



# Lecture 25: Final Review

CSE 373: Data Structures and Algorithms



# Announcements

## Asks of you

- Course evals by Sunday
- Course survey for 3 Socratic ec points
- TA nominations

## Grades

- HW4 & HW6 scores go out later today
- HW

## Logistics

- Final is on TUESDAY March 19th 8:30-10:20am here in Smith
- Erik Review: TODAY 3:30-5:30

## What comes after 373?

- 417
- Projects
- TA
- internships

# On the exam

## Graphs

### Graph definitions

- Directed vs undirected
- Weighted vs unweighted
- Walks vs paths vs cycles
- Self-loops and parallel edges
- Simple vs non-simple graphs (e.g. multigraphs)
- Trees, DAGs
- Strongly connected components

### Graph representations

- Adjacency list
- Adjacency matrix
- Pros and cons of each

### Graph algorithms

- Graph traversals: BFS and DFS
- Single-source shortest-path algorithms: Dijkstra's algorithm
- Topological sorts
- MST algorithms: Prim and Kruskal
- Disjoint sets

## Sorting

- Quadratic sorts: insertion sort, selection sort
- Faster sorts: heap sort, merge sort, quick sort
- Understand the runtimes of all of the above (in the best and worst case)

## Memory and Locality

- Basics of memory architecture
- Spatial and temporal locality

## P vs NP

- Definitions of P, NP and NP Complete
- Understand what a reduction is

## Design Decisions

- Given a scenario, what ADT, data structure implementation and/or algorithm is best optimized for your goals?
  - What is the purpose of the ADTs we've learned?
  - Given a scenario, determine whether one data structure would be a better fit than another (and explain why)
  - What is the optimal implementation of an ADT for a given situation?
- What is the runtime of a data structure's implementations of each ADT behavior?
- How can you leverage an algorithm to answer a given question?

## NOT on the exam

- Java generics and Java interfaces.
- JUnit.
- Java syntax.

# Algorithms you're responsible for

For each of the listed algorithms make sure you understand:

## In what situations it is useful

- What will this tell us about the data
- What state should the data be in to use it?

## What the pros and cons of applying that algorithm are

- Runtime
- Memory usage

## Heaps

- percolateUp
- percolateDown
- Floyd's Build Heap

## Sorting

- Insertion
- Selection
- Merge
- Quick
- Heap

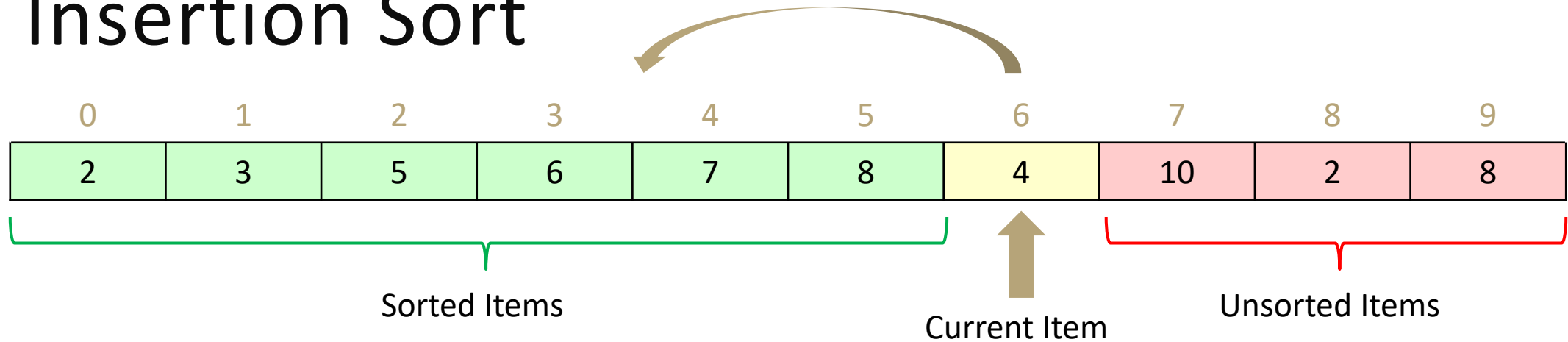
## Graphs

- Breadth First Search (BFS)
- Depth First Search (DFS)
- Dijkstra's
- Topological Sort
- Prim's MST
- Kruskal's MST

## Disjoint Sets

- Union by rank
- Path compression

# Insertion Sort



```
public void insertionSort(collection) {  
    for (entire list)  
        if(currentItem is bigger than nextItem)  
            int newIndex = findSpot(currentItem);  
            shift(newIndex, currentItem);  
}  
public int findSpot(currentItem) {  
    for (sorted list)  
        if (spot found) return  
}  
public void shift(newIndex, currentItem) {  
    for (i = currentItem > newIndex)  
        item[i+1] = item[i]  
    item[newIndex] = currentItem  
}
```

Worst case runtime?  $O(n^2)$

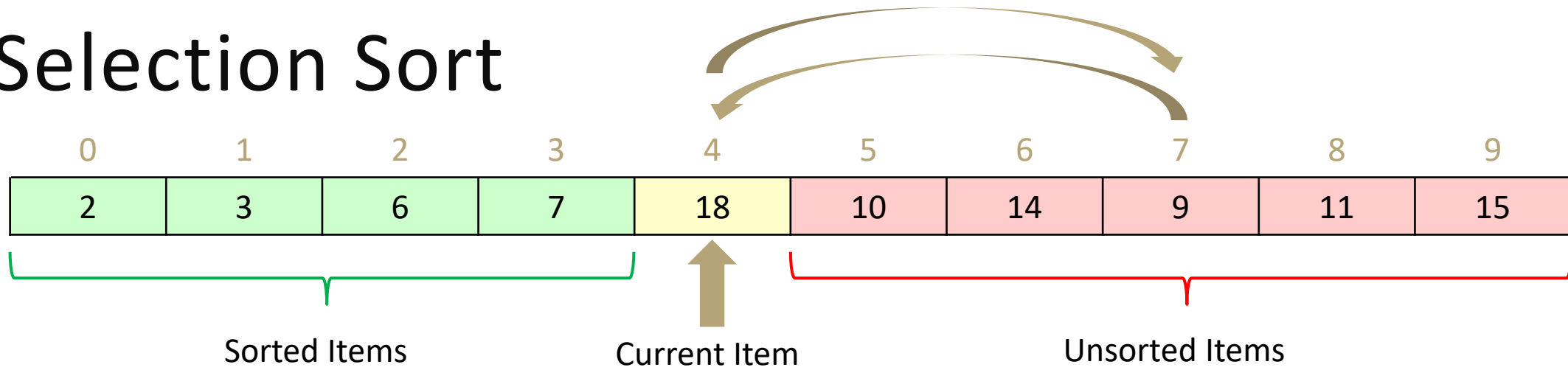
Best case runtime?  $O(n)$

Average runtime?  $O(n^2)$

Stable? Yes

In-place? Yes

# Selection Sort



```
public void selectionSort(collection) {  
    for (entire list)  
        int newIndex = findNextMin(currentItem);  
        swap(newIndex, currentItem);  
}  
public int findNextMin(currentItem) {  
    min = currentItem  
    for (unsorted list)  
        if (item < min)  
            min = currentItem  
    return min  
}  
public int swap(newIndex, currentItem) {  
    temp = currentItem  
    currentItem = newIndex  
    newIndex = temp  
}
```

Worst case runtime?  $O(n^2)$

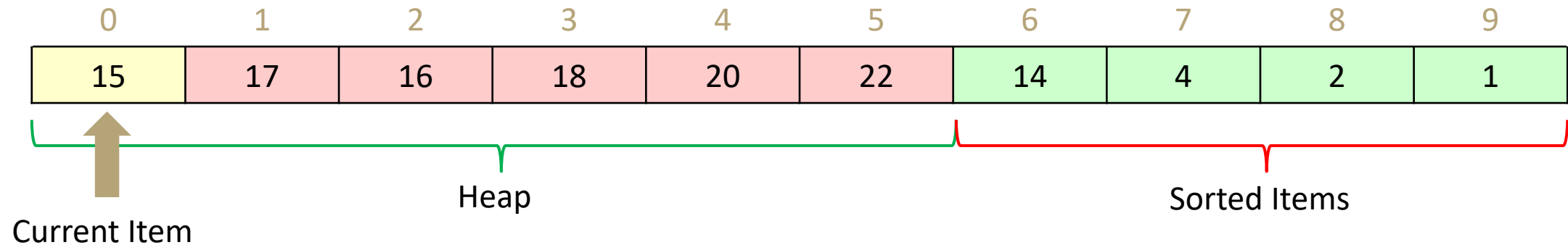
Best case runtime?  $O(n^2)$

Average runtime?  $O(n^2)$

Stable? Yes

In-place? Yes

# In Place Heap Sort



```
public void inPlaceHeapSort(collection) {  
    E[] heap = buildHeap(collection)  
    for (n)  
        output[n - i - 1] = removeMin(heap)  
}
```

Complication: final array is reversed!

- Run reverse afterwards ( $O(n)$ )
- Use a max heap
- Reverse compare function to emulate max heap

Worst case runtime?  $O(n \log n)$

Best case runtime?  $O(n \log n)$

Average runtime?  $O(n \log n)$

Stable? No

In-place? Yes

# Merge Sort

```
mergeSort(input) {
  if (input.length == 1)
    return
  else
    smallerHalf = mergeSort(new [0, ..., mid])
    largerHalf = mergeSort(new [mid + 1, ...])
    return merge(smallerHalf, largerHalf)
}
```

Worst case runtime?

Best case runtime?

$T(n) =$

$\begin{cases} 1 & \text{if } n \leq 1 \\ 2T(n/2) + n & \text{otherwise} \end{cases}$   
 $= O(n \log n)$

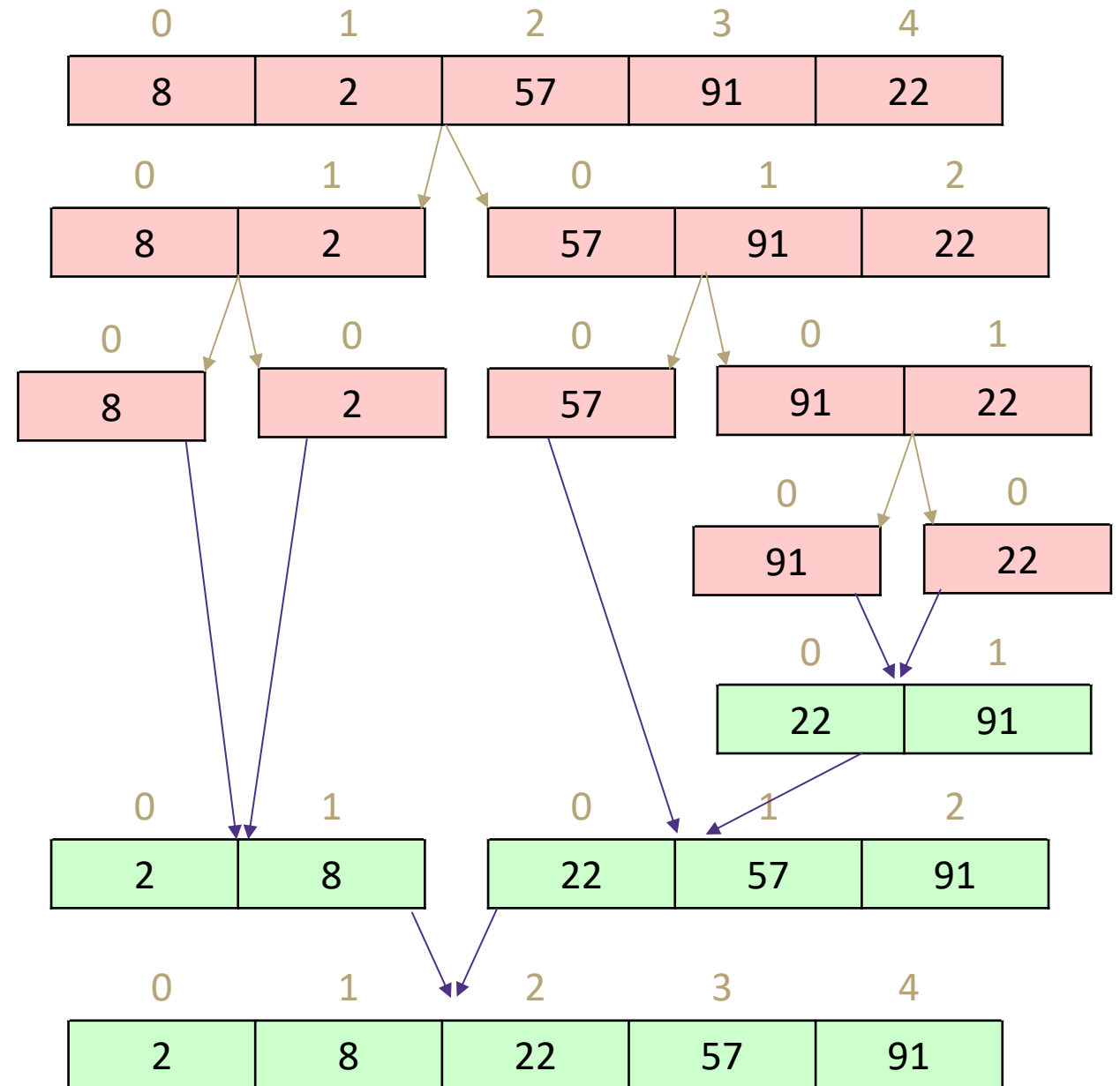
Average runtime?

Stable?

Yes

In-place?

