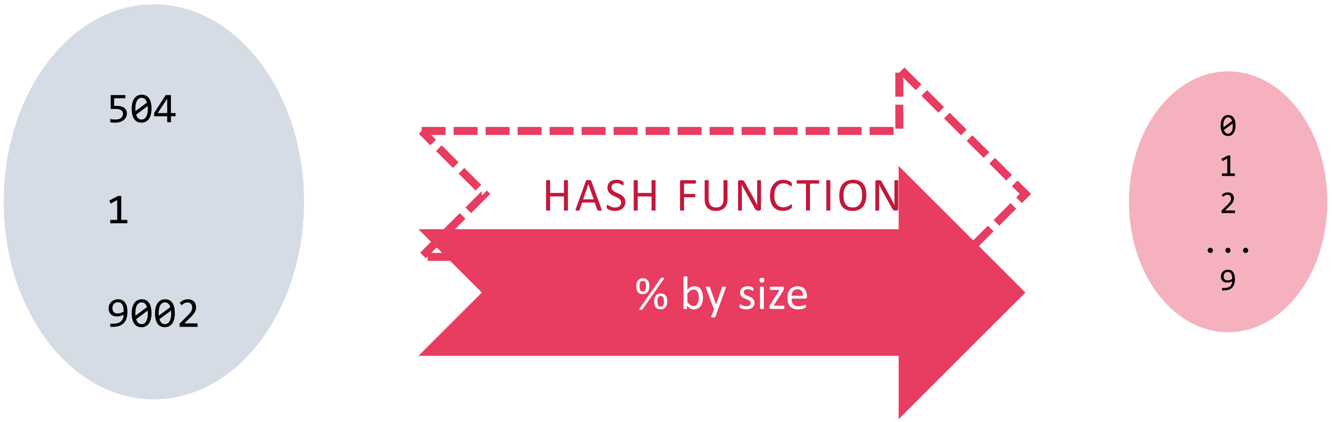

## IDEA 2

- Create a smaller array, with a translation from integer keys into available indices
- Problems:
  - How can we construct a translation?



# Hash Functions

- **Hash Function**: any function that can be used to map data of an arbitrary size to fixed-size values.
  - We want to translate from the set of all integers to the set of valid indexes in our array




9002 % 10 = 2 (so store it in index 2 of the array)

- One simple approach: take the key and % (mod) it by size of the array

# Mod: Remainder

- The `%` operator computes the remainder from integer division.

**`14 % 4` is `2`** **`218 % 5` is `3`**

```
       3
    ____
4 ) 14
    12
    __
     2
```

```
       43
    _____
5 ) 218
    20
    __
     18
     15
     __
      3
```

Equivalently, to find `a % b` (for `a,b > 0`):

```
while(a > b-1)
      a -= b;
return a;
```

- Applications of `%` operator:
  - Obtain last digit of a number: `230857 % 10` is `7`
  - See whether a number is odd: `7 % 2` is `1`, `42 % 2` is `0`
  - Limit integers to specific range: `8 % 12` is `8`, `18 % 12` is `6`

Limit keys to indices within array

For more review/practice, check out https://www.khanacademy.org/computing/computer-science/cryptography/modarithmetic/a/what-is-modular-arithmetic

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**SeparateChaining HashMap**

**YOUR BEST FRIEND**: Addresses limitations with hash collisions, but still fast!

# SimpleHashMap: “% by size” as Hash Function

```
put(0, “I”)        0 % 10 = 0
put(8, “Maps”)     8 % 10 = 8
put(11, “<3”)     11 % 10 = 1
put(23, “Hash”)   23 % 10 = 3
```

**IMPLEMENTATION**

```
public void put(int key, int value) {
    data[hashToValidIndex(key)] = value;
}

public V get(int key) {
    return data[hashToValidIndex(key)];
}

public int hashToValidIndex(int k) {
   return k % this.data.length;
}
```

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| data | I | <3 | | Hash | | | | | Maps | |

Itempool

Pause Video when Prompted

# What input will cause a problem?

```
put(0, “I”)         0 % 10 = 0
put(8, “Maps”)      8 % 10 = 8
put(11, “<3”)      11 % 10 = 1
put(23, “Hash”)    23 % 10 = 3
```

**IMPLEMENTATION**

```
public void put(int key, int value) {
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public V get(int key) {
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}

public int hashToValidIndex(int k) {
   return k % this.data.length;
}
```

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| data | I | <3 | | Hash | | | | | Maps | |

# SimpleHashMap: Collisions?!

```
put(0, “I”)       0 % 10 = 0
put(8, “Maps”)    8 % 10 = 8
put(11, “<3”)    11 % 10 = 1
put(23, “Hash”)  23 % 10 = 3
put(20, “We”)    20 % 10 = 0
```

**IMPLEMENTATION**

```
public void put(int key, int value) {
    data[hashToValidIndex(key)] = value;
}

public V get(int key) {
    return data[hashToValidIndex(key)];
}

public int hashToValidIndex(int k) {
   return k % this.data.length;
}
```

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| data | We<br>I | <3 | | Hash | | | | | Maps | |

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# Handling Collisions

- Two common strategies to handle collisions:

