LEC 14

CSE 373

Graphs

BEFORE WE START

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Announcements

- P2 late cutoff tonight at 11:59pm
- P3 due just under weeks on Friday, 11/13
  - Start early!
  - Remember that changePriority and contains aren't efficient on a heap alone – you should **use an extra data structure**!
  - Recommendation: just get it working first, then analyze where inefficiencies are – what data structure could help?
- EX3 published this Friday, 11/06
  - Focusing on post-Exam I content, especially this week

LEC 12 PQs & Heaps

LEC 13 Heaps II, Interviews

P3 Heap

LEC 14 Graphs

LEC 15 BFS, DFS, SP

LEC 16 Dijkstra's

EX3 BFS/DFS/Dijkstra's

Learning Objectives

After this lecture, you should be able to...

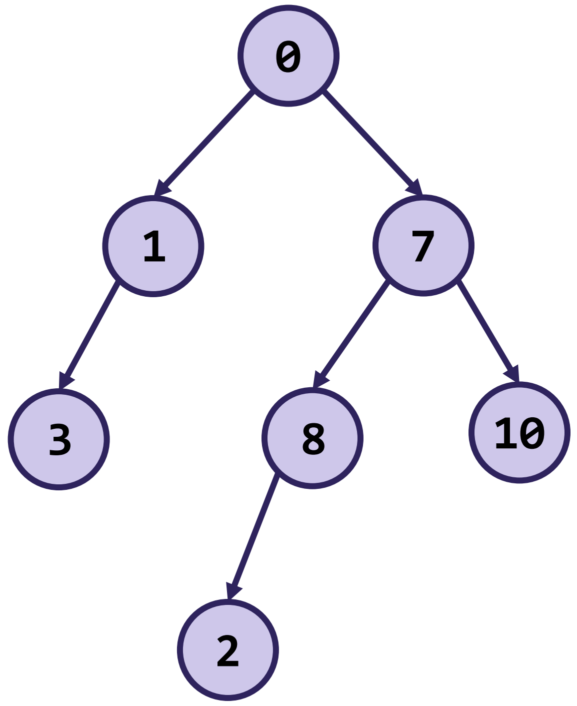
1. Categorize graph data structures based on which properties they exhibit
2. Select which properties of a graph would be most appropriate to model a scenario (e.g. Directed/Undirected, Cyclic/Acyclic, etc.)
3. Compare the runtimes of Adjacency Matrix and Adjacency List graph implementations, and select the most appropriate one for a particular problem
4. Describe the high-level algorithm for solving the s-t Connectivity Problem, and be prepared to expand on it going forward

Lecture Outline

- Graphs
  - Definitions
  - Choosing Graph Types
- Graph Implementations
- s-t Connectivity Problem

Review Trees

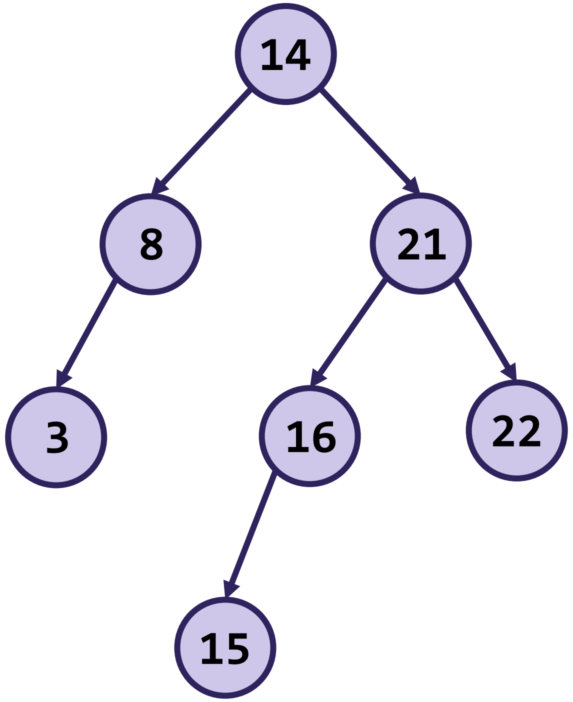
- A **tree** is a collection of nodes where each node has at most 1 parent and at least 0 children
  - A **binary tree** is a tree where each node has at most 2 children