No





# Quick Sort

```
quickSort(input) {
  if (input.length == 1)
    return
  else
    pivot = getPivot(input)
    smallerHalf = quickSort(getSmaller(pivot, input))
    largerHalf = quickSort(getBigger(pivot, input))
    return smallerHalf + pivot + largerHalf
}
```

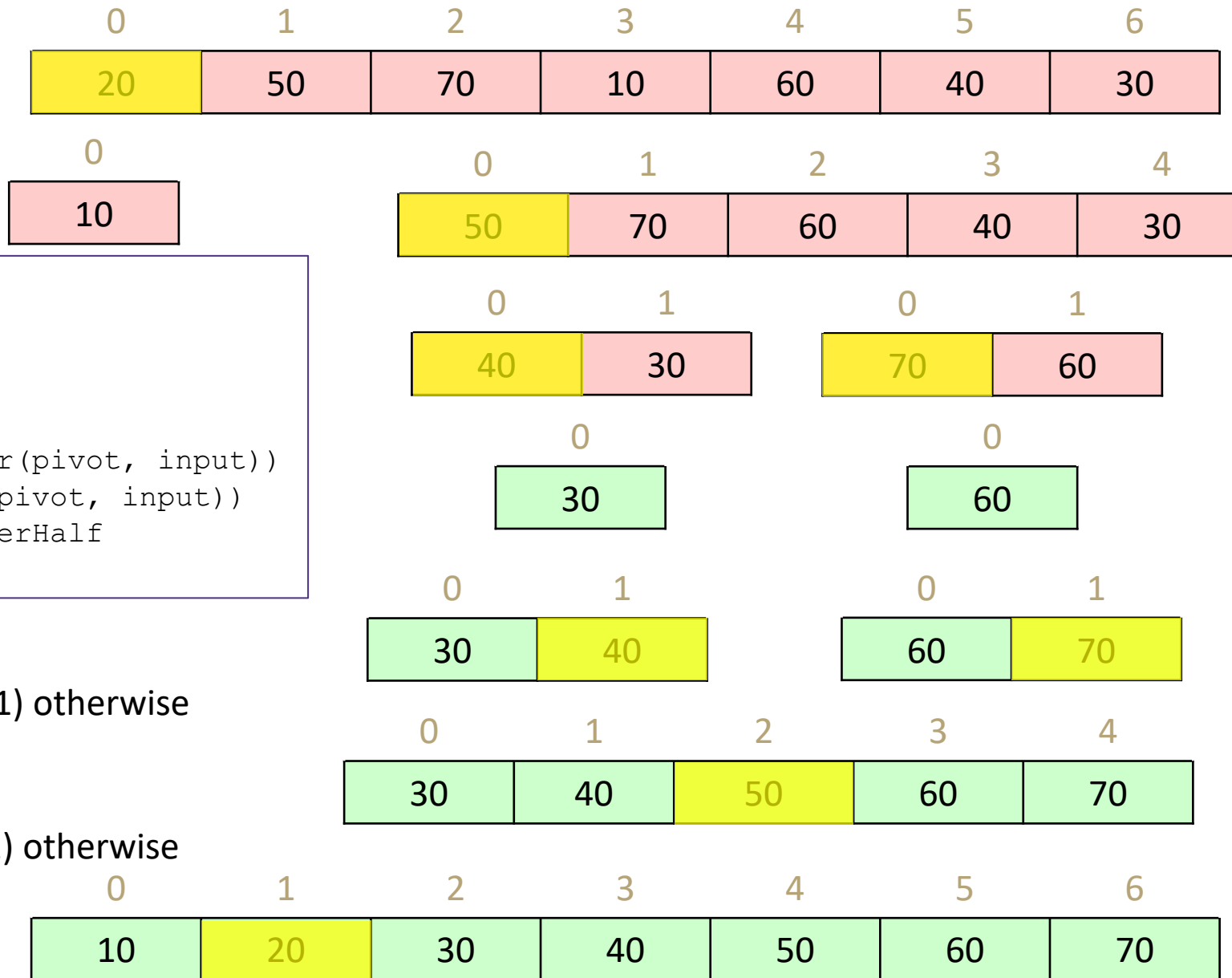
Worst case runtime?  $T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ n + T(n - 1) & \text{otherwise} \end{cases}$

Best case runtime?  $T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ n + 2T(n/2) & \text{otherwise} \end{cases}$

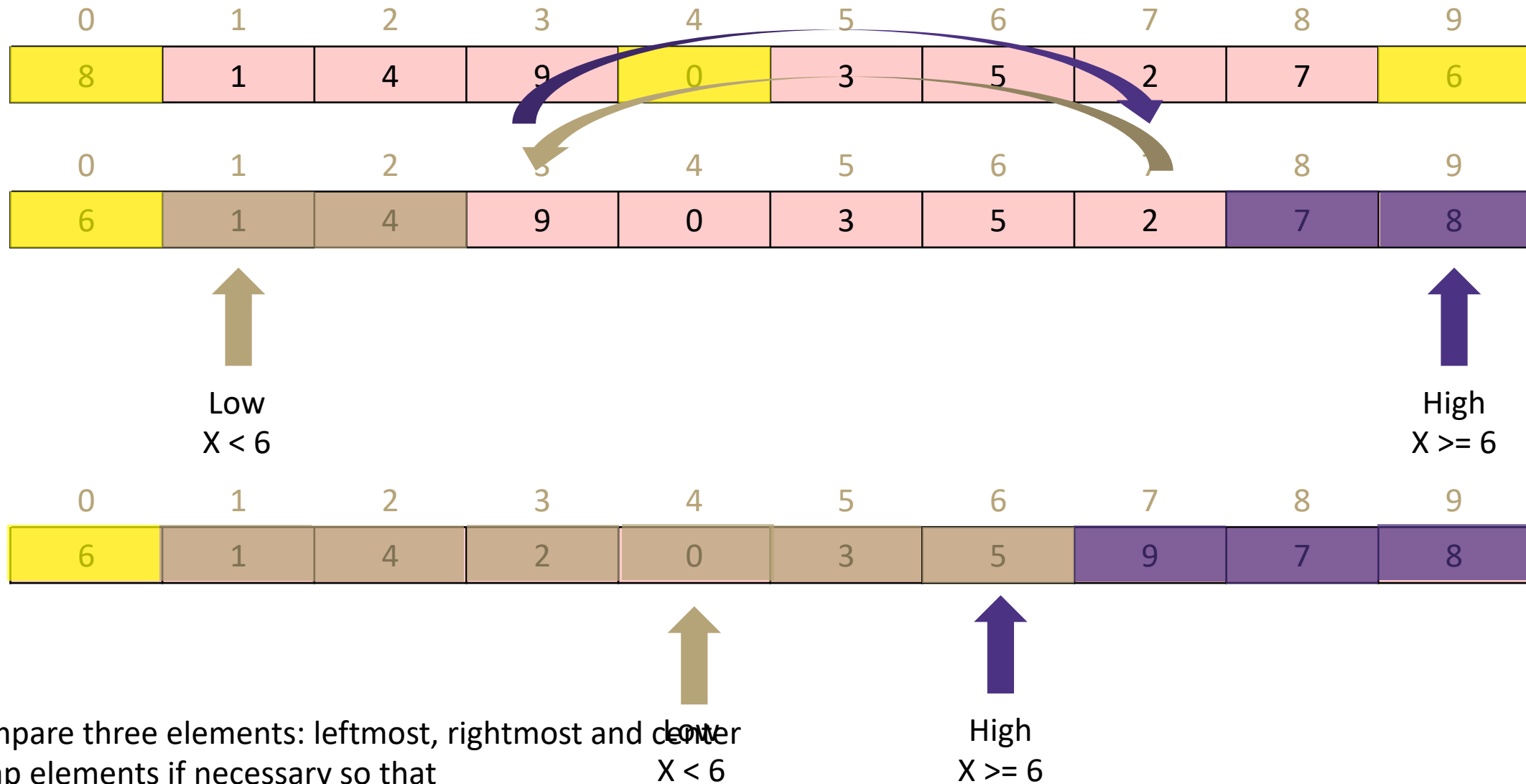
Average runtime?

Stable? No

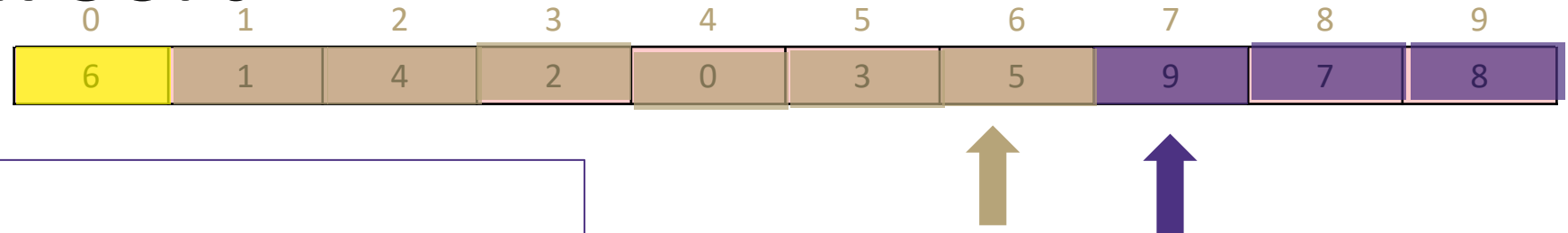
In-place? No



# Better Quick Sort



# Better Quick Sort



```
quickSort(input) {  
  if (input.length == 1)  
    return  
  else  
    pivot = getPivot(input)  
    smallerHalf = quickSort(getSmaller(pivot, input))  
    largerHalf = quickSort(getBigger(pivot, input))  
    return smallerHalf + pivot + largerHalf  
}
```

Worst case runtime?

Best case runtime?  $T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ n + 2T(n/2) & \text{otherwise} \end{cases}$

Average runtime?

Stable? No

In-place? Yes

# Graph: Formal Definition

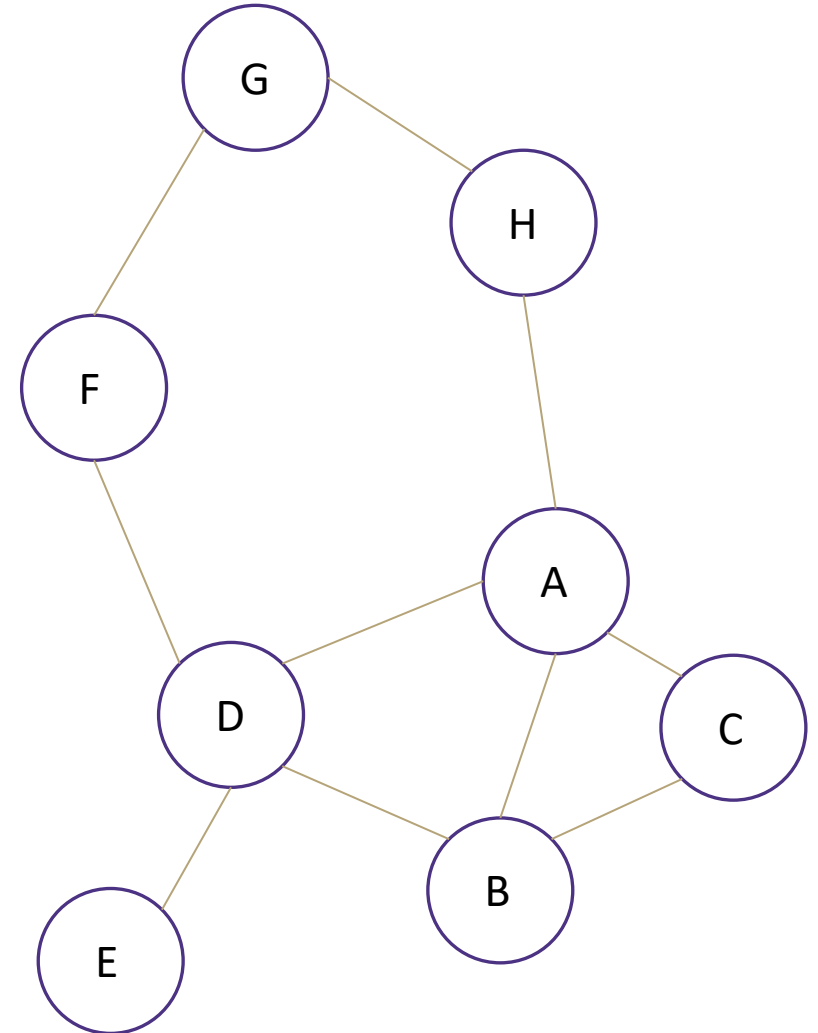
A **graph** is defined by a pair of sets  $G = (V, E)$  where...

- $V$  is a set of **vertices**
  - A vertex or “node” is a data entity

$V = \{A, B, C, D, E, F, G, H\}$

- $E$  is a set of **edges**
  - An edge is a connection between two vertices

$E = \{(A, B), (A, C), (A, D), (A, H),$   
 $(C, B), (B, D), (D, E), (D, F),$   
 $(F, G), (G, H)\}$



# Graph Vocabulary

## Graph Direction

- **Undirected graph** – edges have no direction and are two-way

$$V = \{ A, B, C \}$$

$$E = \{ (A, B), (B, C) \} \text{ inferred } (B, A) \text{ and } (C, B)$$

- **Directed graphs** – edges have direction and are thus one-way

$$V = \{ A, B, C \}$$

$$E = \{ (A, B), (B, C), (C, B) \}$$

## Degree of a Vertex

- **Degree** – the number of edges containing that vertex

$$A : 1, B : 1, C : 1$$

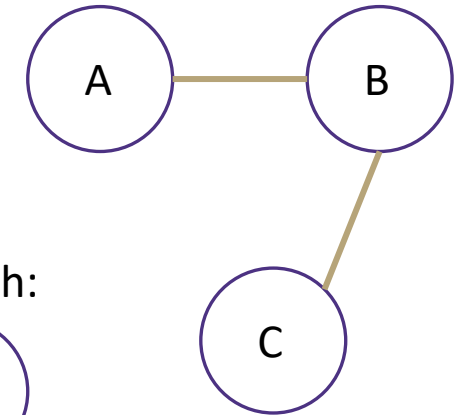
- **In-degree** – the number of directed edges that point to a vertex

$$A : 0, B : 2, C : 1$$

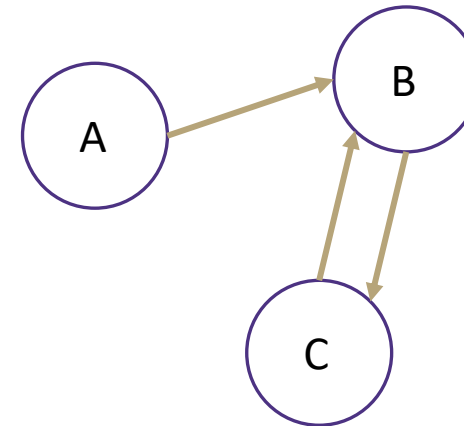
- **Out-degree** – the number of directed edges that start at a vertex

$$A : 1, B : 1, C : 1$$

Undirected Graph:



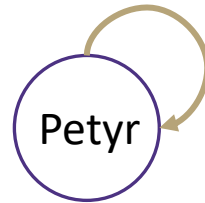
Undirected Graph:



