CSE 373 18wi: Summation Identities

Splitting a sum

$$\sum_{i=a}^{b}(x+y) = \sum_{i=a}^{b} x + \sum_{i=a}^{b} y$$

Adjusting summation bounds

$$\sum_{i=a}^{b} f(x) = \sum_{i=0}^{b} f(x) - \sum_{i=0}^{a-1} f(x)$$

Factoring out a constant

$$\sum_{i=a}^{b} cf(i) = c\sum_{i=a}^{b} f(i)$$

Summation of a constant

$$\sum_{i=0}^{n-1} c = \underbrace{c + c + \ldots + c}_{n \text{ times}} = cn$$

Note: this rule is a special case of the rule on the left

Gauss's identity

$$\sum_{i=0}^{n-1} i = 0 + 1 + \ldots + n - 1 = \frac{n(n-1)}{2}$$

Sum of squares

$$\sum_{i=0}^{n-1} i^2 = \frac{n(n-1)(2n-1)}{6}$$

Finite geometric series

$$\sum_{i=0}^{n-1} x^i = \frac{x^n - 1}{x - 1}$$

Infinite geometric series

$$\sum_{i=0}^{\infty} x_i = \frac{1}{1-x}$$

Note: applicable only when $-1 < x < 1$