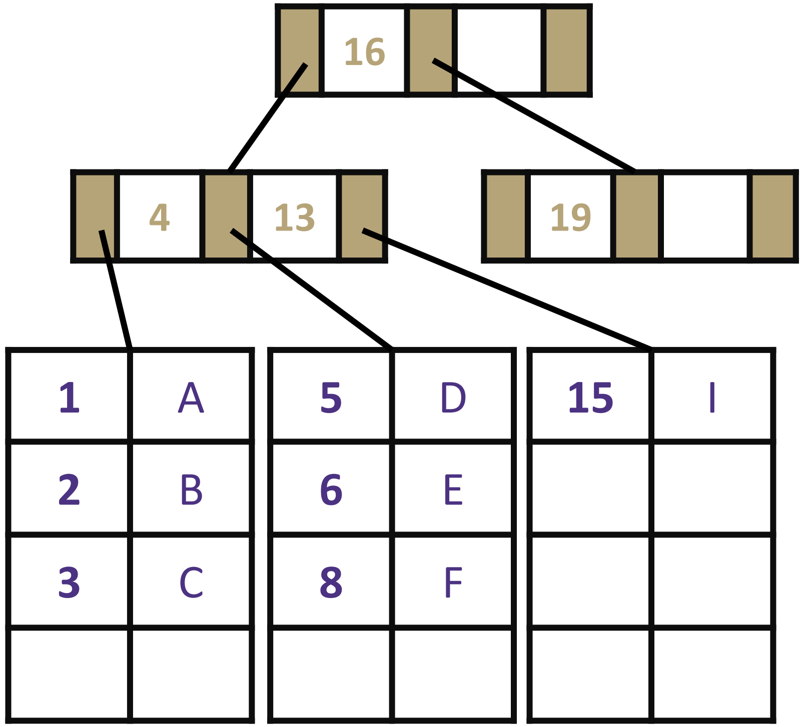
- **Root node:** the single node with no parent, "top" of the tree
- **Leaf node:** a node with no children
- **Subtree:** a node and all its descendants
- **Edge:** connection between parent and a child

Review Trees We've Seen So Far

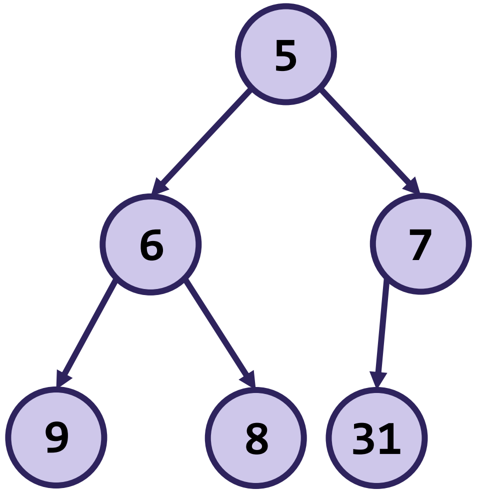
Binary Search Trees
- And variant: **AVL Trees**



B+ Trees



Binary Min-Heaps



Inter-data Relationships

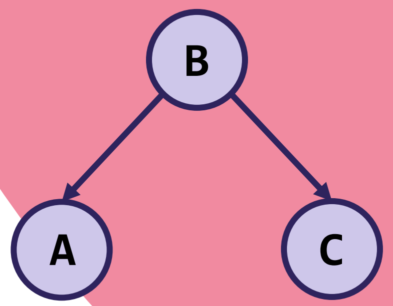
Arrays

- Elements only store pure data, no connection info
- Only relationship between data is order

0	1	2
A	B	C

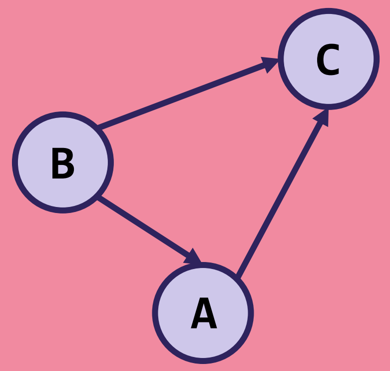
Trees

- Elements store data and connection info
- Directional relationships between nodes; limited connections



Graphs

- Elements AND connections can store data
- Relationships dictate structure; huge freedom with connections