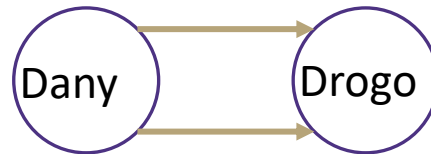


# Graph Vocabulary

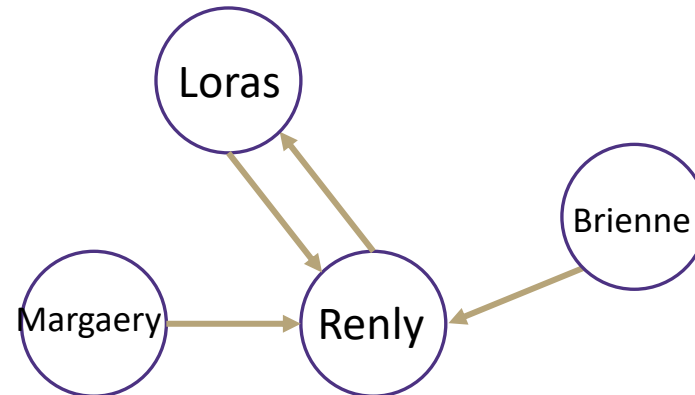
**Self loop** – an edge that starts and ends at the same vertex



**Parallel edges** – two edges with the same start and end vertices



**Simple graph** – a graph with no self-loops and no parallel edges



# Adjacency Matrix

Assign each vertex a number from 0 to  $V - 1$

Create a  $V \times V$  array of Booleans

If  $(x,y) \in E$  then  $arr[x][y] = \text{true}$

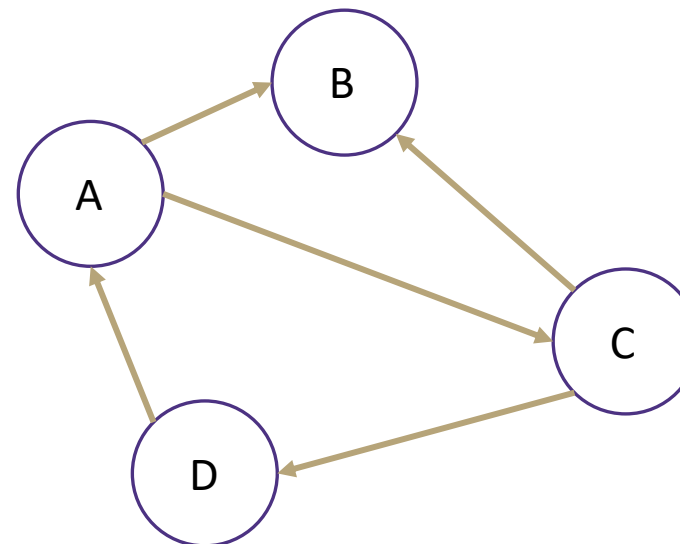
Runtime (in terms of  $V$  and  $E$ )

- get out - edges for a vertex  $O(V)$
- get in - edges for a vertex  $O(V)$
- decide if an edge exists  $O(1)$
- insert an edge  $O(1)$
- delete an edge  $O(1)$
- delete a vertex
- add a vertex

How much space is used?

$V^2$

	A	B	C	D
A		T	T	
B				
C		T		T
D	T			



# Graph Vocabulary

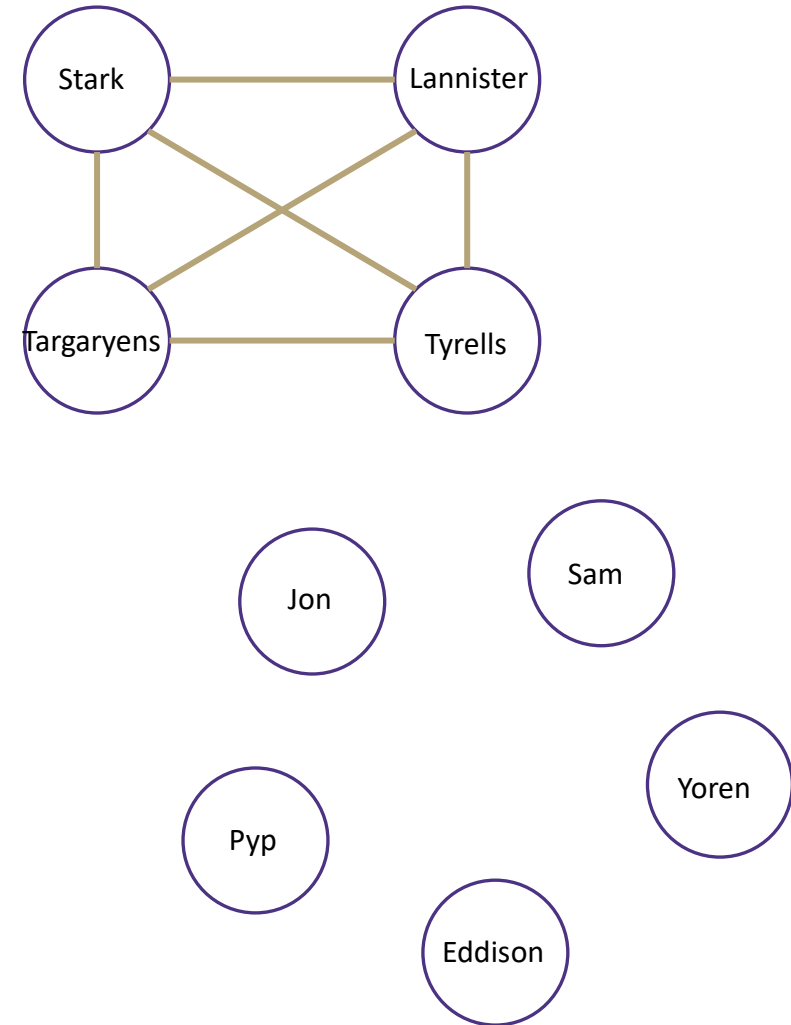
**Dense Graph** – a graph with a lot of edges

$$E \in \Theta(V^2)$$

**Sparse Graph** – a graph with “few” edges

$$E \in \Theta(V)$$

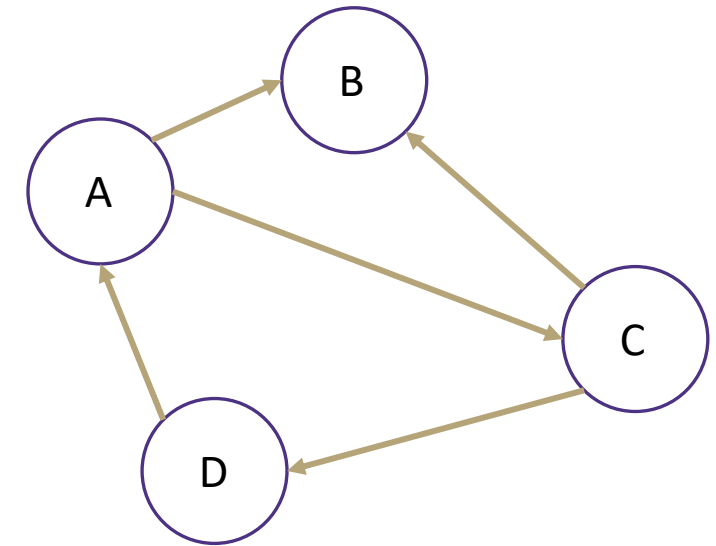
An Adjacency Matrix seems a waste for a sparse graph...



# Adjacency List

Create a Dictionary of size  $V$  from type  $V$  to Collection of  $E$

If  $(x,y) \in E$  then add  $y$  to the set associated with the key  $x$

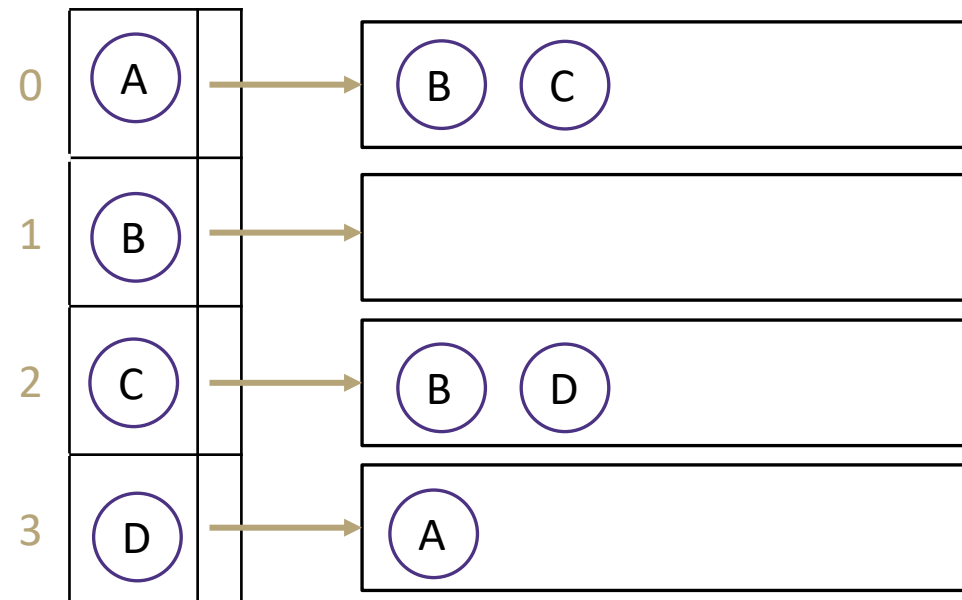


Runtime (in terms of  $V$  and  $E$ )

- get out - edges for a vertex  $O(1)$
- get in - edges for a vertex  $O(V + E)$
- decide if an edge exists  $O(1)$
- insert an edge  $O(1)$
- delete an edge  $O(1)$
- delete a vertex
- add a vertex

How much space is used?

$V + E$

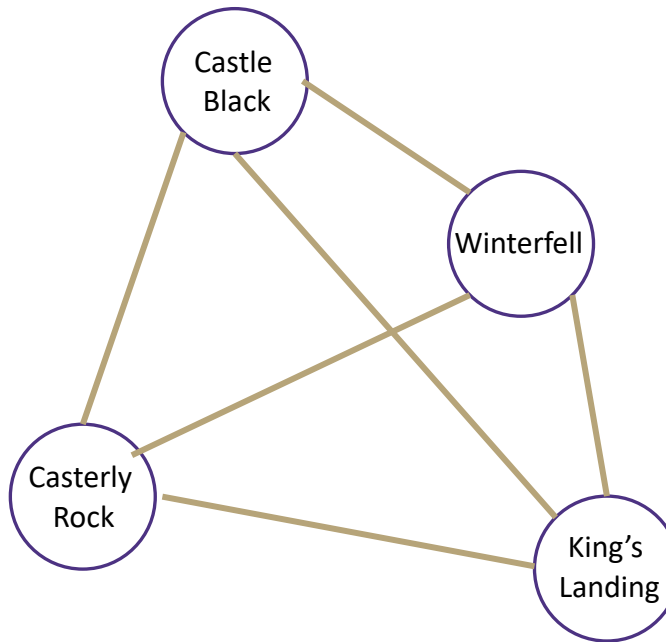
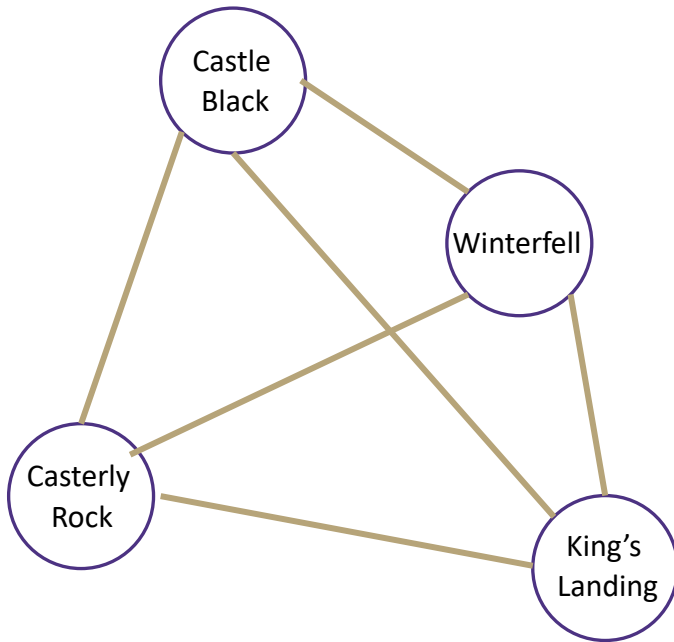


# Walks and Paths

Walk – continuous set of edges leading from vertex to vertex

A list of vertices where if  $l$  is some int where  $0 < l < V_n$  every pair  $(V_i, V_{i+1})$  in  $E$  is true

Path – a walk that never visits the same vertex twice

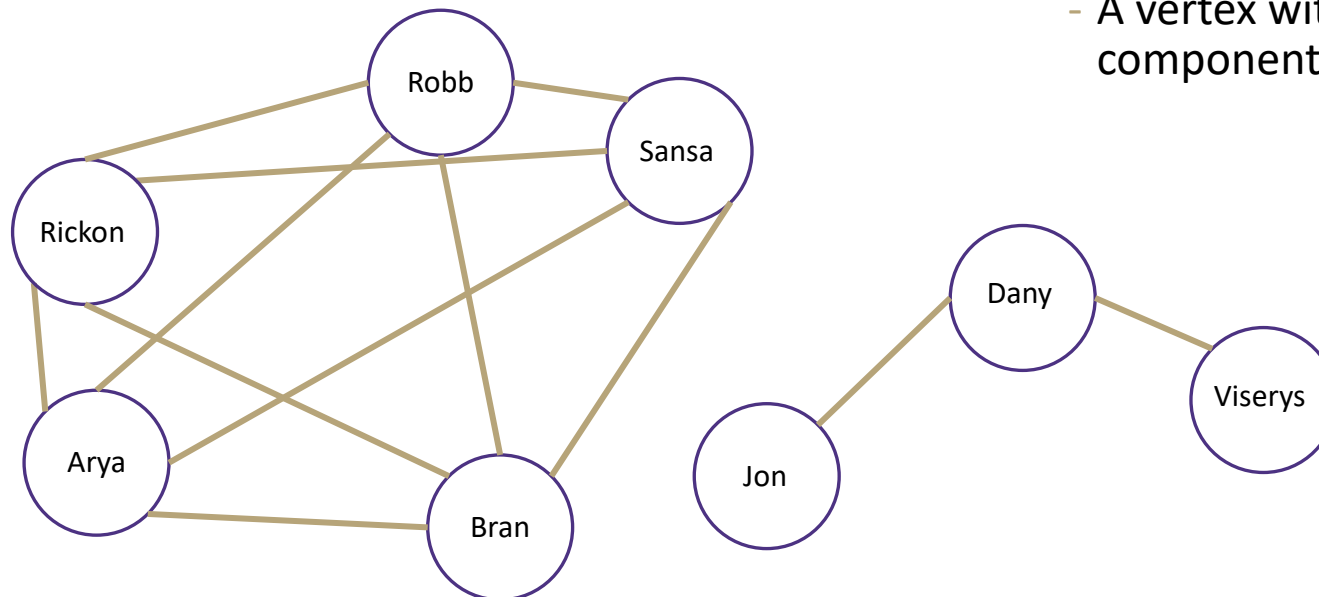




# Connected Graphs

**Connected graph** – a graph where every vertex is connected to every other vertex via some path. It is not required for every vertex to have an edge to every other vertex