**1. Separate Chaining**

”Chain” together multiple values stored in a single bucket

**2. Open Addressing**

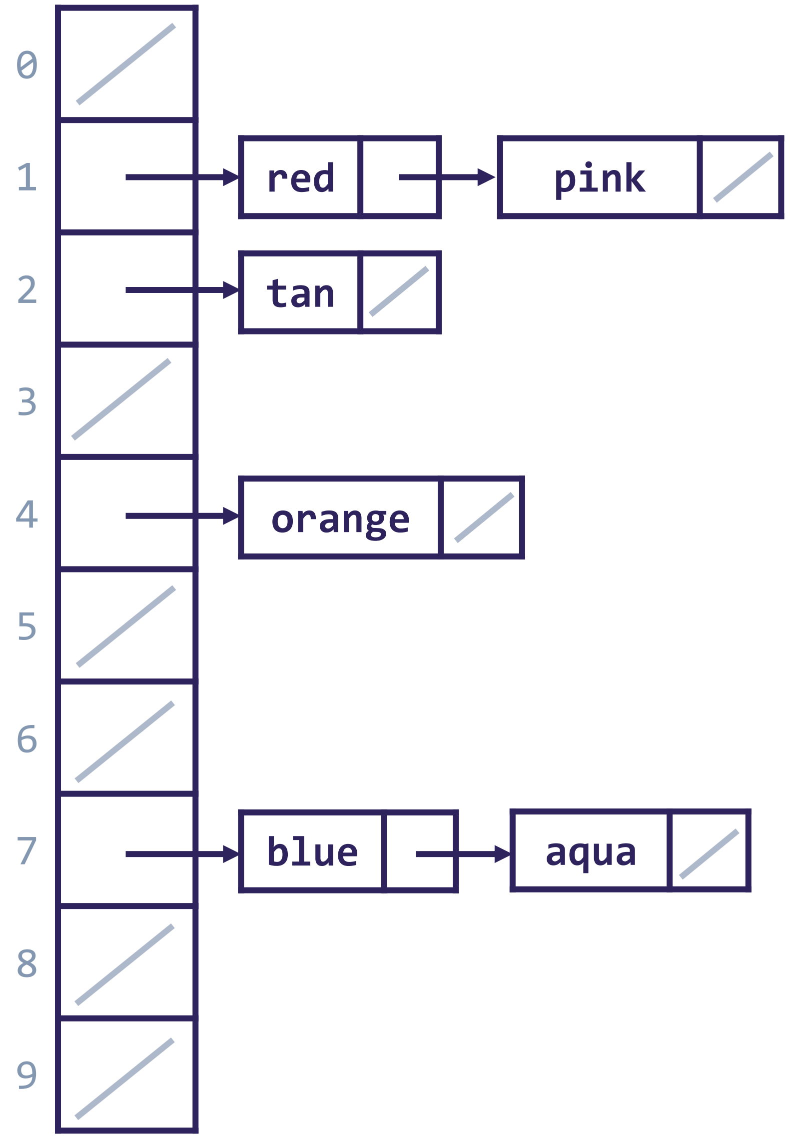
If a bucket is taken, find a new bucket using some strategy:
Linear Probing
Quadratic Probing
Double Hashing