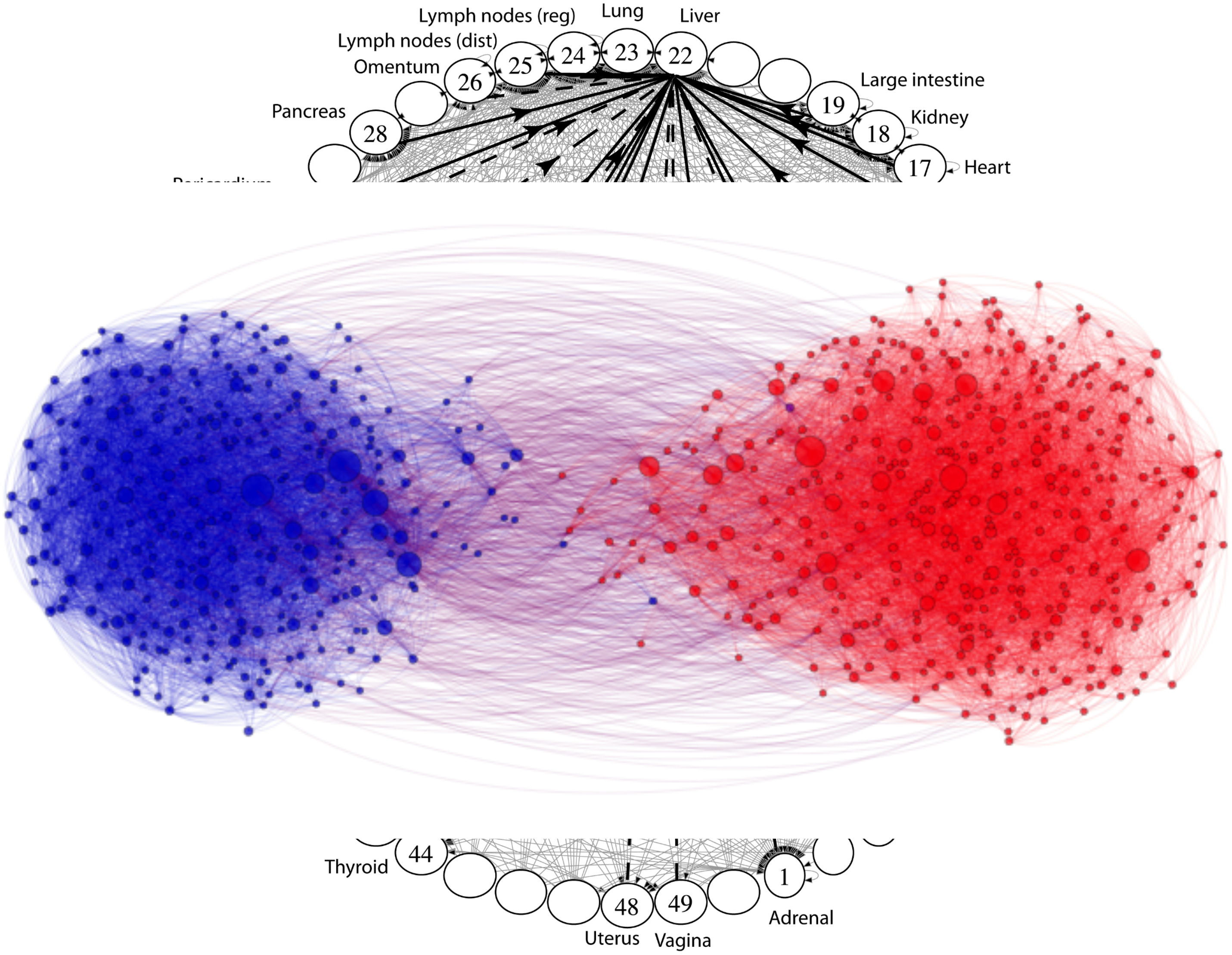
Everything is Graphs

- **Everything** is graphs.
- Most things we've studied this quarter can be represented by graphs.
  - BSTs are graphs
  - Linked lists? Graphs.
  - Heaps? Also can be represented as graphs.
  - Those trees we drew in the tree method? Graphs.
- But it's not just data structures that we've discussed…
  - Google Maps database? Graph.
  - Facebook? They have a "graph search" team. Because it's a graph
  - Gitlab's history of a repository? Graph.
  - Those pictures of prerequisites in your program? Graphs.
  - Family tree? That's a graph

Applications

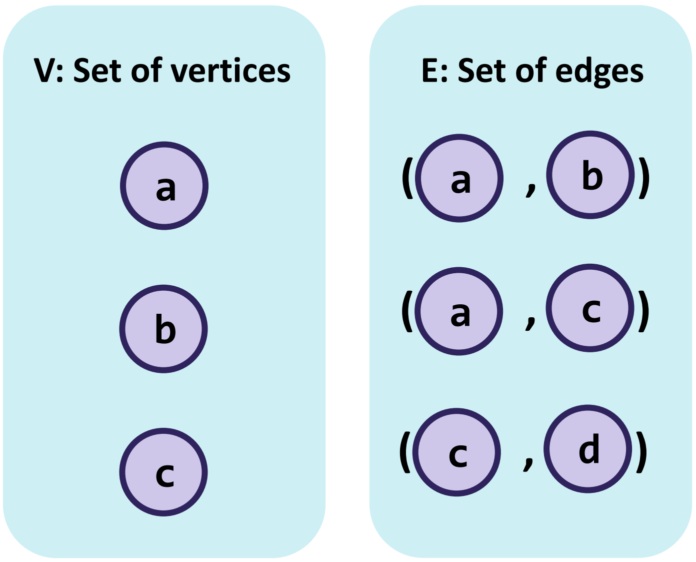
- Physical Maps
  - Airline maps
    - Vertices are airports, edges are flight paths
  - Traffic
    - Vertices are addresses, edges are streets
- Relationships
  - Social media graphs
    - Vertices are accounts, edges are follower relatior
  - Code bases
    - Vertices are classes, edges are usage
- Influence
  - Biology
    - Vertices are cancer cell destinations, edges are m
- Related topics
  - Web Page Ranking
    - Vertices are web pages, edges are hyperlinks
  - Wikipedia
    - Vertices are articles, edges are links