There exists some way to get from each vertex to every other vertex



**Connected Component** – a *subgraph* in which any two vertices are connected via some path, but is connected to no additional vertices in the *supergraph*

- There exists some way to get from each vertex within the connected component to every other vertex in the connected component
- A vertex with no edges is itself a connected component

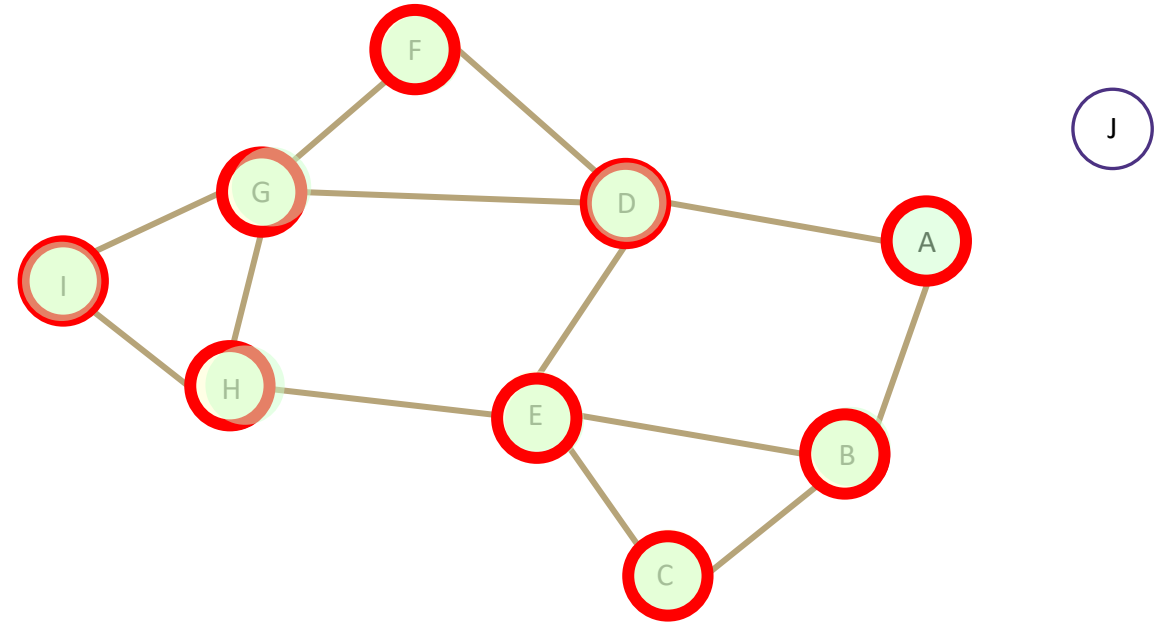
# Breadth First Search

```
search(graph)
  toVisit.enqueue(first vertex)
  while(toVisit is not empty)
    current = toVisit.dequeue()
    for (V : current.neighbors())
      if (V is not in queue)
        toVisit.enqueue(v)
    visited.add(current)
```

Current node: I

Queue: B D E C F G H I

Visited: A B D E C F G H I



# Depth First Search

```
dfs(graph)
  toVisit.push(first vertex)
  while(toVisit is not empty)
    current = toVisit.pop()
    for (V : current.neighbors())
      if (V is not in stack)
        toVisit.push(v)
    visited.add(current)
```

Current node: D

Stack: D B E I H G

Visited: A B E H G F I C D

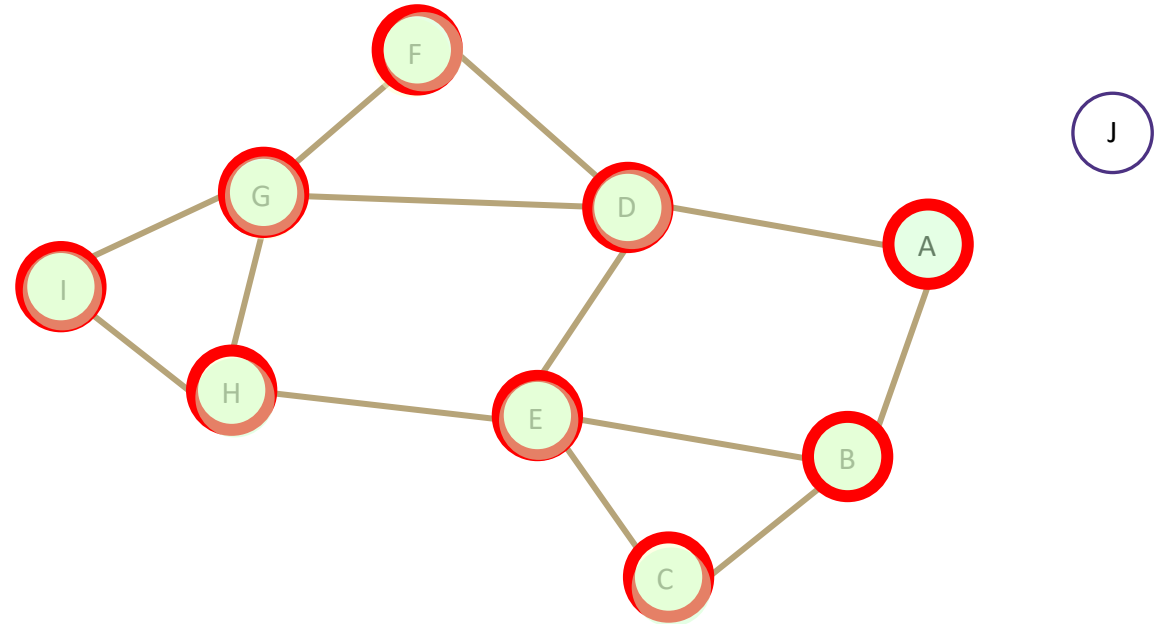
How many times do you visit each node?

1 time each

How many times do you traverse each edge?

Max 2 times each

- Putting them into toVisit
- Checking if they're in toVisit

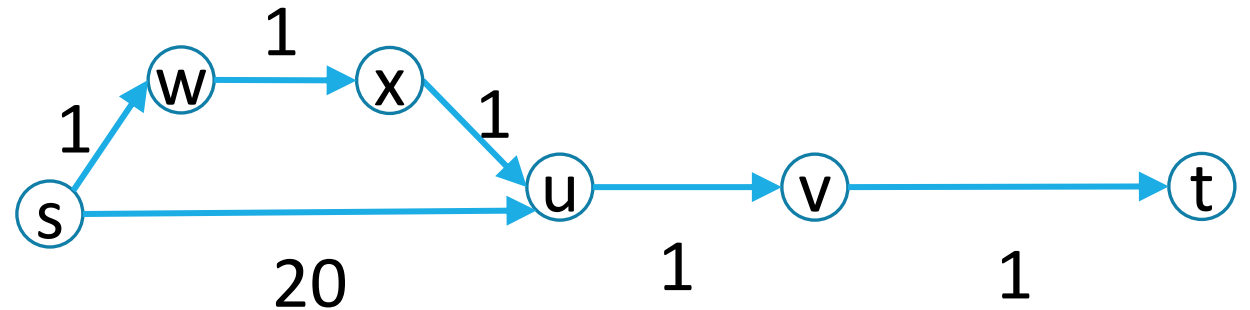


Runtime?  $O(V + 2E) = O(V + E)$  "graph linear"

# Dijkstra's Algorithm

```
Dijkstra(Graph G, Vertex source)
  initialize distances to  $\infty$ 
  mark source as distance 0
  mark all vertices unprocessed
  while(there are unprocessed vertices){
    let u be the closest unprocessed vertex
    foreach(edge (u,v) leaving u){
      if(u.dist+weight(u,v) < v.dist){
        v.dist = u.dist+weight(u,v)
        v.predecessor = u
      }
    }
    mark u as processed
  }
```

Vertex	Distance	Predecessor	Processed
s	0	--	Yes
w	1	s	Yes
x	2	w	Yes
u	<del>20</del> 3	<del>s</del> x	Yes
v	4	u	Yes
t	5	v	Yes



# Dijkstra's Runtime

```
Dijkstra(Graph G, Vertex source)
```

```
+V for (Vertex v : G.getVertices()) { v.dist = INFINITY; }
```

```
+C1 {  
    G.getVertex(source).dist = 0;  
    initialize MPQ as a Min Priority Queue, add source
```

```
while(MPQ is not empty){
```

```
    u = MPQ.removeMin(); +logV
```

```
    for (Edge e : u.getEdges(u)){
```

```
        oldDist = v.dist; newDist = u.dist+weight(u,v)
```

```
        if(newDist < oldDist){
```

```
            v.dist = newDist
```

```
            v.predecessor = u
```

```
            if(oldDist == INFINITY) { MPQ.insert(v) } +logV
```

```
            else { MPQ.updatePriority(v, newDist) } +?
```

```
        }
```

```
    }
```

```
}
```

**Code Model =  $C_1 + V + V(\log V + E(C_2 + 2\log V))$**   
**=  $C_1 + V + V\log V + VEC_2 + VEC_3\log V$**

**Tight O Bound =  $O(V\log V)$**

How often do we actually update  
the MPQ thanks to this if  
statement?

E times!

**Tight O Bound =  $O(V\log V + E\log V)$**

(assume logV)



# How Do We Find a Topological Ordering?

TopologicalSort(Graph G, Vertex source)

count how many incoming edges each vertex has

Collection toProcess = new Collection()

foreach(Vertex v in G){

if(v.edgesRemaining == 0)

toProcess.insert(v)

}

topOrder = new List()

while(toProcess is not empty){

u = toProcess.remove()

topOrder.insert(u)

foreach(edge (u,v) leaving u){

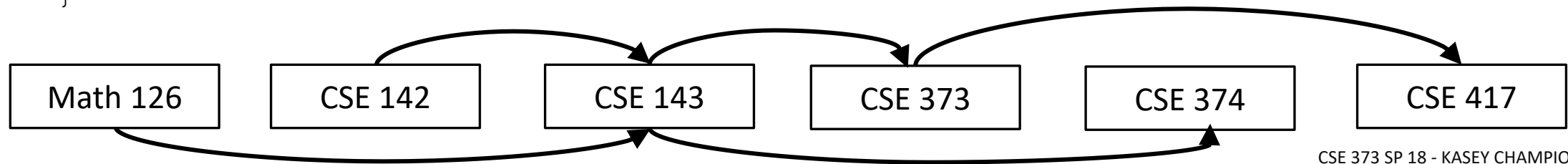
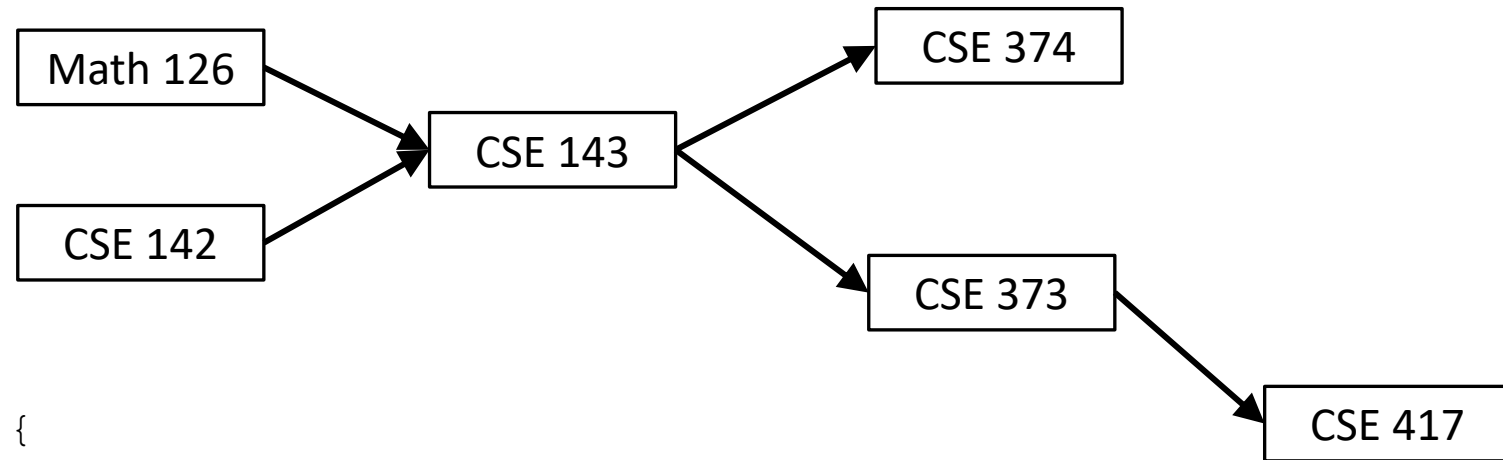
v.edgesRemaining--

if(v.edgesRemaining == 0)

toProcess.insert(v)

}

}



# How Do We Find a Topological Ordering?

```
TopologicalSort(Graph G, Vertex source)
```

```
    count how many incoming edges each vertex has ← BFS
```

```
    Collection toProcess = new Collection()
```

```
    foreach(Vertex v in G){
```

```
        if(v.edgesRemaining == 0)
```

```
            toProcess.insert(v)
```

```
    }
```

```
    topOrder = new List()
```

```
    while(toProcess is not empty){
```

```
        u = toProcess.remove()
```

```
        topOrder.insert(u)
```

```
        foreach(edge (u,v) leaving u){
```

```
            v.edgesRemaining--
```

```
            if(v.edgesRemaining == 0)
```

```
                toProcess.insert(v)
```

```
        }
```

```
    }
```

Graph linear  
+ V + E

**$O(V + E)$**

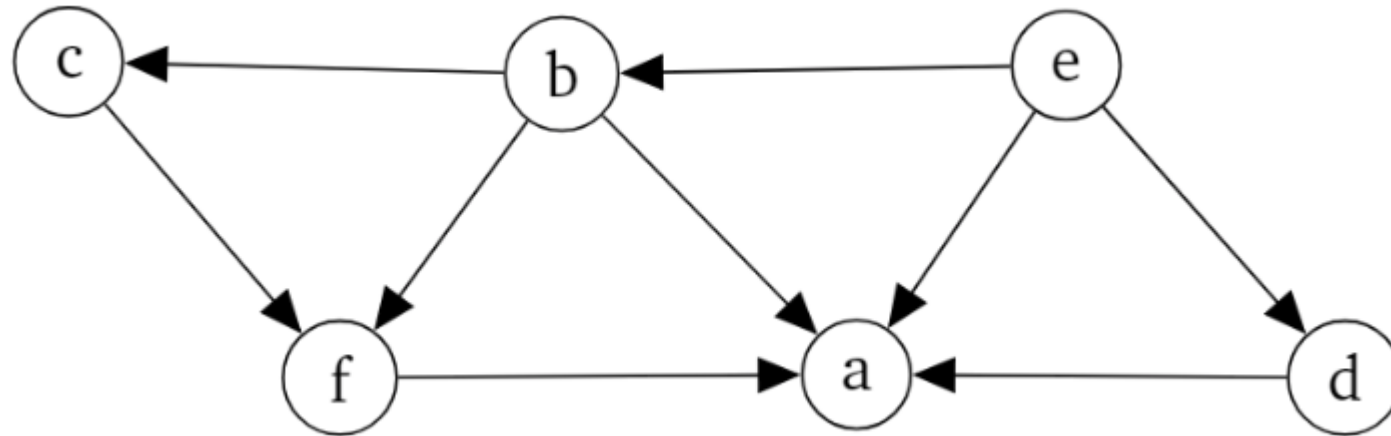
+V

Runs as most once per edge  
+E

Pick something with  
 $O(1)$  insert / removal

# Practice

What is a possible ordering of the graph after a topological sort?