We’ll focus on separate chaining this quarter, much more common in practice

Bonus topic beyond the scope of the class

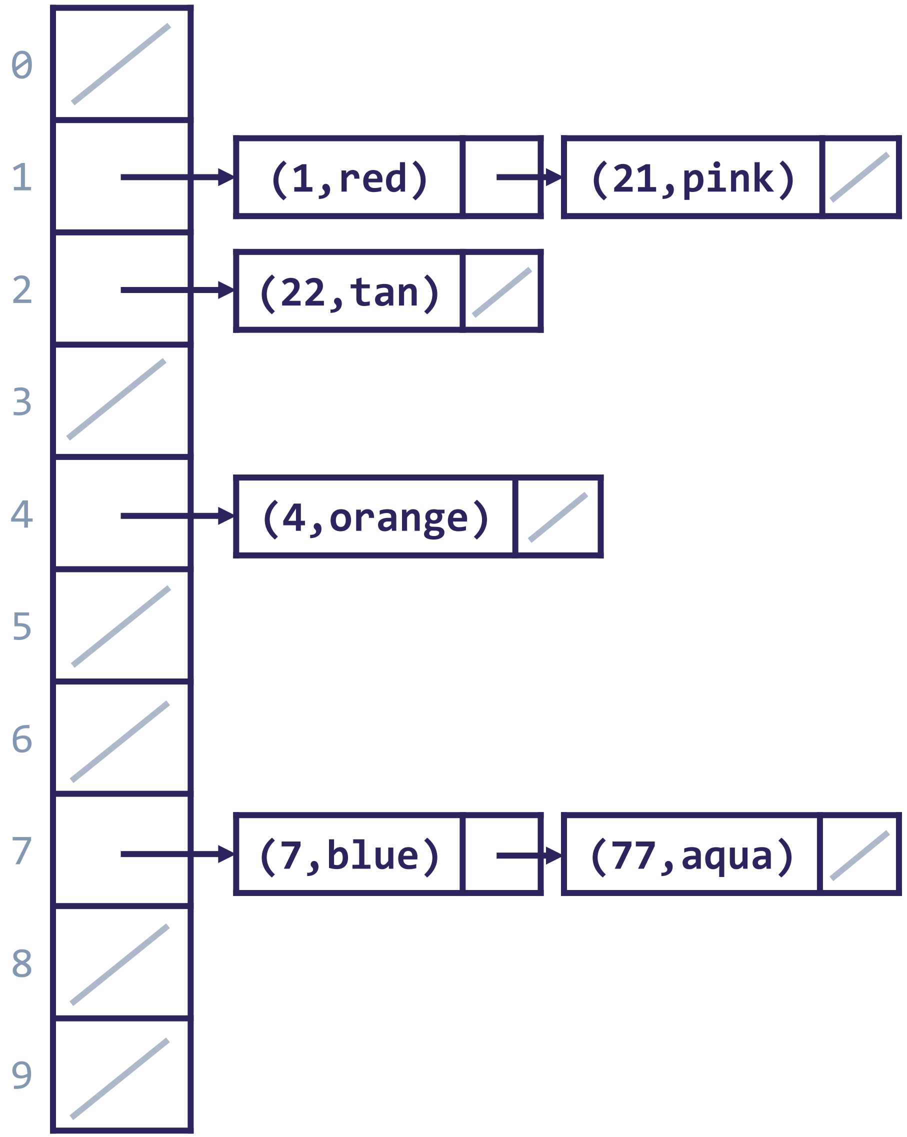
# Separate Chaining

- If two values want to live in the same index, let's just let them be roommates!
- Each index is a "bucket"
  - Linked Nodes are a common implementation for these bucket "chains"
- When item x hashes to index h:
  - If bucket at h is empty, create new list with x
  - Else, add x to the list



# Separate Chaining

- If two values want to live in the same index, let's just let them be roommates!
- Each index is a "bucket"
  - Linked Nodes are a common implementation for these bucket "chains"
- When item x hashes to index h:
  - If bucket at h is empty, create new list with x
  - Else, add x to the list
- But if multiple keys can hash to the same index, need to store the key too!