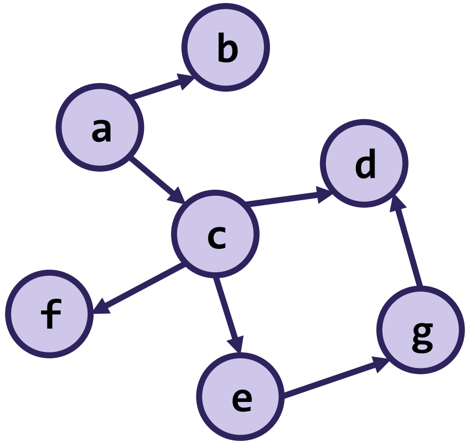
So many more:
www.allthingsgraphed.com

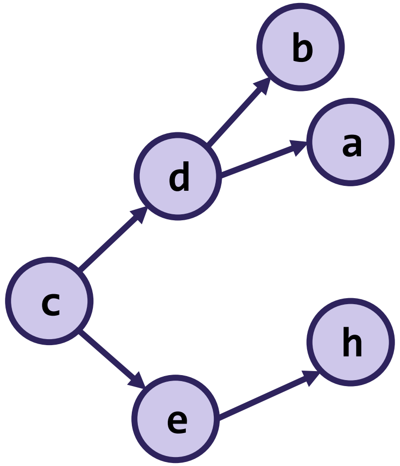


Graphs

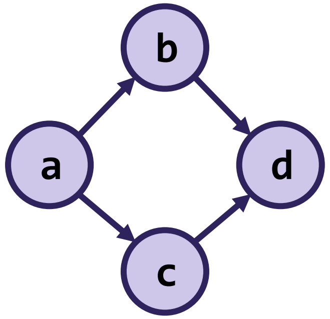
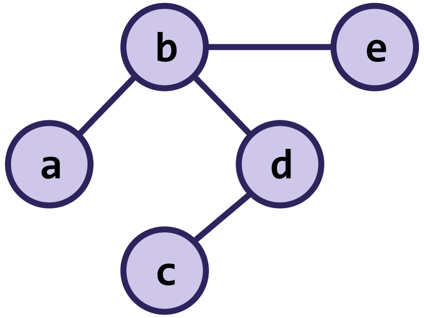
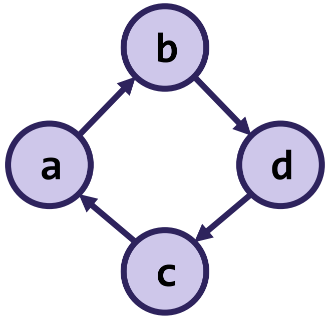
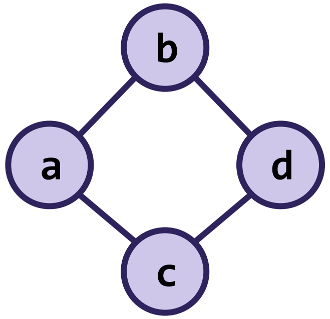
- A **Graph** consists of two sets, V and E:
  - V: Set of **vertices** (aka **nodes**)
  - E: Set of **edges** (pairs of vertices)
  - |V|: Size of V (also called n)
  - |E|: Size of E (also called m)





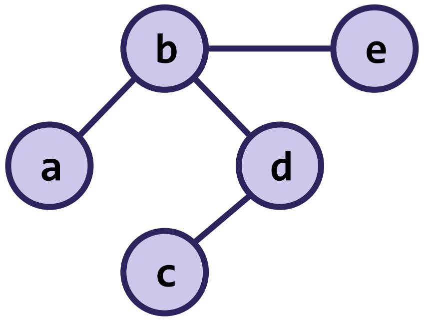


Directed vs Undirected; Acyclic vs Cyclic

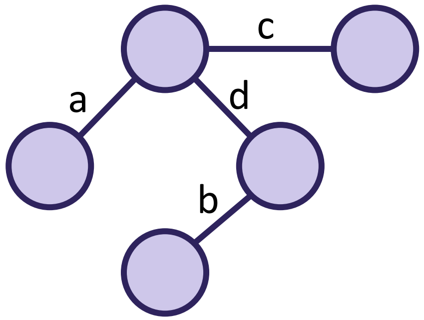
	Directed:	Undirected:
Acyclic:		
Cyclic:		

