1. **General: True or False**[1]
   For each statement below, state that its true or false. Explain.

   (a) Binary search in a sorted array is worst case O(n).

   (b) Master Theorem can be used to find the big-Theta of any recurrence.

   (c) To implement a dictionary, you should always use a hash table over an AVL tree if your main priority is speed.

   (d) The number of collisions in a hash table is solely dependent on the table capacity and the hash function.

   (e) The specific order that values are inserted into a heap will affect both the internal ordering and runtime for future operations.

   (f) AVL trees and BSTs never have the same tight big-O runtime for inserting elements.

   (g) 3-Heaps have better tight big-O runtime for insert than 2-heaps or 4 heaps.

   (h) For a graph with negative edge weights, adding a constant to make all edge weights positive and then running Dijkstra's will give the same shortest path from a node $s$ to $g$ as the one with negative edge weights.

   (i) MSTs can be used to find the minimum cost path between any two nodes.

   (j) Graphs can be implemented with a Dictionary.

   (k) To find the shortest path from one vertex to another in an unweighted graph, you should use Dijkstra's algorithm as it is the most efficient solution.

[1] We ran out of space on this page to put in a title, but this document is **Week 9: Final Review**.

2. **ADTs and Implementations**

For each of the following ADTs, list the implementations that we covered in class and an advantage of each implementation over the others. (Hint: in what situations would you use each one?)

(a) List

(b) Dictionary/Map

(c) Union-Find

3. **Design Decisions: Sorting**

For each of the following scenarios, choose the most appropriate sorting algorithm from the list, then briefly explain (1-2 sentences) why your choice is the best.

**Selection Sort, Insertion Sort, Merge Sort, Quick Sort, Heap Sort**

(a) At class pictures the photographers usually sort the children by height before assigning places. The photographer doesn't care who was originally where in line but he/she does need to get them sorted as fast as possible and there is a lot of extra space in the room to arrange them if necessary.

(b) A company has lists of numbers that each need to be sorted, and they want their algorithm to be reliably and consistently fast so they can anticipate when the job will be done.

(c) When you were younger, you may have had a box of Crayola crayons. When you first buy them however, all the colors are not sorted in order. Which sort can you use to sort the crayons by gradient color in the box such that you only take one or two of the crayons out of the box at any given time?

4.

```java
public static void printReachingFriendships(ChainedHashDictionary<String, IList<String>> graph) {
    IPriorityQueue<String> heap = new ArrayHeapPriorityQueue<>();
    for (KVPair<String, IList<String>> pair : graph) {
        heap.add(pair.getKey());
    }


    String start = heap.removeMin();
    ISet<String> seen = new ChainedHashSet<>();
    DoubleLinkedList<String> toProcess = new DoubleLinkedList<>();
    toProcess.add(start);
    while (!toProcess.isEmpty()) {
        String currentVertex = toProcess.get(0);
        toProcess.delete(0);
        System.out.println("currentVertex person: " + currentVertex);
        for (String neighbor : graph.get(currentVertex)) {
            if (!seen.contains(neighbor)) {
                seen.add(neighbor);
                toProcess.add(neighbor);
            }
            mysterySort(toProcess, 0, toProcess.size() - 1);
        }
    }
}

public static void mysterySort(DoubleLinkedList<String> list, int low, int high) {
    if (low < high) {
       /* pi is partitioning index, list.get(pi) is
        now at right place */
        int pi = partition(list, low, high);
        // Recursively sort elements before
        // partition and after partition
        mysterySort(list, low, pi-1);
        mysterySort(list, pi+1, high);
    }
}

public static int partition(DoubleLinkedList<String> list, int low, int high) {
    String pivot = list.get(low);
    int i = low - 1;
    int j = high + 1;
    while (true) {
        i++;
        while (list.get(i).compareTo(pivot) < 0) {
            i++;
        }

        j--;
        while (list.get(j).compareTo(pivot) > 0) {
            j--;
        }

        if (i >= j) { // if i has become big enough and j has become small enough that they cross
            return j;
        }

        // swap list.get(i) and list.get(j)
        String temp = list.get(i);
        list.set(i, list.get(j));
        list.set(j, temp);
    }
}
```

The above method printReachingFriendships takes in a graph in the form of an adjacency list. The vertices represent people and edges between two people represent that those people are friends. This means when you call graph.get("Robbie") a list of Robbie's friends is returned. The graph stored inside the dictionary parameter represents an undirected and unweighted graph.