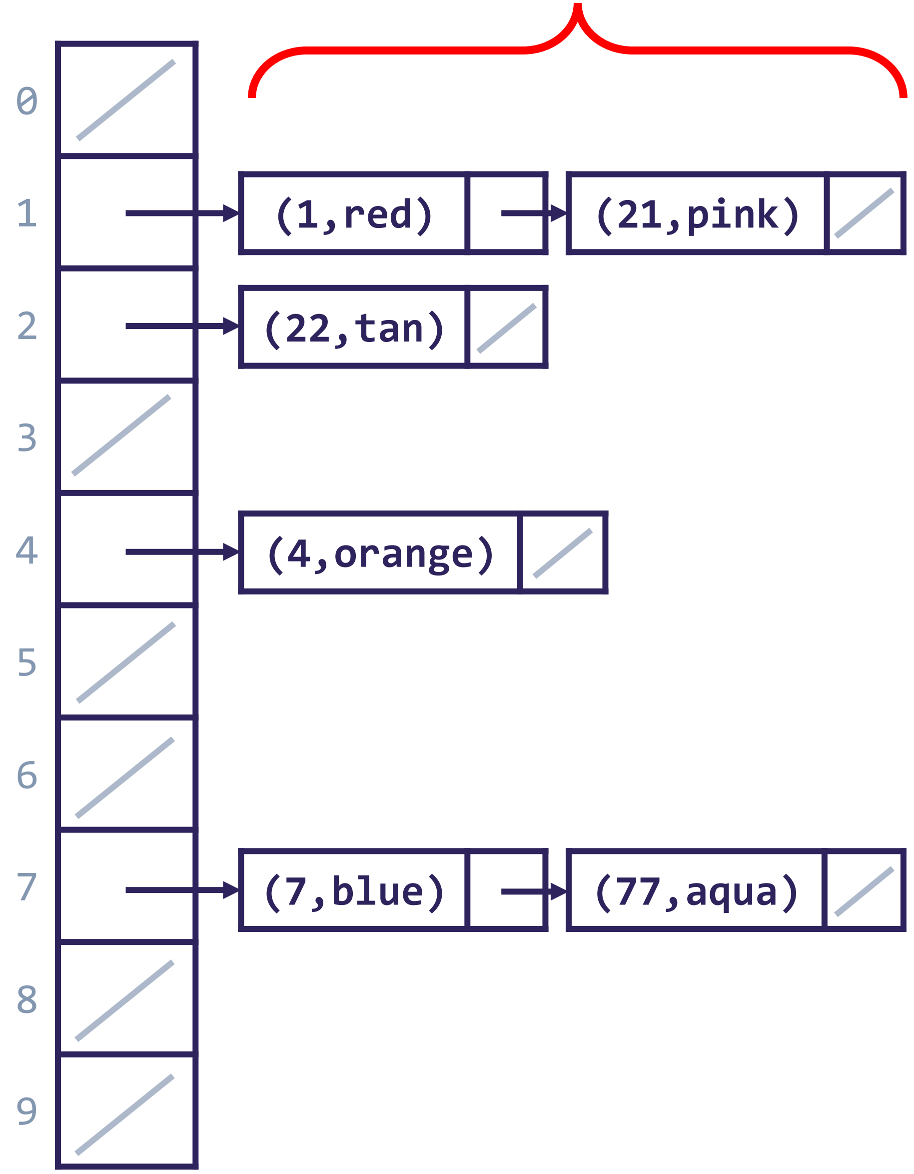


# Separate Chaining

- Implementation of get/put/containsKey very similar

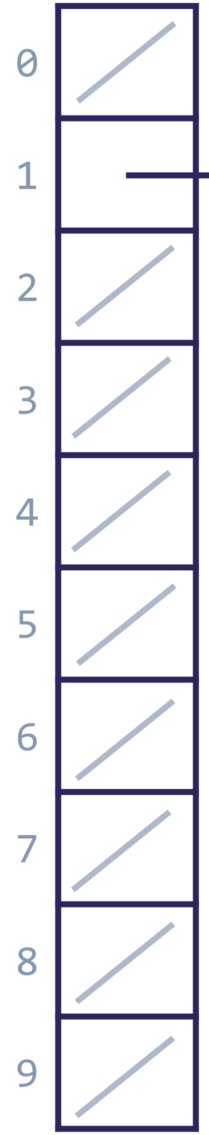
**PSEUDOCODE**

```
public boolean get(int key) {
    int bucketIndex = key % data.length;
    loop through each pair in data[bucketIndex]
        if pair.key == key
            return pair.value
    return null if we get here
}
```




Let's analyze the runtime. First, are there different possible states for this HashMap to make the code faster or slower, assuming n key/value pairs are already stored?

# Separate Chaining Worst Case



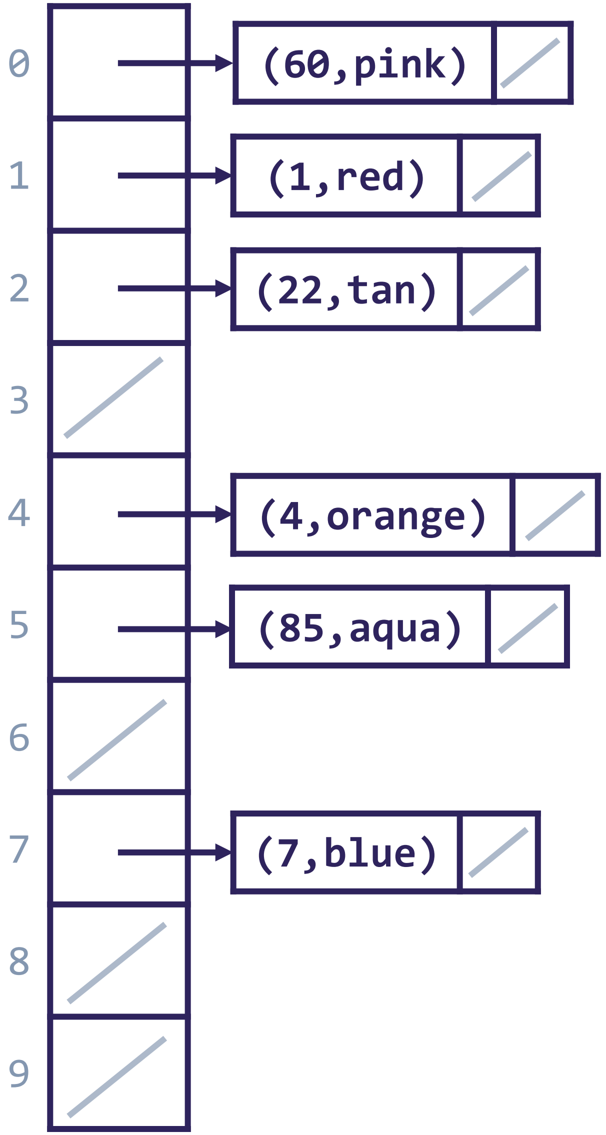


- It’s possible that everything hashes to the same bucket by chance!
  - get would take Θ(n) time ☹
- Consider `get(51)`
  - Use hash function (% 10) to get index (5)
  - Check every element in bucket for key 51
- We’ve lost that Θ(1) runtime

**PSEUDOCODE**

```
public boolean get(int key) {
    int bucketIndex = key % data.length;
    loop through each pair in data[bucketIndex]
        if pair.key == key
            return pair.value
    return null if we get here
}
```

# Separate Chaining Best Case




- However, if everything is spread evenly across the buckets, get takes Θ(1)
- Consider `get(22)`
  - Use hash function (% 10) to get index (2)
  - Check the single element in bucket for key 22 – a constant time operation!
- Key to a successful Hash Map implementation: how can we keep the buckets as close to this distribution as possible?