Labeled and Weighted Graphs

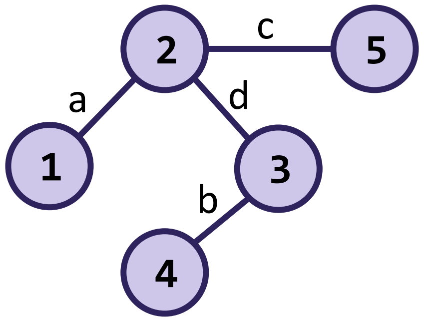
Vertex Labels



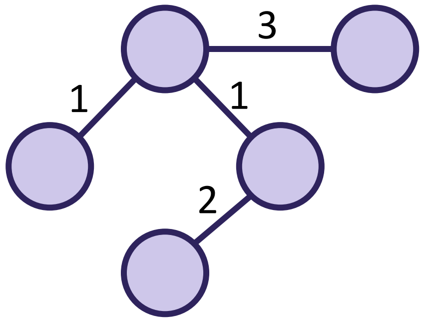
Edge Labels



Vertex & Edge Labels

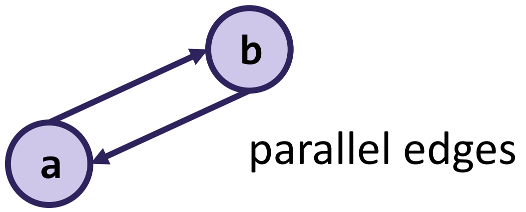


Numeric Edge Labels
(**Edge Weights**)



More Graph Terminology

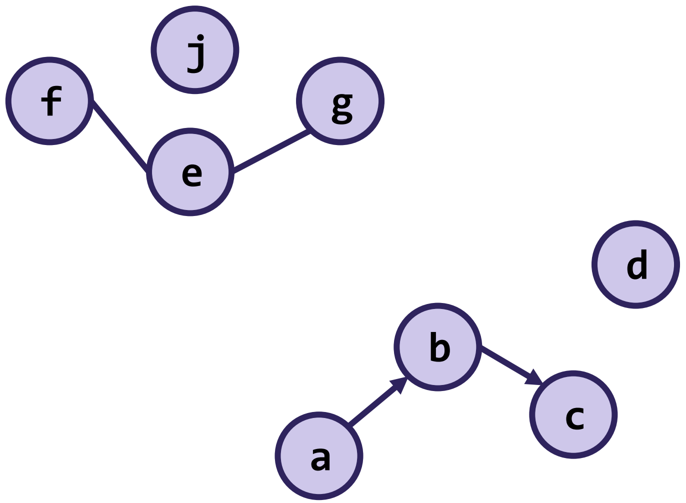
- A **Simple Graph** has no **self-loops** or **parallel edges**
  - In a simple graph, |E| is O(|V|^2)
  - Unless otherwise stated, all graphs in this course are simple



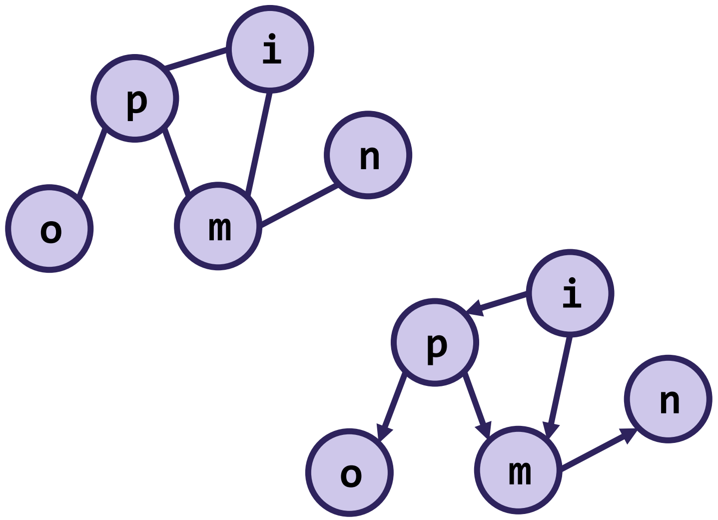
- Vertices with an edge between them are **adjacent**
  - Vertices or edges may have optional **labels**
    - Numeric edge labels are sometimes called **weights**

More More Graph Terminology

- Two vertices are **connected** if there is a path between them
  - If all the vertices are connected, we say the graph is **connected**
  - The number of edges leaving a vertex is its **degree**



- A **path** is a sequence of vertices connected by edges
  - A **simple path** is a path without repeated vertices
  - A **cycle** is a path whose first and last vertices are the same
    - A graph with a cycle is **cyclic**



Lecture Outline

- **Graphs**
  - Definitions
  - **Choosing Graph Types**
- Graph Implementations
- s-t Connectivity Problem

Some examples

- For each of the following: what should you choose for vertices and edges? Directed?
- **Webpages on the Internet**
- **Ways to walk between UW buildings**
- **Course Prerequisites**

