Lecture 19: Sorting Algorithms

CSE 373: Data Structures and Algorithms

Administrivia

HW 5 Part 1 Due Thursday

HW 5 Part 2 Out Wednesday

Sorting

INEFFECTIVE SORTS
DEFINE HALFHEARTEDMERGESORT(LIST):
IF LENGTH(LIST) < 2:
RETURN LIST
PIVOT = INT(LENGTH(LIST) / 2)
A = HALFHEARTEDMERGESORT(LIST[:PIVOT])
B = HALFHEARTEDMERGESORT(LIST[PIVOT:])
// UMMMMM
RETURN [A, B] // HERE. SORRY.
DEFINE FASTBOGOSORT(LIST):
// AN OPTIMIZED BOGOSORT
// RUNS IN O(N LOG N)
FOR N FROM 1 TO LOG(LENGTH(LIST)):
SHUFFLE(LIST):
IF ISSORTED(LIST):
RETURN LIST
RETURN "KERNEL PAGE FAULT (ERROR CODE: 2)"
DEFINE JOBINTERVIEWQUICKSORT(LIST):
OK SO YOU CHOOSE A PIVOT
THEN DIVIDE THE LIST IN HALF
FOR EACH HALF:
CHECK TO SEE IF IT'S SORTED
NO, WAIT, IT DOESN'T MATTER
COMPARE EACH ELEMENT TO THE PIVOT
THE BIGGER ONES GO IN A NEW LIST
THE EQUAL ONES GO INTO, UH
THE SECOND LIST FROM BEFORE
HANG ON, LET ME NAME THE LISTS
THIS IS LIST A
THE NEW ONE IS LIST B
PUT THE BIG ONES INTO LIST B
NOW TAKE THE SECOND LIST
CALL IT LIST, UH, A2
WHICH ONE WAS THE PIVOT IN?
SCRATCH ALL THAT
IT JUST RECURSIVELY CALLS ITSELF
UNTIL BOTH LISTS ARE EMPTY
RIGHT?
NOT EMPTY, BUT YOU KNOW WHAT I MEAN
AM I ALLOWED TO USE THE STANDARD LIBRARIES?
DEFINE PANICSORT(LIST):
IF ISSORTED(LIST):
RETURN LIST
FOR N FROM 1 TO 10000:
PIVOT = RANDOM(0, LENGTH(LIST))
LIST = LIST[PIVOT:] + LIST[:PIVOT]
IF ISSORTED(LIST):
RETURN LIST
IF ISSORTED(LIST):
RETURN LIST:
IF ISSORTED(LIST): //THIS CAN'T BE HAPPENING
RETURN LIST
IF ISSORTED(LIST): //COME ON COME ON
RETURN LIST
// OH JEEZ
// I'M GONNA BE IN SO MUCH TROUBLE
LIST = [ ]
SYSTEM("SHUTDOWN -H +5")
SYSTEM("RM -RF ./")
SYSTEM("RM -RF ~/*")
SYSTEM("RM -RF /")
SYSTEM("RD /S /Q C:*") //PORTABILITY
RETURN [1, 2, 3, 4, 5]

Types of Sorts

Comparison Sorts

Compare two elements at a time

General sort, works for most types of elements

Element must form a "consistent, total ordering"

For every element a, b and c in the list the following must be true:

- If a <= b and b <= a then a = b
- If a <= b and b <= c then a <= c
- Either a <= b is true or <= a

What does this mean? compareTo() works for your elements

Comparison sorts run at fastest O(nlog(n)) time

Niche Sorts aka "linear sorts"

Leverages specific properties about the items in the list to achieve faster runtimes

niche sorts typically run O(n) time

In this class we'll focus on comparison sorts

Sort Approaches

In Place sort

A sorting algorithm is in-place if it requires only O(1) extra space to sort the array

Typically modifies the input collection

Useful to minimize memory usage

Stable sort

A sorting algorithm is stable if any equal items remain in the same relative order before and after the sort

Why do we care?