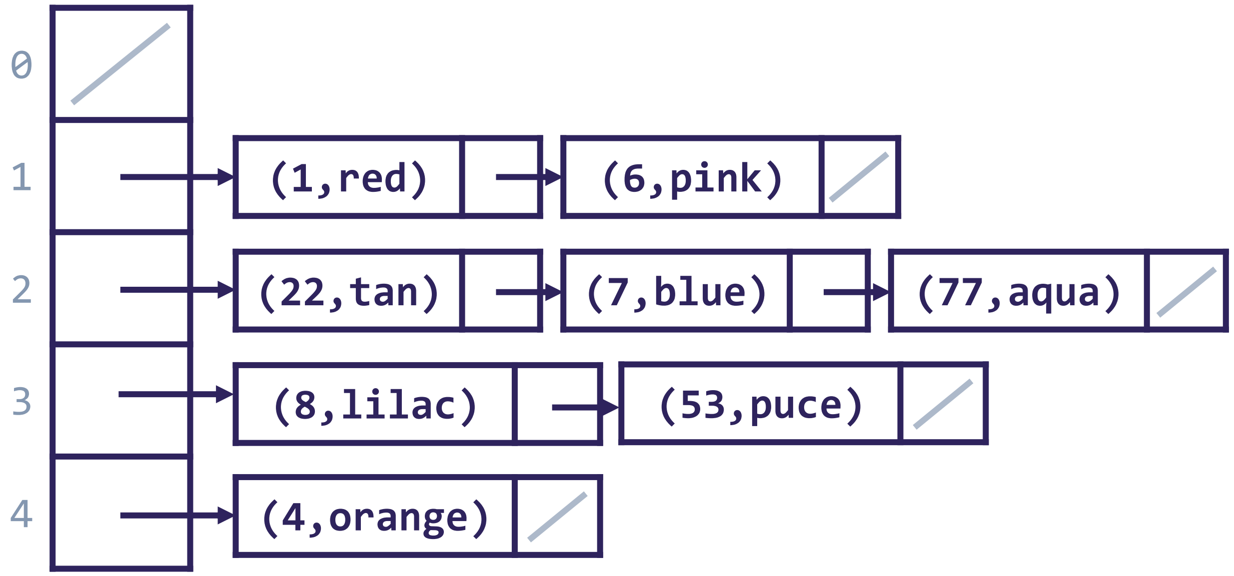
# Separate Chaining... In Practice

- A well-implemented separate chaining hash map will stay very close to the best case
  - Most of the time, operations are fast. Rarely, do an expensive operation that restores the map close to best case.
- How to stay close to best case?
  - Good distribution & Resizing!
- We can describe the "in-practice" case as what *almost always* happens:
  - (1) items are fairly evenly distributed
  - (2) assume resizing doesn't occur
    - This is *similar* to the concept of "amortized"

| Operation | Case | Runtime |
| --- | --- | --- |
| `put(key,value)` | best | $\Theta(1)$ |
| | In-practice | $\Theta(1)$ |
| | worst | $\Theta(n)$ |
| `get(key)` | In-practice | $\Theta(1)$ |
| | average | $\Theta(1)$ |
| | worst | $\Theta(n)$ |
| `remove(key)` | best | $\Theta(1)$ |
| | In-practice | $\Theta(1)$ |
| | worst | $\Theta(n)$ |

# Resizing

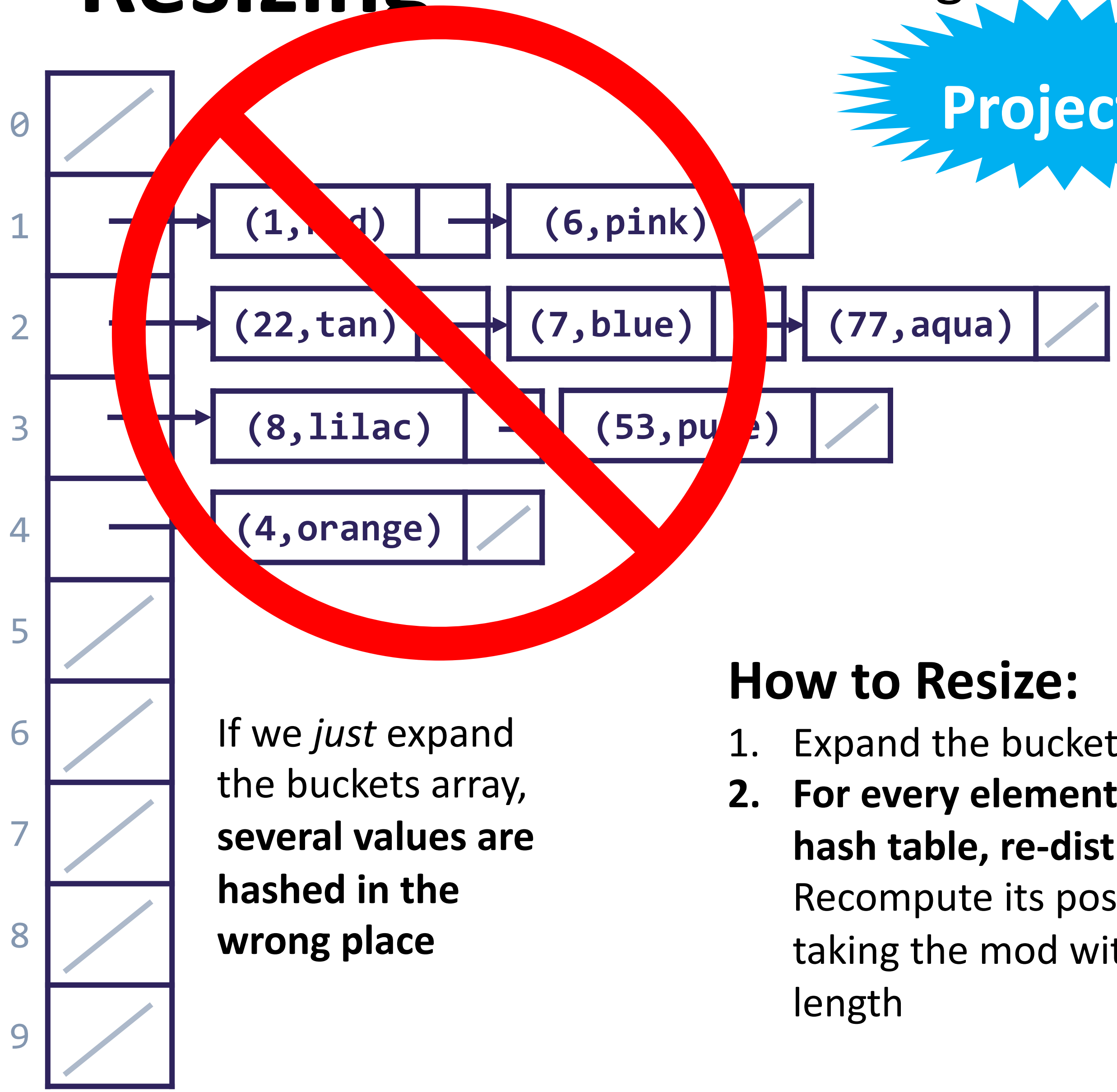
- The runtime to scan each bucket is creeping up
  - If we don't intervene, our in-practice runtime is going to hit Θ(n)
    - number of buckets is a constant, so n / (# buckets) is Θ(n)