Some examples

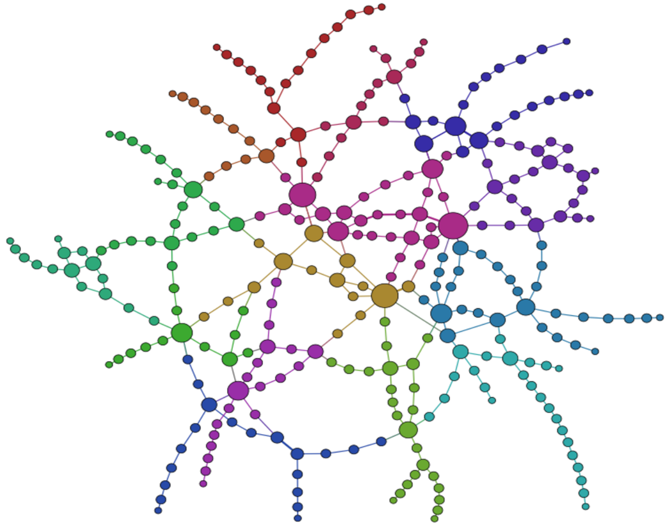
- For each of the following: what should you choose for vertices and edges? Directed?
- **Webpages on the Internet**
  - **Vertices**: webpages. **Edges** from a to b if a has a hyperlink to b.
  - Directed, since hyperlinks go in one direction
- **Ways to walk between UW buildings**
  - **Vertices**: buildings. **Edges**: A street name or walkway that connects 2 buildings
  - Undirected, since each route can be walked both ways
- **Course Prerequisites**
  - **Vertices**: courses. **Edge**: from a to b if a is a prereq for b.
  - Directed, since one course comes before the other

Itempool

Pause Video when Prompted

This schematic map of the Paris Métro is a graph. Which of the following characteristics make sense here?

Introduction to Network Visualization with GEPHI – Martin Grandjean
Examples



A. Undirected / Connected / Cyclic / Vertex-labeled
B. Directed / Connected / Cyclic / Vertex-labeled
C. Undirected / Connected / Cyclic / Edge-labeled
D. Directed / Connected / Cyclic / Edge-labeled
E. I’m not sure …

Lecture Outline

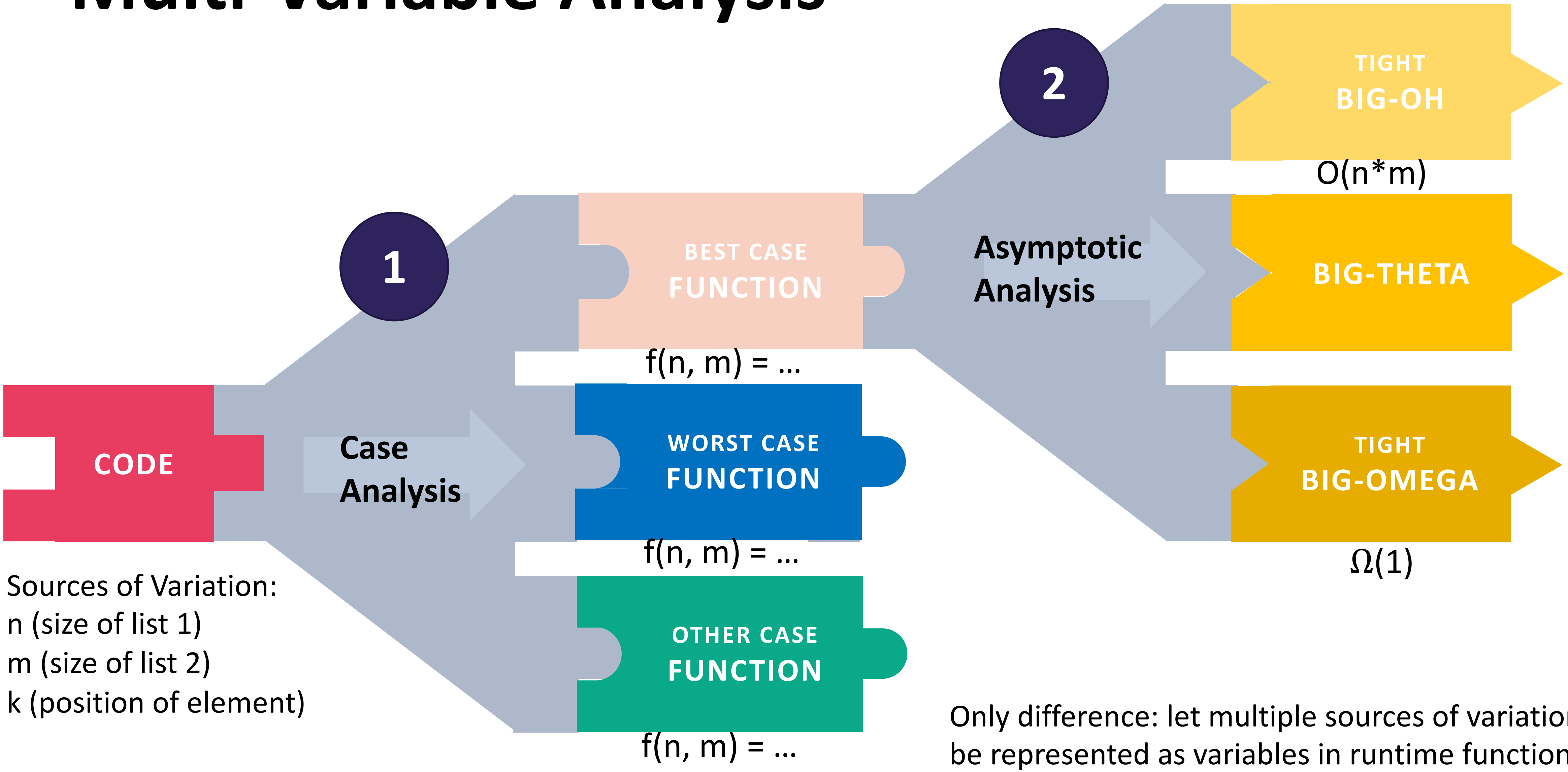
- Graphs
  - Definitions
  - Choosing Graph Types
- **Graph Implementations**
- s-t Connectivity Problem