All possible orderings:

e, d, b, c, f, a

e, b, d, c, f, a

e, b, c, d, f, a

e, b, c, f, a

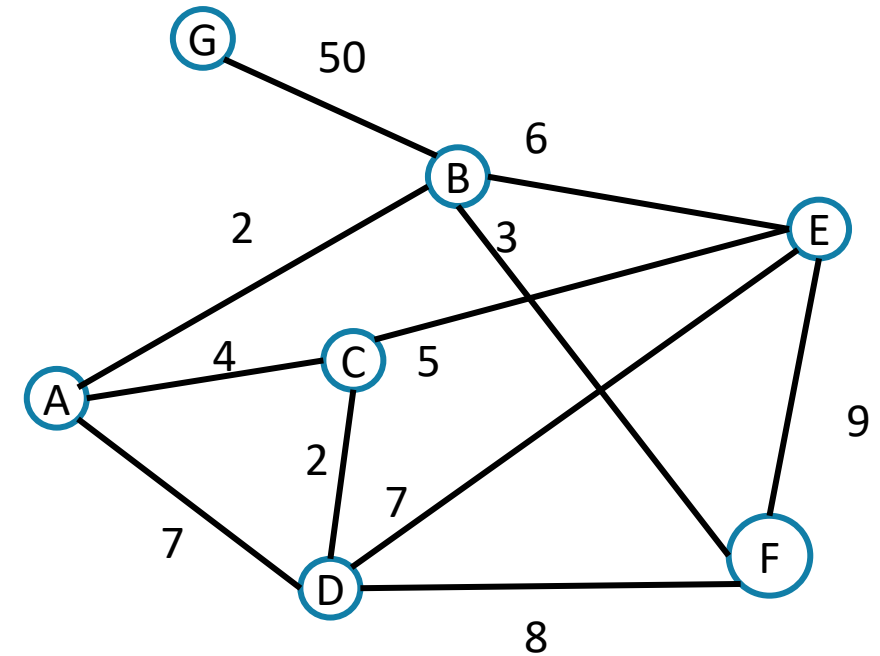
# Try it Out

PrimMST(Graph G)

```

    initialize distances to  $\infty$ 
    mark source as distance 0
    mark all vertices unprocessed
    foreach(edge (source, v) ) {
        v.dist = weight(source,v)
        v.bestEdge = (source,v)
    }
    while(there are unprocessed vertices){
        let u be the closest unprocessed vertex
        add u.bestEdge to spanning tree
        foreach(edge (u,v) leaving u){
            if(weight(u,v) < v.dist && v unprocessed ){
                v.dist = weight(u,v)
                v.bestEdge = (u,v)
            }
        }
        mark u as processed
    }

```



Vertex	Distance	Best Edge	Processed
A	-	X	✓
B	2	(A, B)	✓
C	4	(A, C)	✓
D	<del>7</del> -2	<del>(A, D)</del> (C, D)	✓
E	<del>6</del> -5	<del>(B, E)</del> (C, E)	✓
F	3	(B, F)	✓
G	50	(B, G)	✓

# Try It Out

```
KruskalMST(Graph G)
```

```
  initialize each vertex to be an independent component
```

```
  sort the edges by weight
```

```
  foreach(edge (u, v) in sorted order){
```

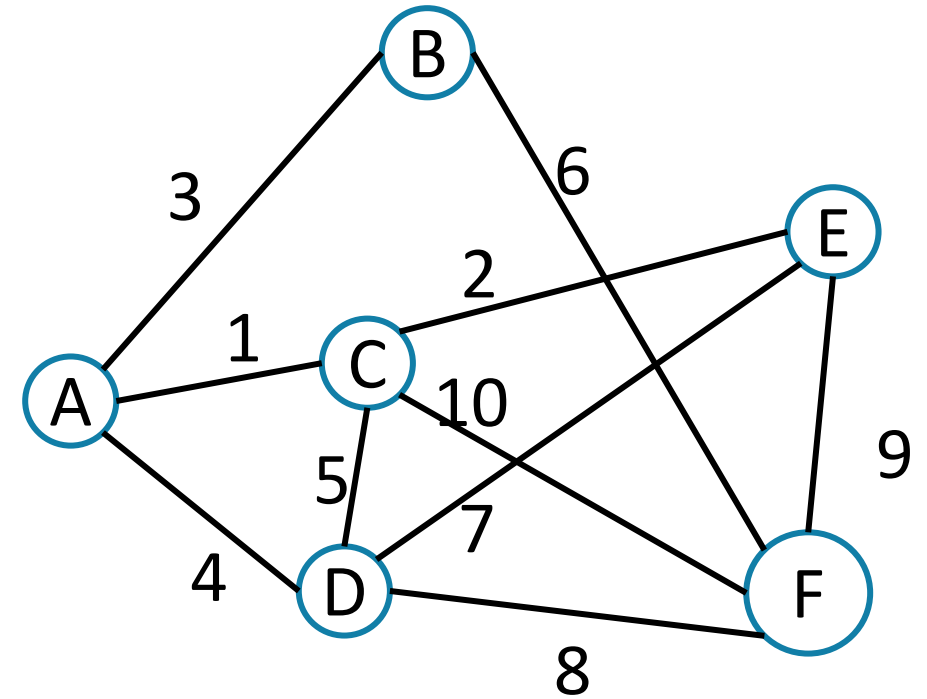
```
    if(u and v are in different components){
```

```
      add (u,v) to the MST
```

```
      Update u and v to be in the same component
```

```
    }
```

```
  }
```



Edge	Include?	Reason
(A,C)	Yes	
(C,E)	Yes	
(A,B)	Yes	
(A,D)	Yes	
(C,D)	No	Cycle A,C,D,A

Edge (cont.)	Inc?	Reason
(B,F)	Yes	
(D,E)	No	Cycle A,C,E,D,A
(D,F)	No	Cycle A,D,F,B,A
(E,F)	No	Cycle A,C,E,F,D,A
(C,F)	No	Cycle C,A,B,F,C



# Kruskal's Algorithm Implementation

```
KruskalMST(Graph G)
  initialize each vertex to be an independent component
  sort the edges by weight
  foreach(edge (u, v) in sorted order){
    if(u and v are in different components){
      add (u,v) to the MST
      update u and v to be in the same component
    }
  }
```

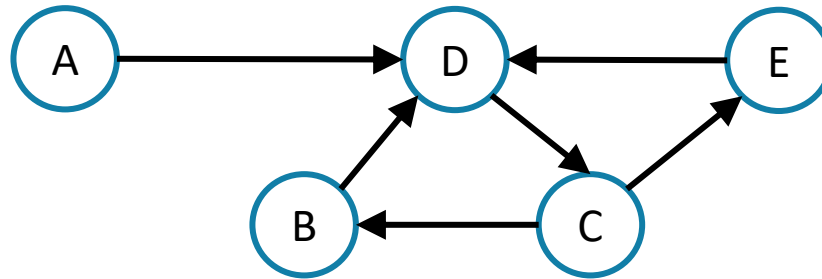
```
KruskalMST(Graph G)
  foreach (V : vertices) {
    makeMST(v); +?
  } +V(makeMST)
  sort edges in ascending order by weight +ElogE
  foreach(edge (u, v)){
    if(findMST(v) is not in findMST(u)) { +?
      union(u, v) +?
    }
  } +E(2findMST + union)
```

How many times will we call union?  
 $V - 1$   
->  **$+V_{\text{union}} + E_{\text{findMST}}$**

# Strongly Connected Components

## Strongly Connected Component

A subgraph  $C$  such that every pair of vertices in  $C$  is connected via some path **in both directions**, and there is no other vertex which is connected to every vertex of  $C$  in both directions.



Note: the direction of the edges matters!

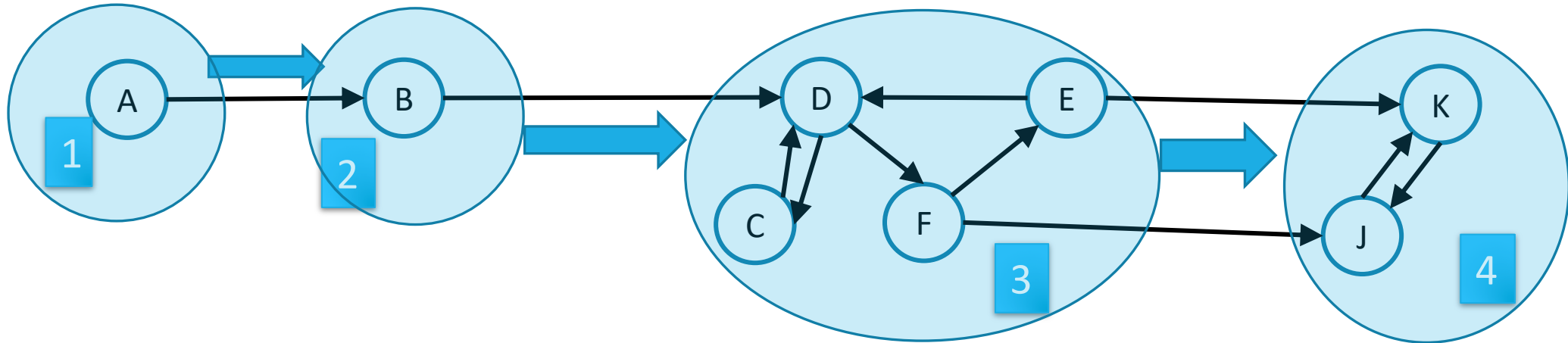
# Why Find SCCs?

Graphs are useful because they encode relationships between arbitrary objects.

We've found the strongly connected components of  $G$ .

Let's build a new graph out of them! Call it  $H$

- Have a vertex for each of the strongly connected components
- Add an edge from component 1 to component 2 if there is an edge from a vertex inside 1 to one inside 2.



# Implement makeSet(x)

makeSet(0)

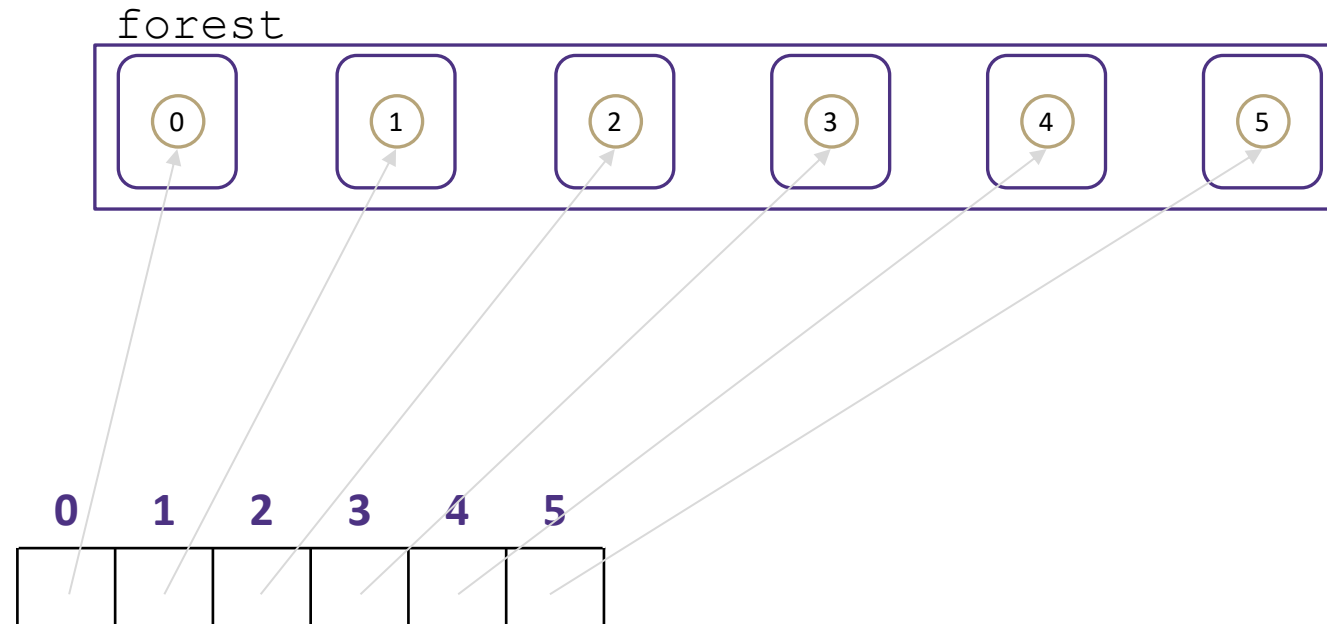
makeSet(1)

makeSet(2)

makeSet(3)

makeSet(4)

makeSet(5)



## TreeDisjointSet<E>

### state

Collection<TreeSet> forest  
Dictionary<NodeValues,  
NodeLocations> nodeInventory

### behavior

`makeSet(x)` - create a new tree of size 1 and add to our forest  
`findSet(x)` - locates node with x and moves up tree to find root  
`union(x, y)` - append tree with y as a child of tree with x

Worst case runtime?