# Resizing

Don't forget to re-distribute your keys!

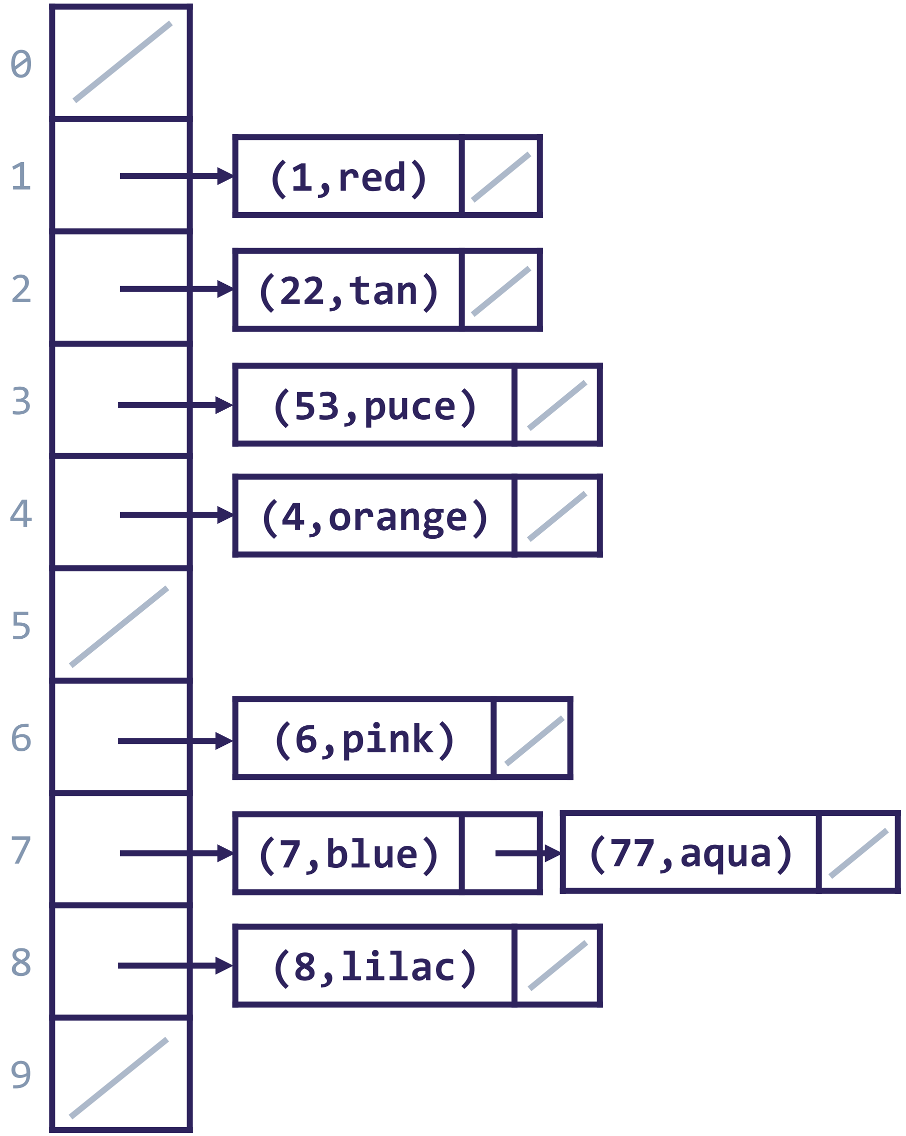
Project 2




If we *just* expand the buckets array, **several values are hashed in the wrong place**

