CSE 373 19su: Final Exam Reference Sheet

Splitting a sum

$$\sum_{i=a}^{b}(x+y) = \sum_{i=a}^{b} x + \sum_{i=a}^{b} y$$

Factoring out a constant

$$\sum_{i=a}^{b} cf(i) = c\sum_{i=a}^{b} f(i)$$

Adjusting Summation Bounds

$$\sum_{i=a}^{b} f(i) = \sum_{i=0}^{b} f(i) - \sum_{i=0}^{a-1} f(i)$$

Summation of a constant

$$\sum_{i=0}^{n-1} c = cn$$

Gauss's identity

$$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$$

Sum of suqares

$$\sum_{i=0}^{n-1} i^2 = \frac{n(n-1)(2n-1)}{6}$$

Finite geometric series

Only applicable when $x \neq 0$ and $x \neq 1$

$$\sum_{i=0}^{n-1} x^i = \frac{x^n - 1}{x - 1}$$

Infinite geometric series

Only applicable when $-1 < x < 1$ and $x \neq 0$

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$$

Log of a product

$$\log_b(x \cdot y) = \log_b(x) + \log_b(y)$$

Log of a fraction

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

Log of a power

$$\log_b(x^y) = y \cdot \log_b(x)$$

Power of a log

$$x^{\log_b(y)} = y^{\log_b(x)}$$

Change-of-base identity

$$\log_a(n) = \frac{\log_b(n)}{\log_b(a)}$$

Power Rules

$$(a^b)^c = a^{b \cdot c}$$

$$a^b \cdot a^c = a^{b+c}$$

Master theorem

Given a recurrence of the form:

$$T(n) = \begin{cases} d & \text{if } n \text{ is at most a constant} \\ aT\left(\frac{n}{b}\right) + f(n) & \text{otherwise} \end{cases}$$

Where $f(n)$ is $\Theta(n^c)$

- If $\log_b(a) < c$ then $T(n) \in \Theta\left(n^c\right)$
- If $\log_b(a) = c$ then $T(n) \in \Theta\left(n^c \log(n)\right)$
- If $\log_b(a) > c$ then $T(n) \in \Theta\left(n^{\log_b(a)}\right)$