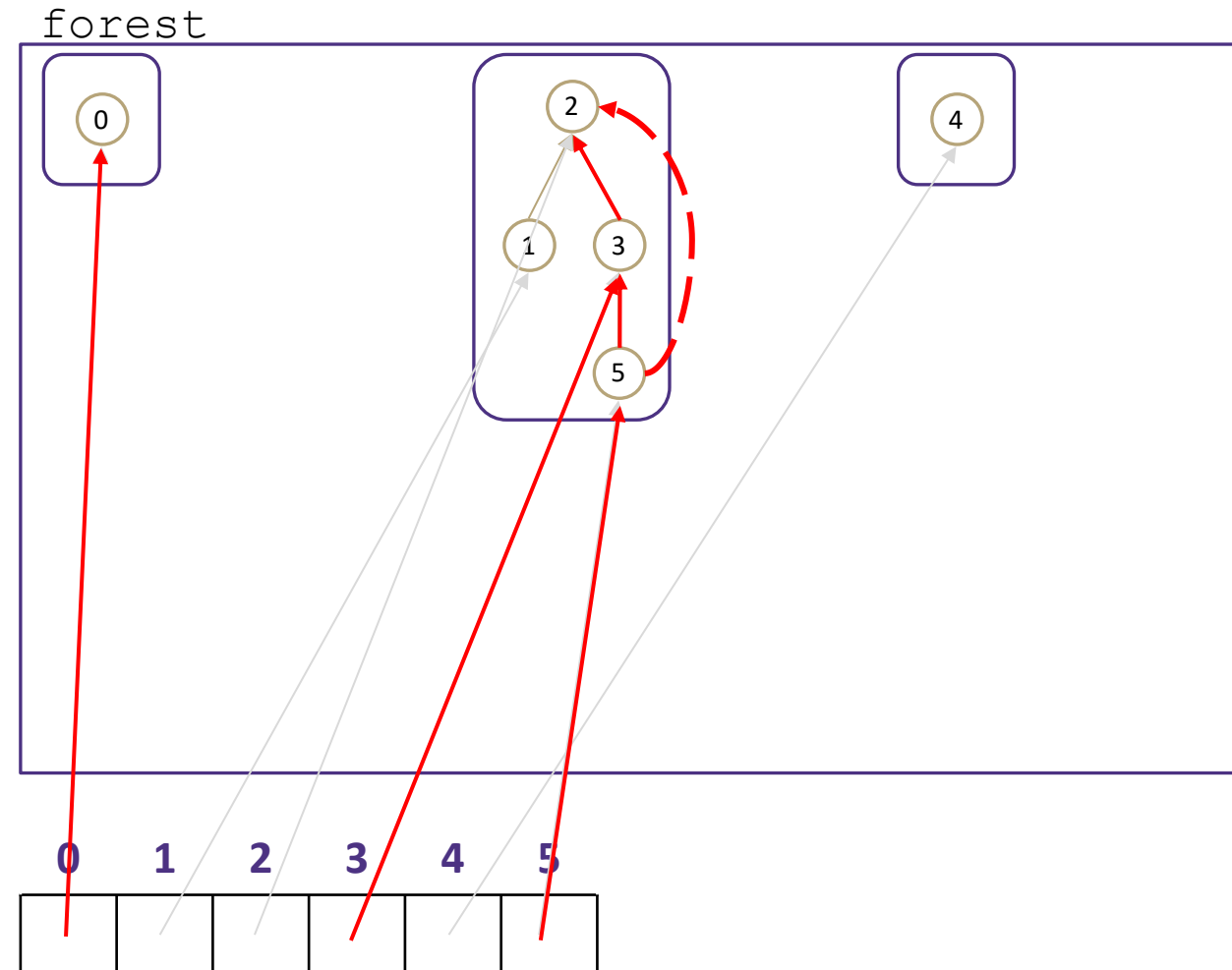
**$O(1)$**

# Implement findSet(x)

findSet(0)

findSet(3)

findSet(5)



Worst case runtime?

$O(n)$

Worst case runtime of union?

$O(n)$

TreeDisjointSet<E>

## state

Collection<TreeSet> forest  
Dictionary<NodeValues,  
NodeLocations> nodeInventory

## behavior

makeSet(x) - create a new tree  
of size 1 and add to our  
forest

findSet(x) - locates node with x  
and moves up tree to find root

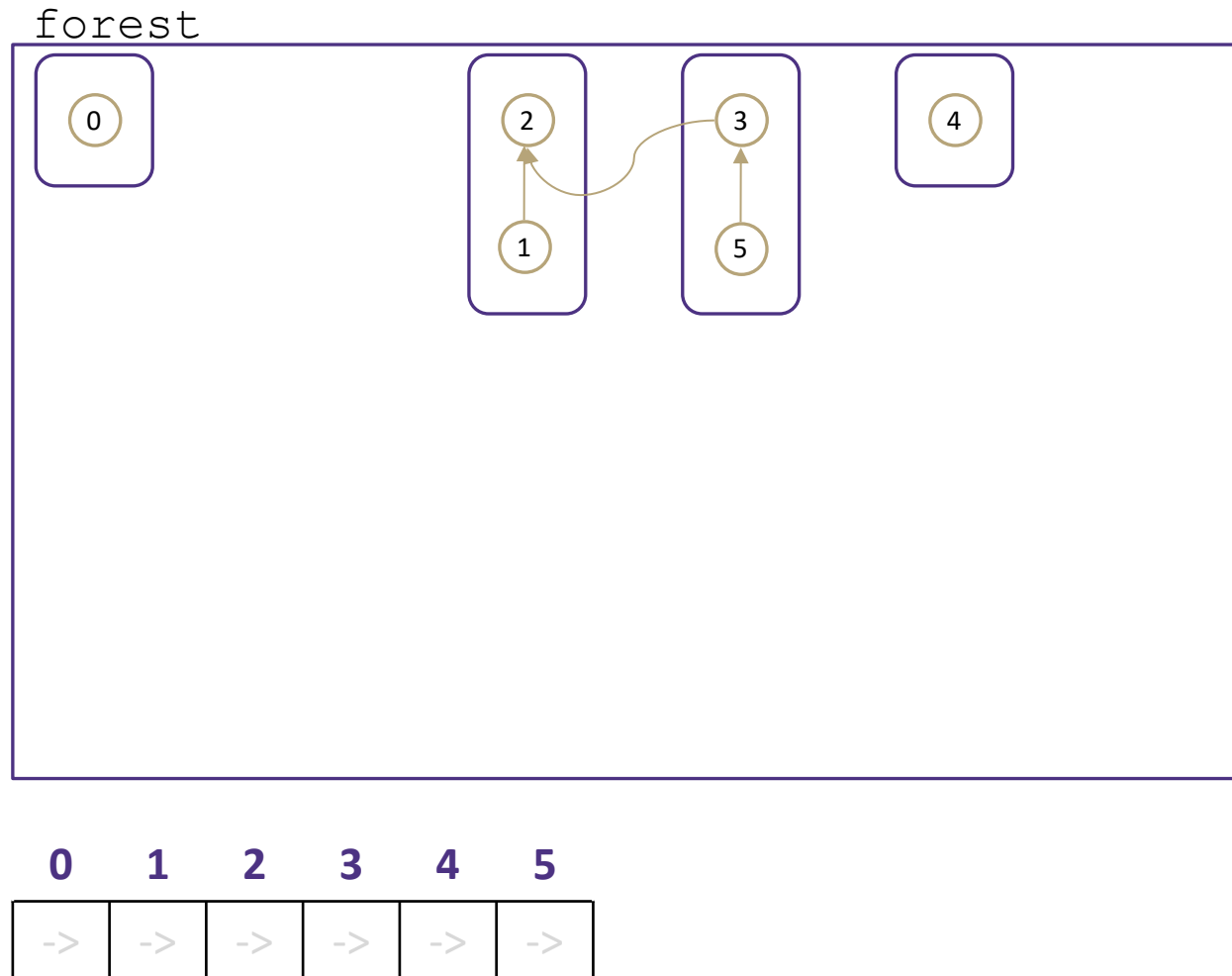
union(x, y) - append tree with y  
as a child of tree with x

# Implement union(x, y)

union(3, 5)

union(2, 1)

union(2, 5)



TreeDisjointSet<E>

## state

Collection<TreeSet> forest  
Dictionary<NodeValues,  
NodeLocations> nodeInventory

## behavior

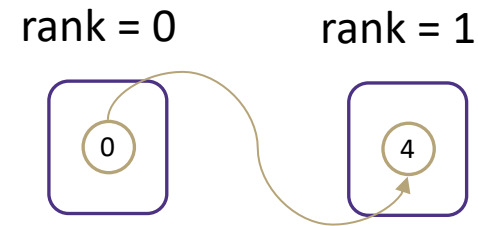
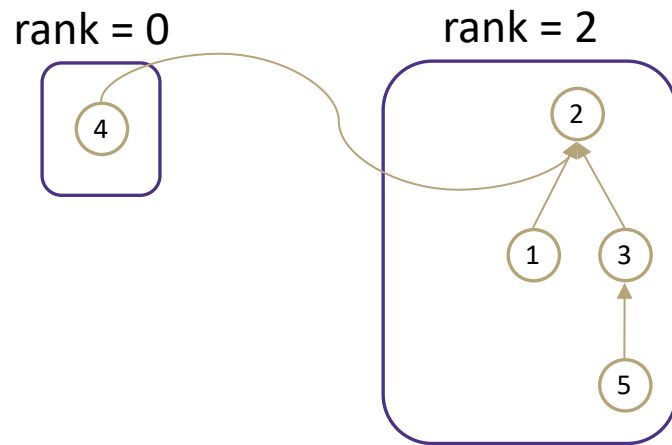
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and moves up tree to find root  
`union(x, y)` - append tree with y  
as a child of tree with x

# Improving union

**Problem:** Trees can be unbalanced

**Solution:** Union-by-rank!

- let  $\text{rank}(x)$  be a number representing the upper bound of the height of  $x$  so  $\text{rank}(x) \geq \text{height}(x)$
- Keep track of rank of all trees
- When unioning make the tree with larger rank the root
- If it's a tie, pick one randomly and increase rank by one



# Improving findSet()

**Problem:** Every time we call findSet() you must traverse all the levels of the tree to find representative

**Solution: Path Compression**

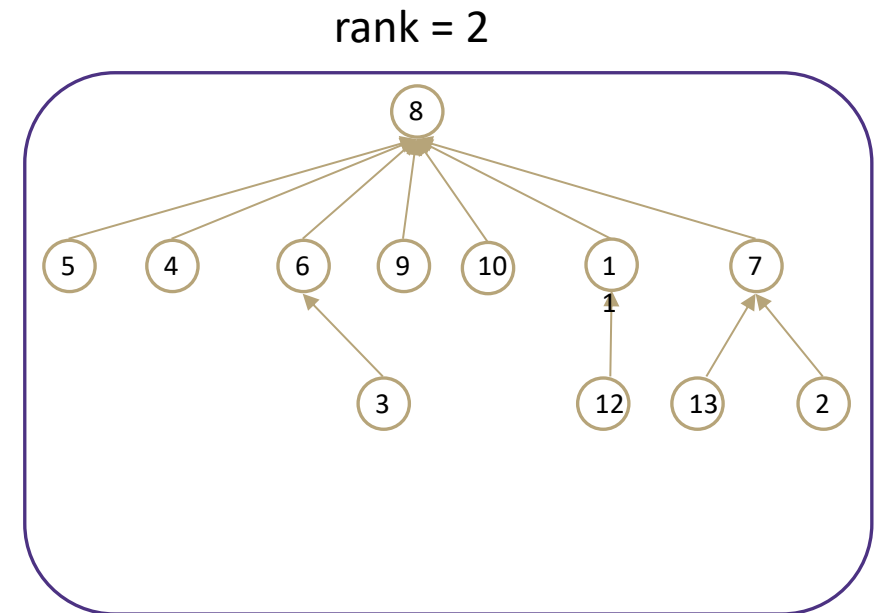
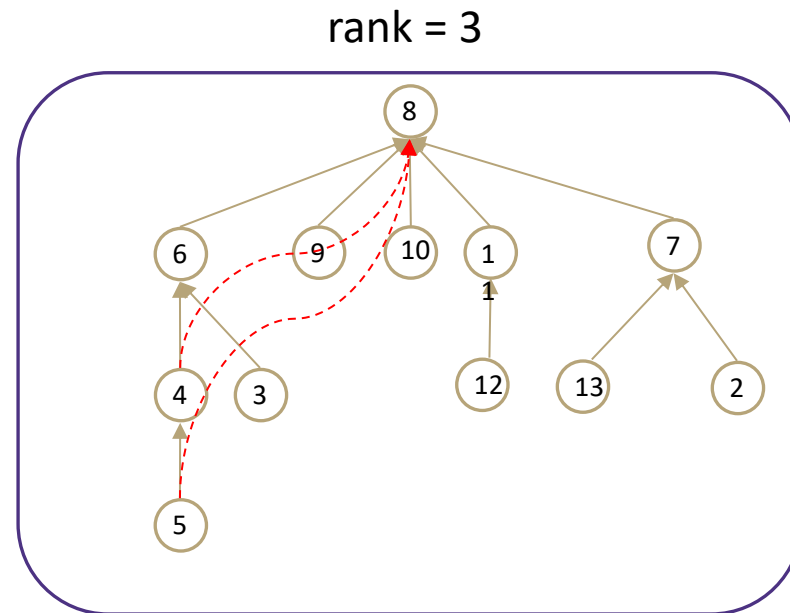
- Collapse tree into fewer levels by updating parent pointer of each node you visit
- Whenever you call findSet() update each node you touch's parent pointer to point directly to overallRoot

findSet(5)

findSet(4)

Does this improve the worst case runtimes?