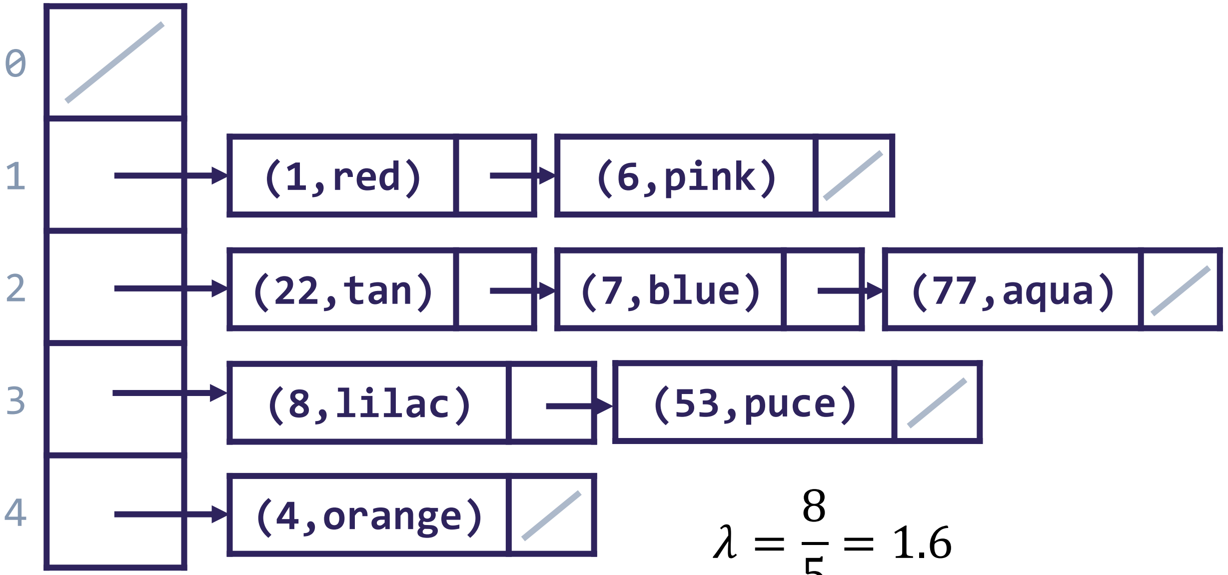
## How to Resize:

1. Expand the buckets array
2. **For every element in the old hash table, re-distribute!** Recompute its position by taking the mod with the new length



# When to Resize?

- In ArrayList, we were forced to resize when we ran out of room
  - In SeparateChainingHashMap, never *forced* to resize, but we want to make sure the buckets don't get too long for good runtime
- How do we quantify "too full"?
  - Look at the average bucket size: number of elements / number of buckets




$$\lambda = \frac{8}{5} = 1.6$$

**LOAD FACTOR λ**

n: total number of key/value pairs
c: capacity of the array (# of buckets)

$$\lambda = \frac{n}{c}$$

# When to Resize?

- In ArrayList, we were forced to resize when we ran out of room
  - In SeparateChainingHashMap, never *forced* to resize, but we want to make sure the buckets don't get too long for good runtime
- How do we quantify "too full"?
  - Look at the average bucket size: number of elements / number of buckets
- If we resize when λ hits some *constant* value like 1:
  - We expect to see 1 element per bucket: **constant runtime**!
  - If we double the capacity each time, the expensive resize operation becomes less and less frequent

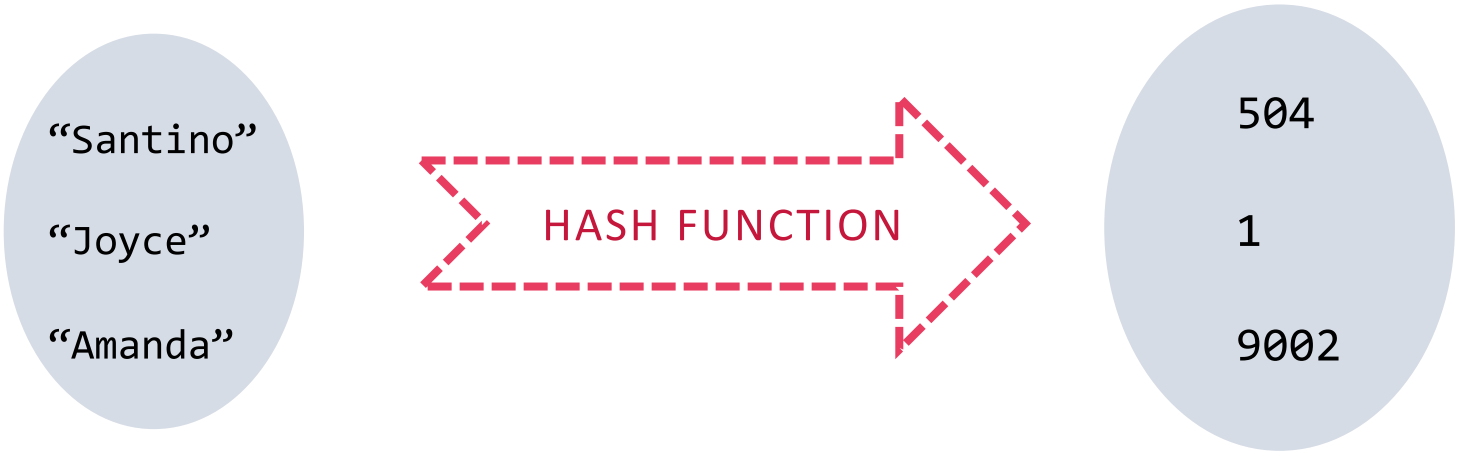
**LOAD FACTOR λ**

n: total number of key/value pairs
c: capacity of the array (# of buckets)

$$\lambda = \frac{n}{c}$$

# Hashing

- What about non-integer data?
  - Remember the definition -- **Hash Function**: any function that can be used to map data of an arbitrary size to fixed-size values.