Multi-Variable Analysis

- So far, we thought of everything as being in terms of some single argument “n” (sometimes its own parameter, other times a size)
  - But there’s no reason we can’t do reasoning in terms of multiple inputs!
- Why multi-variable?
  - Remember, algorithmic analysis is just a tool to help us understand code. Sometimes, it helps our understanding more to build a Oh/Omega/Theta bound for multiple factors, rather than handling those factors in case analysis.

- With graphs, we usually do our reasoning in terms of:
  - n (or |V|): total number of vertices (sometimes just call it V)
  - m (or |E|): total number of edges (sometimes just call it E)
  - deg(u): degree of node u (how many outgoing edges it has)

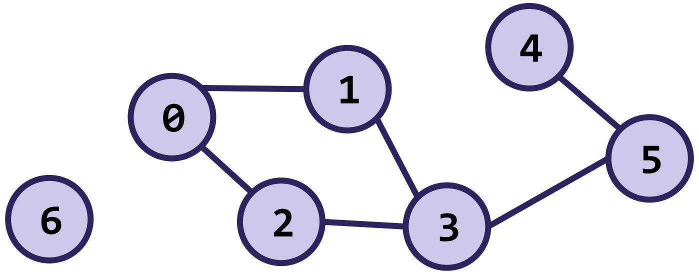
Multi-Variable Analysis



Sources of Variation:
n (size of list 1)
m (size of list 2)
k (position of element)

Only difference: let multiple sources of variation be represented as variables in runtime functions, instead of wrapping them up into cases!

Adjacency Matrix



- Create a 2D matrix that is |V| x |V|
- In an adjacency matrix, a[u][v] is 1 if there is an edge (u,v), and 0 otherwise.
- Symmetric for undirected graphs

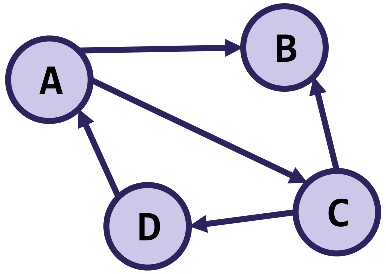
	0	1	2	3	4	5	6
0	0	1	1	0	0	0	0
1	1	0	0	1	0	0	0
2	1	0	0	1	0	0	0
3	0	1	1	0	0	1	0
4	0	0	0	0	0	1	0
5	0	0	0	1	1	0	0
6	0	0	0	0	0	0	0

Add Edge	$\Theta(1)$
Remove Edge	$\Theta(1)$
Check if edge (u, v) exists	$\Theta(1)$
Get out-neighbors of u	$\Theta(n)$
Get in-neighbors of v	$\Theta(n)$
(Space Complexity)	$\Theta(n^2)$

(|V| = n, |E| = m)

Adjacency List

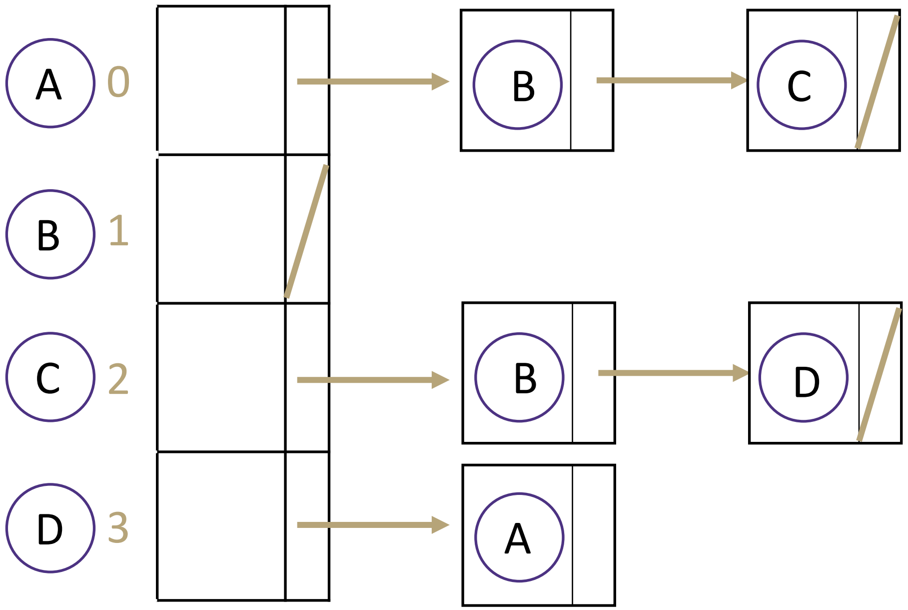
- Create a Map from V to some Collection of E
- In an adjacency list, if (u,v) ∈ E, then v is found in the collection under key u
- Since each node maps to a list of its neighbors, in undirected graph every edge will be included twice
    - In directed graph, every edge from u is in list associated with key u.