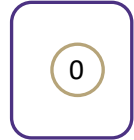
findSet is more likely to be  $O(1)$  than  $O(\log(n))$



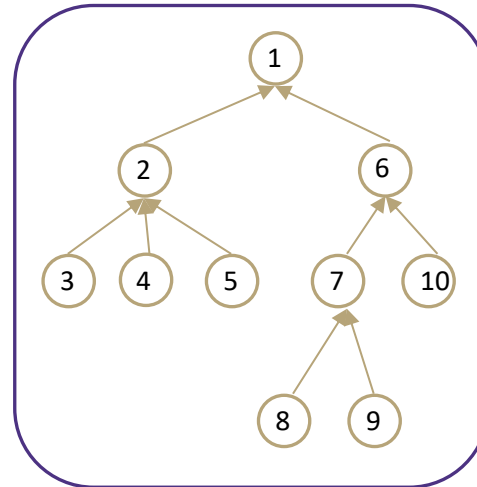


# Array Implementation

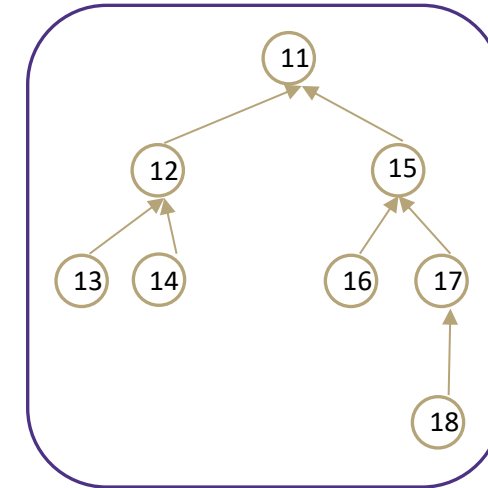
rank = 0



rank = 3



rank = 3



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
-1	-4	1	2	2	2	1	6	7	7	7	-4	11	12	12	11	15	15	17

Store  $(\text{rank} * -1) - 1$

Each “node” now only takes 4 bytes of memory instead of 32

# Optimized Disjoint Set Runtime

## makeSet(x)

Without Optimizations  $O(1)$

With Optimizations  $O(1)$

## findSet(x)

Without Optimizations  $O(n)$

With Optimizations Best case:  $O(1)$  Worst case:  $O(\log n)$

## union(x, y)

Without Optimizations  $O(n)$

With Optimizations Best case:  $O(1)$  Worst case:  $O(\log n)$

# Scenario #1

You are going to Disneyland for spring break!  
You've never been, so you want to make sure  
you hit ALL the rides.

Is there a graph algorithm that would help?

BFS or DFS

How would you draw the graph?

- What are the vertices?

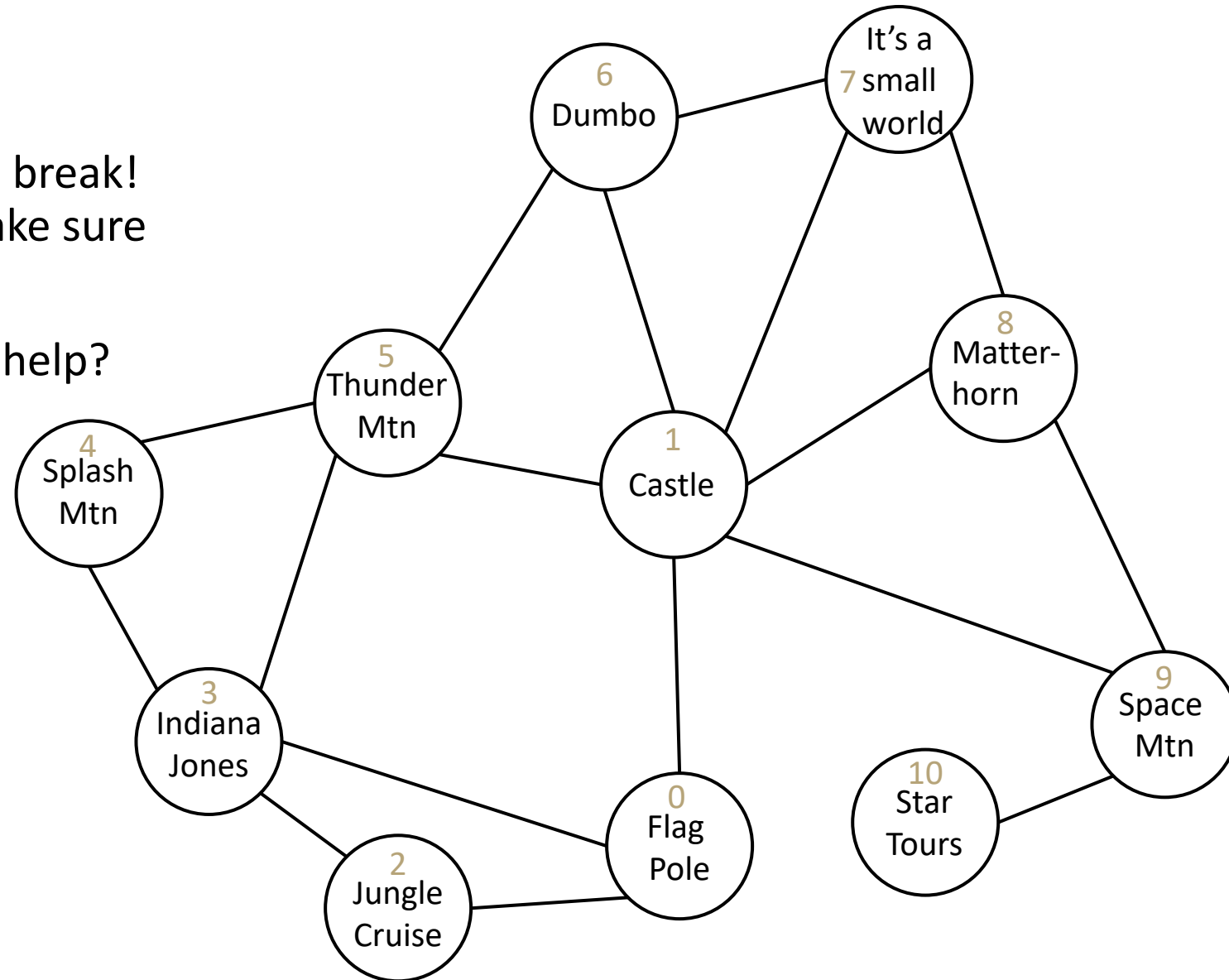
Rides

- What are the edges?

Walkways

BFS = 0 1 2 3 5 6 7 8 9 4 10

DFS = 0 3 5 6 7 8 9 10 1 4 2



# Scenario #1 continued

Now that you have your basic graph of Disneyland what might the following graph items represent in this context?

## Weighted edges

- Walkway distances
- Walking times
- Foot traffic

## Directed edges

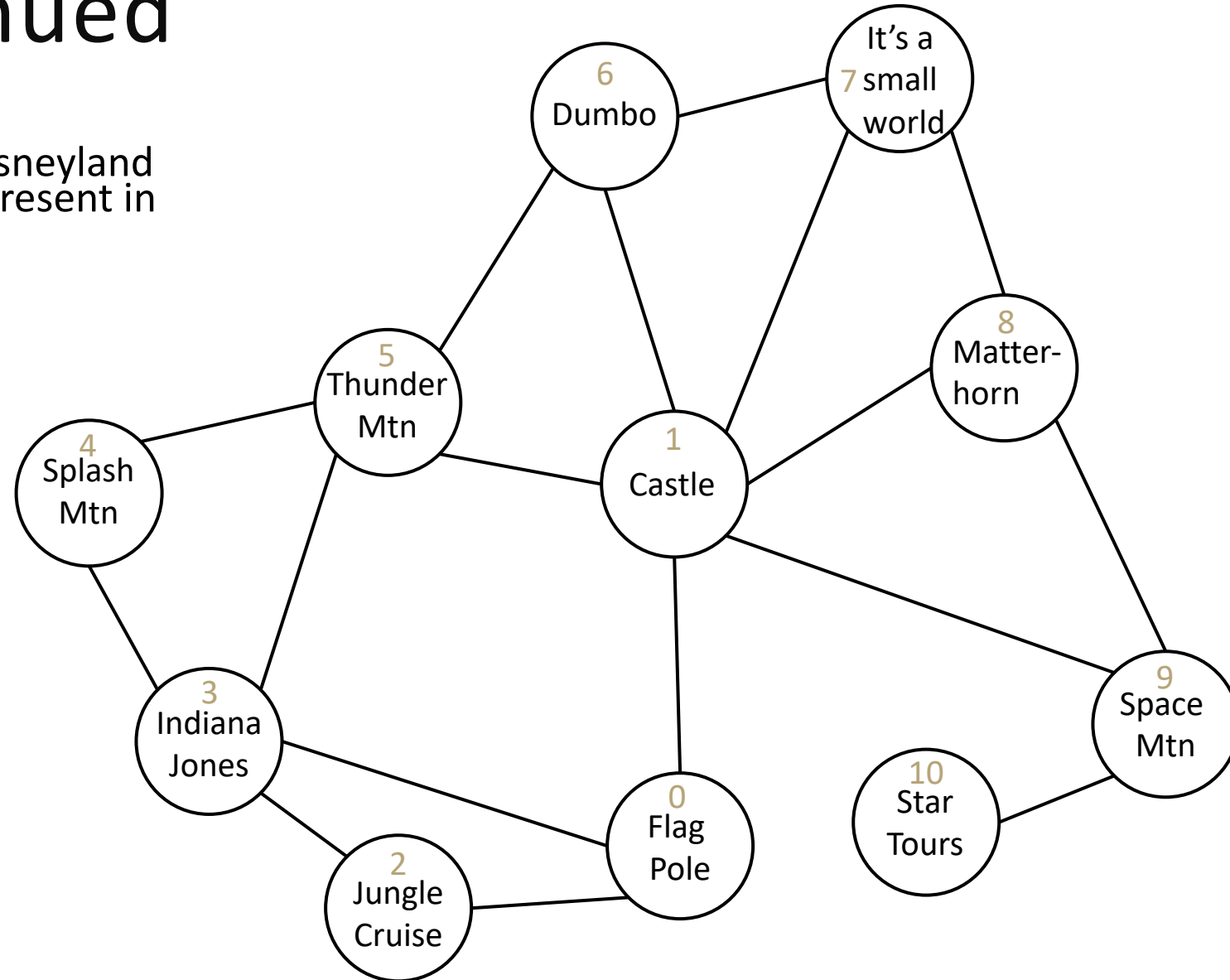
- Entrances and exits
- Crowd control for fireworks
- Parade routes

## Self Loops

- Looping a ride

## Parallel Edges

- Foot traffic at different times of day
- Walkways and train routes



# Scenario #2

You are a Disneyland employee and you need to rope off as many miles of walkways as you can for the fireworks while leaving guests access to all the rides.

Is there a graph algorithm that would help?

MST

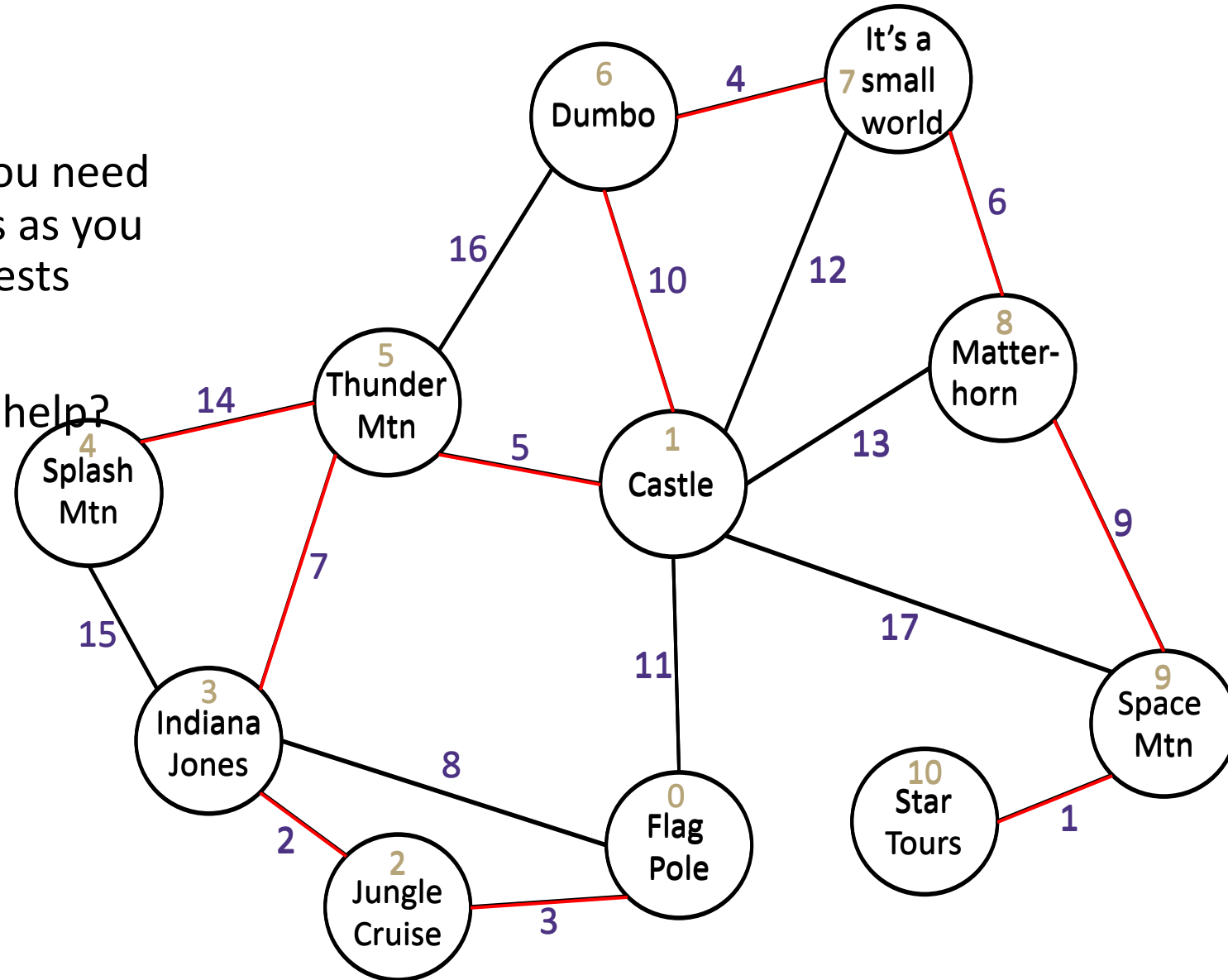
How would you draw the graph?

- What are the vertices?

Rides

- What are the edges?

Walkways with distances



# Scenario #3

You arrive at Disneyland and you want to visit all the rides, but do the least amount of walking possible. If you start at the Flag Pole, plan the shortest walk to each of the attractions.

Is there a graph algorithm that would help?