- Considerations for Hash Functions:
  1. **Deterministic** – same input should generate the same output
  2. **Efficient** – reasonable runtime
  3. **Uniform** – inputs spread "evenly" across output range

# Hashing

**Implementation 1: Simple aspect of values**

```
public int hashCode(String input) {
   return input.length();
}
```

**Pro:** super fast
**Con:** lots of collisions!

**Implementation 2: More aspects of value**

```
public int hashCode(String input) {
   int output = 0;
   for(char c : input) {
      out += (int)c;
   }
   return output;
}
```

**Pro:** still really fast
**Con:** some collisions

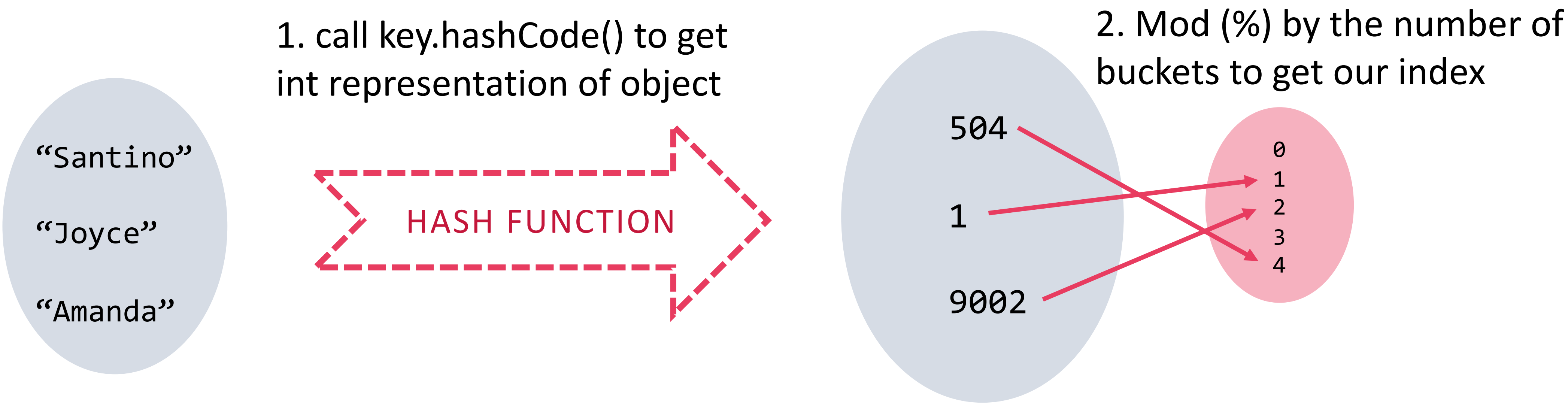
**Implementation 3: Multiple aspects of value + math!**

```
public int hashCode(String input) {
   int output = 1;
   for (char c : input) {
      int nextPrime = getNextPrime();
      out *= Math.pow(nextPrime, (int)c);
    }
   return Math.pow(nextPrime, input.length());
}
```

**Pro:** few collisions
**Con:** slower, gigantic integers

# Hashing

- Fortunately, experts have made most of these design decisions for us!
  - All objects in Java have a `.hashCode()` method that does some magic to make a “good” hash for any object type (e.g. String, ArrayList, Scanner)
  - The built-in hashCode() has a good distribution/not a lot of collisions
- More precisely, hashCode() just gets us an int representation: *then* we % by size



# *Review* Iterators

- **Iterator**: a Java interface that dictates how a collection of data should be traversed. Can only move forward and in a single pass.

**Iterator Interface**

Behavior

`hasNext()` – true if elements remain
`next()` – returns next element

**hasNext()** – returns true if the iteration has more elements yet to be examined

**next()** – returns the next element in the iteration and moves the iterator forward to next item

**Two ways to *use* an iterator in Java:**

```
ArrayList<Integer> list;

Iterator itr = list.iterator();
while (itr.hasNext()) {
   int item = itr.next();
}
```

```
ArrayList<Integer> list;

for (int i : list) {
   int item = i;
}
```

# P2 Reminders

- Implementing an iterator for a Hash Map is *complex*!
  - You need to iterate through the elements of a bucket, but when you reach the end of the chain, *have to move to the next bucket*
  - "you're not iterating over some linear data structure, you're playing 2D chess" – Howard Xiao
- Start early! P2 is out for over 1.5 weeks, but for good reason!
  - Especially the ChainedHashMap iterator
- Remember to read the entire Tips section of the instructions!