

Quickcheck 02: Solutions

Name:

Definition: Dominated by

A function $f(n)$ is dominated by $g(n)$ when...

- There exists two constants $c > 0$ and $n_0 > 0$...
- Such that for all values of $n \geq n_0$...
- $f(n) \leq c \cdot g(n)$ is true.

Demonstrate that $2n^3 - 3 + 9n^2$ is dominated by n^3 by finding a c and n_0 . Show your work.

Solution:

We'll go term by term in the first function.

$$2n^3 \leq 2 \cdot n^3 \text{ for all } n.$$

$$-3 \leq 0 \cdot n^3 \text{ for all } n.$$

$$9n^2 \leq 1 \cdot n^3 \text{ for all } n \geq 9$$

All inequalities are true as long as $n \geq \max\{0, 0, 9\} = 9$. If all the inequalities are true we can sum them to get

$$2n^3 - 3 + 9n^2 \leq (2 + 0 + 1)n^3 = 3n^3$$

for all $n \geq 9$.

Thus we take $n_0 = 9$ and $c = 3$. This is not the only solution; many others are possible.