Dijkstra's

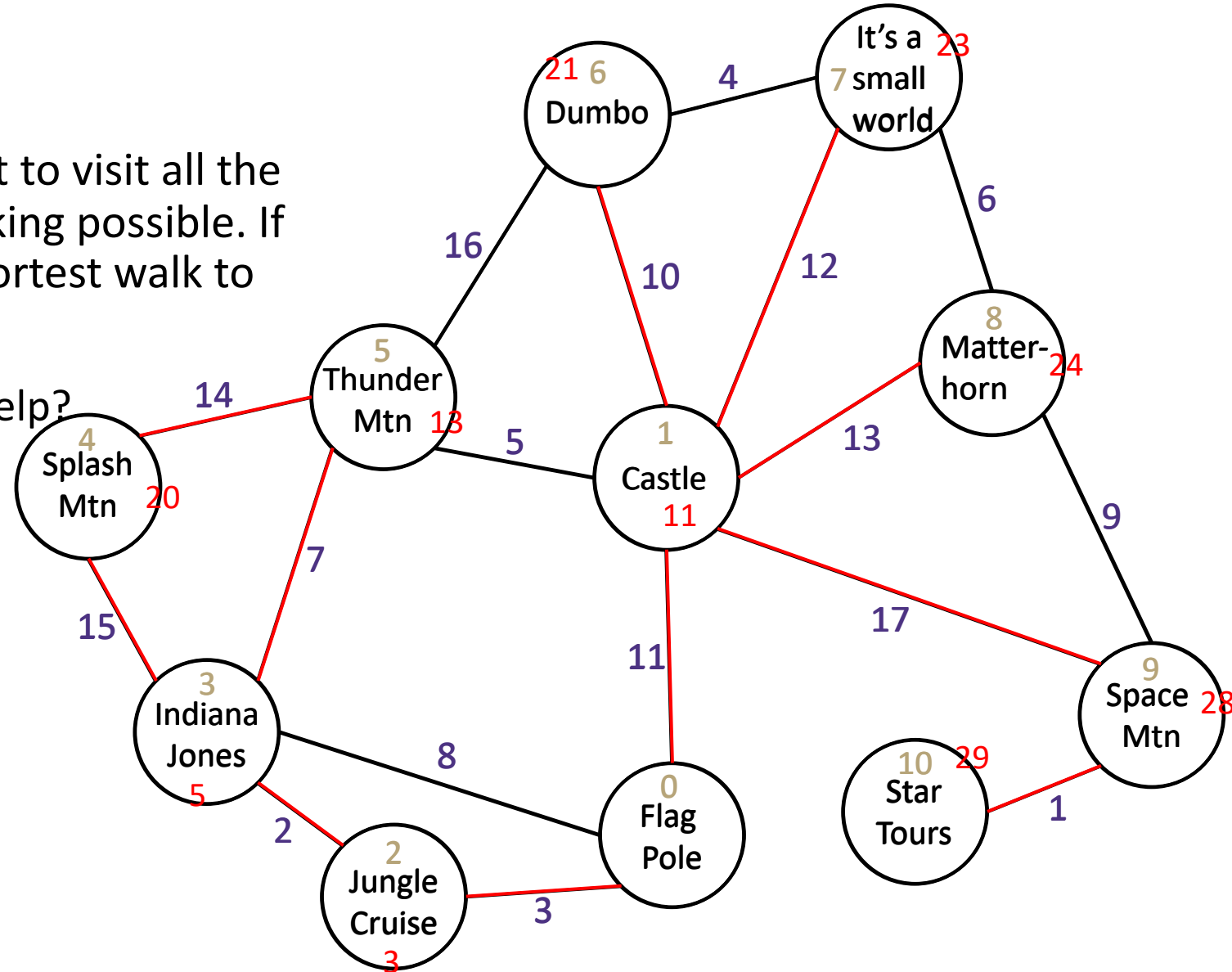
How would you draw the graph?

- What are the vertices?

Rides

- What are the edges?

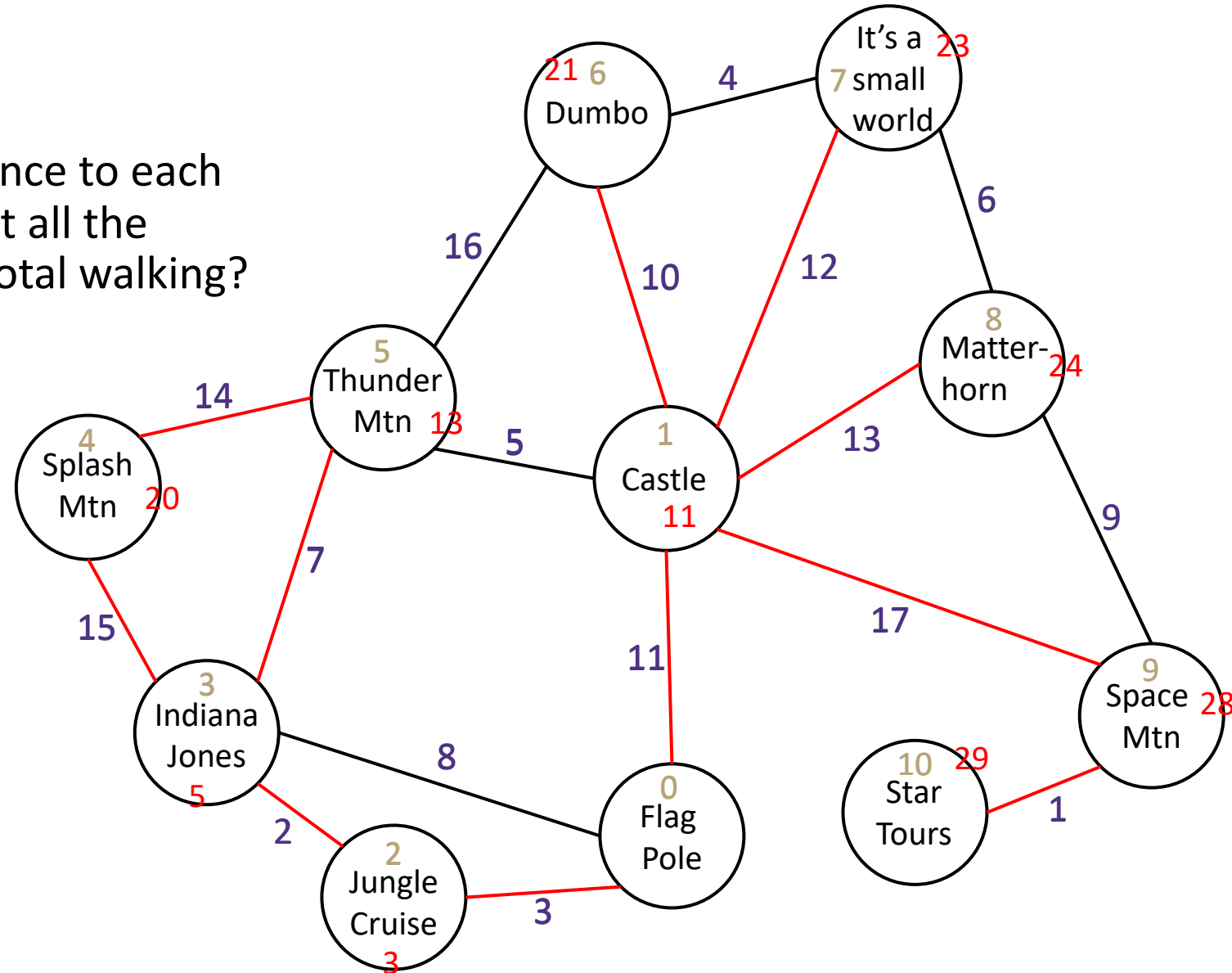
Walkways with distances



# Scenario #2b

Now that you know the shortest distance to each attraction, can you make a plan to visit all the attractions with the least amount of total walking?

Nope! This is the travelling salesman problem which is much more complicated than Dijkstra's.  
(NP Hard, more on this later)



# Scenario #3

You have great taste so you are riding Space Mountain. Your friend makes poor choices so they are riding Splash Mountain. You decide to meet at the castle, how long before you can meet up?

Is there a graph algorithm that would help?

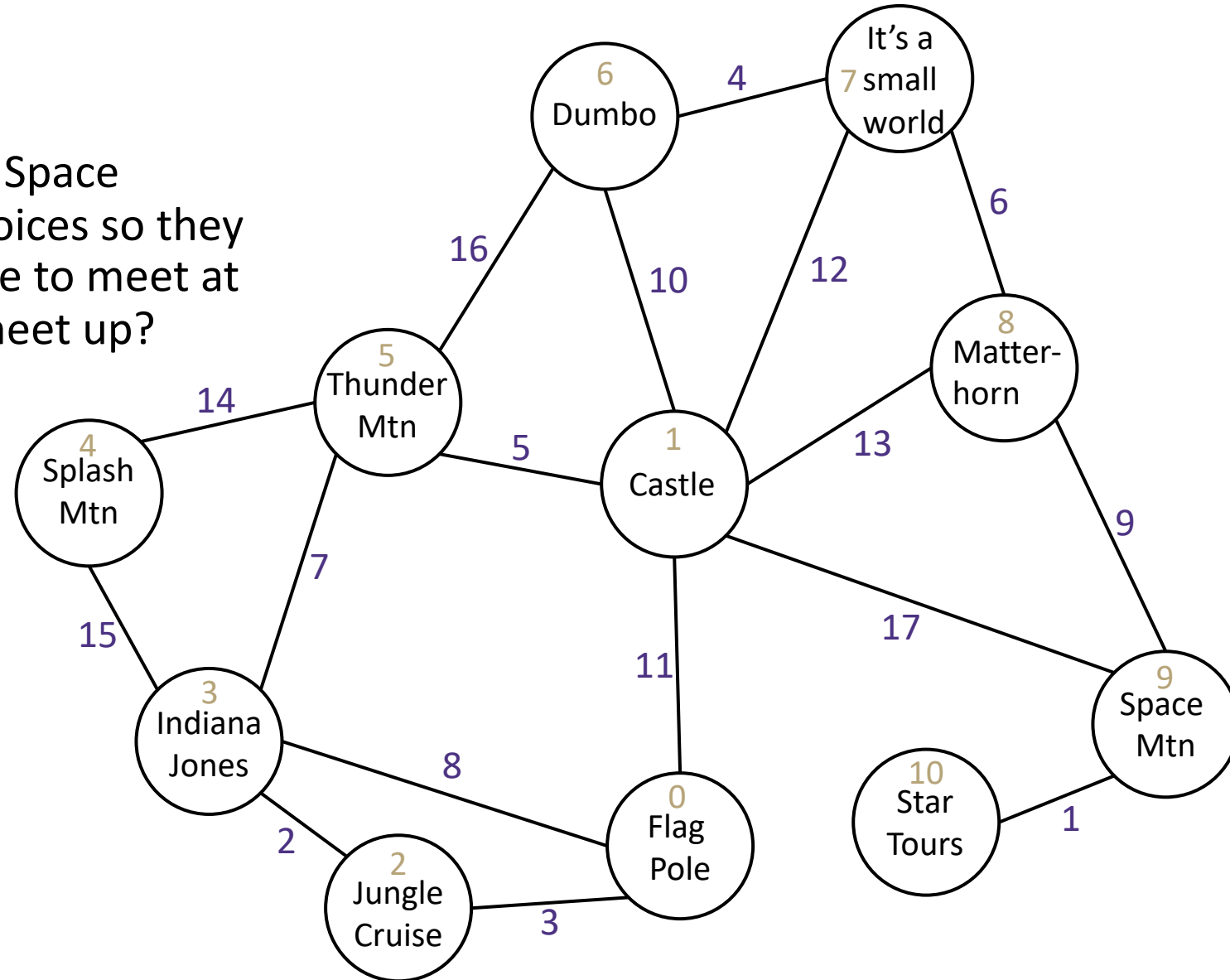
Dijkstra's

What information do our edges need to store?

Walking times

How do we apply the algorithm?

- Run Dijkstra's from Splash Mountain.
- Run Dijkstra's from Space Mountain.
- Take the larger of the two times.





# Types of Problems

**Decision Problem** – any arbitrary yes-or-no question on an infinite set of inputs. Resolution to problem can be represented by a Boolean value.

- IS-PRIME: is  $X$  a prime number? (where  $X$  is some input)
- IS-SORTED: is this list of numbers sorted?
- EQUAL: is  $X$  equal to  $Y$ ? (for however  $X$  and  $Y$  define equality)

**Solvable** – a decision problem is solvable if there exists some algorithm that given any input or instance can correctly produce either a “yes” or “no” answer.

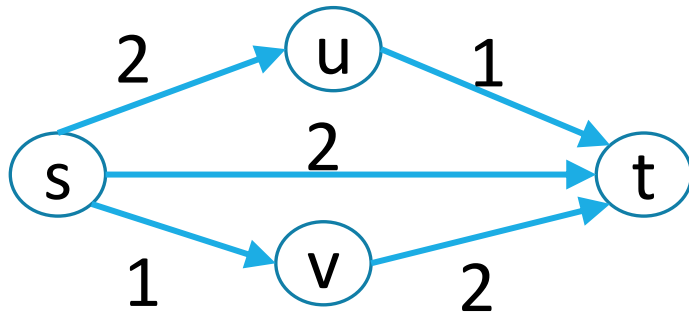
- Not all problems are solvable!
  - Example: Halting problem

**Efficient algorithm** – an algorithm is efficient if the worst case bound is a polynomial. The growth rate of this is such that you can actually run it on a computer in practice.

- Definitely efficient:  $O(1)$ ,  $O(n)$ ,  $O(n \log n)$ ,  $O(n^2)$
- Technically efficient:  $O(n^{1000000})$ ,  $O(1000000000000000n^2)$

Everything we’ve talked about in class so far has been **solvable** and **efficient**...

# Weighted Graphs: A Reduction



Transform Input

Transform Output

Unweighted Shortest Paths

# P

## P (stands for “Polynomial”)

The set of all decision problems that have an algorithm that runs in time  $O(n^k)$  for some constant  $k$ .

The decision version of all problems we’ve solved in this class are in P.

P is an example of a “complexity class”

A set of problems that can be solved under some limitations (e.g. with some amount of memory or in some amount of time).

# I'll know it when I see it.

More formally,

**NP (stands for “nondeterministic polynomial”)**

The set of all decision problems such that if the answer is YES, there is a proof of that which can be verified in polynomial time.

It's a common misconception that NP stands for “not polynomial”  
Please never ever ever ever say that.

Please.

Every time you do a theoretical computer scientist sheds a single tear.  
(That theoretical computer scientist is me)