Answer the following questions about the above method to model the runtime of the above code. You can assume that System.out.println and compareTo run in constant time. Assume that all ChainedHashDictionary and ChainedHashSet operations will run in their in-practice runtimes (even for worst/best case analysis here). All your answers for this part should be defined in terms of $n$ and $m$, where they represent the total number of nodes (vertices) and total number of edges in the graph parameter.

(a) For the loop on lines 3-5 give a simplified theta bound for:

i. the best case runtime

ii. the worst case runtime

(b) For the loop from lines 12 to 23:

i. how many times are lines 13-15 executed in the worst case? Bonus question: best case?

ii. how many times will mysterySort get called in the worst case?

iii. how many times are lines 18 and 19 executed in the worst case?

iv. What is the worst case runtime for the loop from 12 to 23? Give your answer as a simplified theta bound. For now, say the runtime of mysterySort is S runtime.

(c) for mysterySort:

i. What sort does mysterySort look the most like?

ii. Imagine the first time partition is called, and the difference between low and high is the size of the list parameter, called p. What is the worst case runtime for partition in terms of p?

iii. What is the worst case (describe it, not the runtime) for mysterySort? Hint: model the code as a recurrence, consider what sort you think this is and the worst case for this sort?

iv. What is the best case (describe it, not the runtime) for mysterySort?

v. How many times will we recurse in the worst case of mysterySort (aka how many times do we call partition)? Assume that the size of the list parameter is called p.

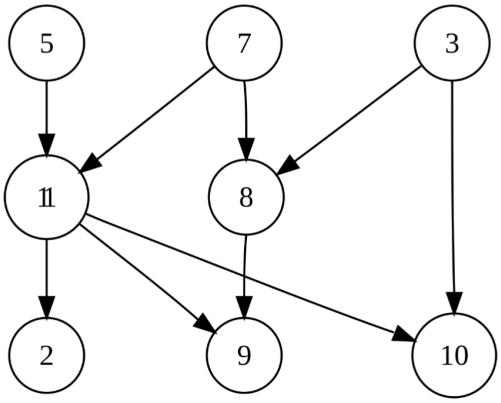
vi. Putting that together, what's the worst case runtime for mysterySort? Assume that the size of the list parameter is called p.

(d) Consider what inputs are being passed to mysterySort on line 21 in printReachingFriendships. Is this triggering situations more like the best case or the worst case runtime?

(e) What is the complexity class of the max size of toProcess?

5. **Topo Sort**

Consider the graph below:




(a) What are the characteristics of this graph? (Hint: what is graph terminology is required for it to have a valid topological ordering?)

(b) Give two possible orderings produced using Topological sort.

6. **Graph Modeling**

As a pilot for PrimAir, you have been assigned a charter flight that will take a rich customer from Shanghai, China to Jackson Hole, Wyoming. Unfortunately, the trip is about 7,000 miles direct, while your second-hand Cessna 152 only has a range of 500 miles. This means that you must pick airports along the way to refuel. However, it is a bad idea to land at every single every airport that you see along the way, because every landing adds about 50 miles of wasted flying (in addition to the ground distance that you are covering).

After taking CSE 373, you wrote your own flight computer and you decided to use your own software to model this flight. Your flight computer contains information about all the airports in the world. Assume each airport has enough jet fuel for your plane. **You want to perform the least amount of flying while not running out of fuel at any time during your trip.**

(a) Explain how you would model this scenario as a graph. Answer the following questions in bullet points or short sentences.

i. What do your vertices represent? If you store extra information in each vertex, what is it?

ii. What do your edges represent (your answer may be a real-world object, or an abstract description of what edges will exist)? If you store extra information in each edge, what is it?

iii. Is your graph directed? Briefly explain (1 sentence).

iv. Is your graph weighted? If so what are the weights? Briefly explain (1 sentence).

v. Do you permit self-loops (i.e. edges from a vertex to itself)? Parallel edges (i.e. more than one copy of an edge between the same location)? Why?

(b) Consider the following information: Everett/Paine Field is 50 miles north of Seattle-Tacoma, Vancouver is 100 miles north of Everett/Paine Field, Anchorage is 1500 miles northwest of Vancouver. Using your graph model, draw out a portion of your graph using these four airports.

(c) Explain how you would run an algorithm taught in this course to find the best route from Shanghai to Jackson Hole. If applicable, say how you would modify the standard algorithm to take landing/takeoff overhead into consideration.

(d) Using the answer above, write a simplified tight Big-O for the worst case runtime of your chosen algorithm in terms of $n$ (number of nodes/vertices) and $m$ (number of edges).