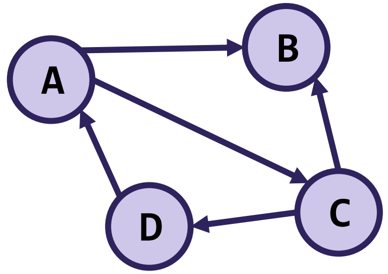
Add Edge	$\Theta(1)$
Remove Edge	$\Theta(\deg(u))$
Check if edge (u, v) exists	$\Theta(\deg(u))$
Get out-neighbors of u	$\Theta(\deg(u))$
Get in-neighbors of v	$\Theta(n+m)$
(Space Complexity)	$\Theta(n+m)$

(|V| = n, |E| = m)

Linked Lists



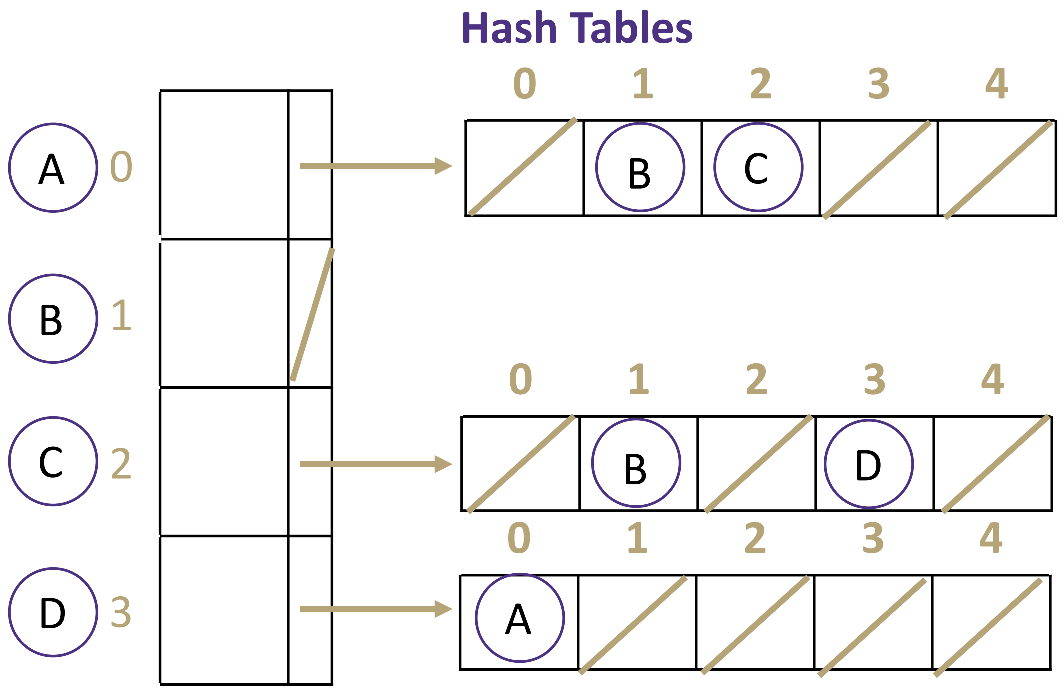
Best of Both Worlds?



- Can use Hashing as an in-between solution
- Represent the Adjacency Matrix as a `Map<Node, Map<Node, EdgeLabel>>`
- Not quite as much space as Adjacency Matrix but get the constant-time "in practice" lookup.

Add Edge	$\Theta(1)$
Remove Edge	$\Theta(1)$
Check if edge (u, v) exists	$\Theta(1)$
Get out-neighbors of u	$\Theta(\deg(u))$
Get in-neighbors of v	$\Theta(n)$
(Space Complexity)	$\Theta(n+m)$

(|V| = n, |E| = m)



Tradeoffs

- Adjacency Matrices take more space, why would you use them?
  - For **dense** graphs (where m is close to n^2), the running times will be close
  - And the constant factors can be much better for matrices than for lists.
  - Sometimes the matrix itself is useful ("spectral graph theory")
- What's the tradeoff between using linked lists and hash tables for the list of neighbors?
  - A hash table still *might* hit a worst-case
  - And the linked list might not
    - Graph algorithms often just need to iterate over all the neighbors, so you might get a better guarantee with the linked list.

373: Graph Implementations

- For this class, unless we say otherwise, we'll assume the hash tables operations **on graphs** are all $O(1)$.
  - Because you can probably control the keys.
- Unless we say otherwise, assume we're using the hash table approach.