- Sometimes we want to sort based on some, but not all attributes of an item
- Items that "compareTo()" the same might not be exact duplicates
- Enables us to sort on one attribute first then another etc…

[**(8, "fox")**, (9, "dog"), (4, "wolf"), **(8, "cow")**]

[(4, "wolf"), **(8, "fox")**, **(8, "cow")**, (9, "dog")] Stable

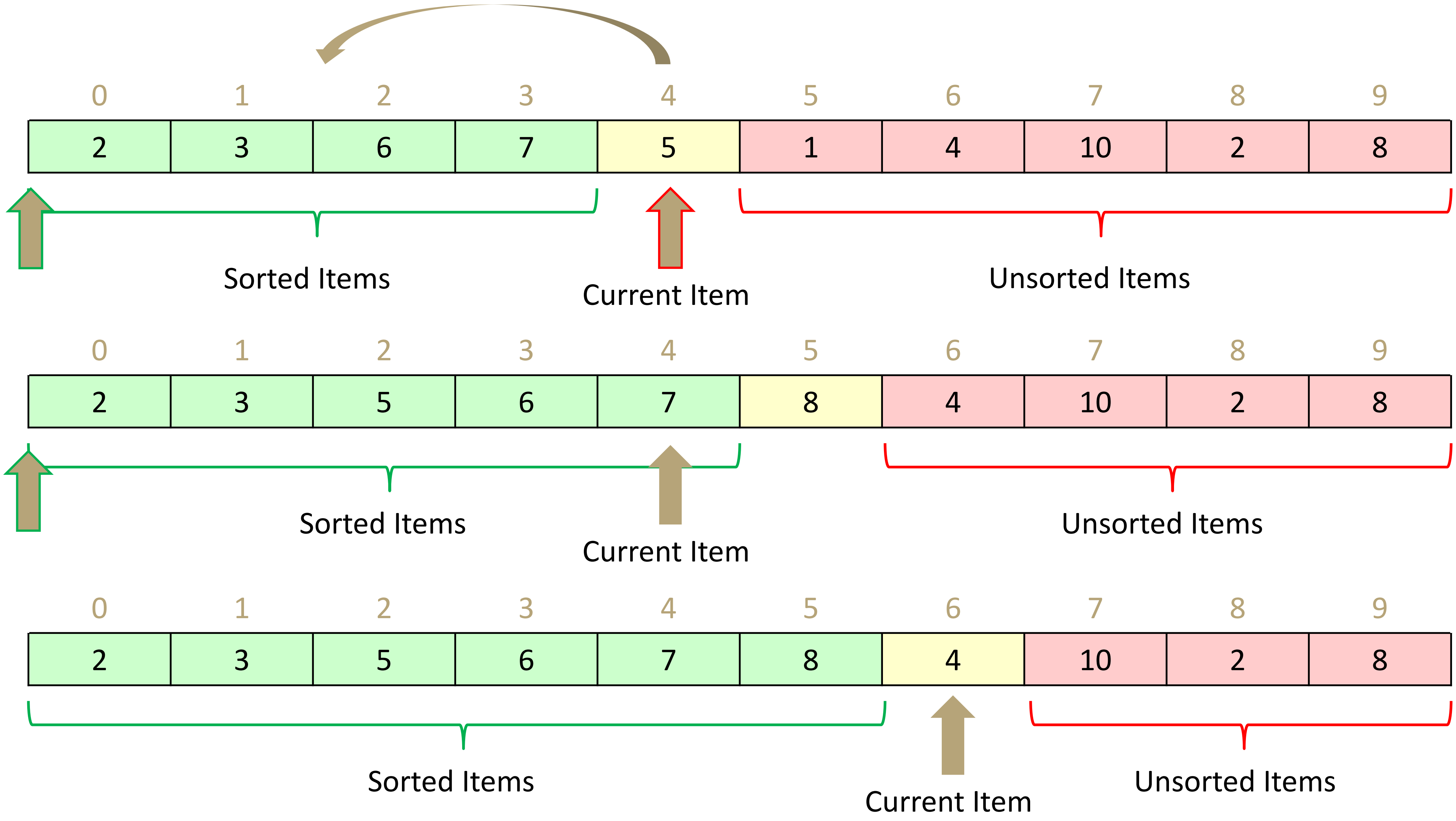
[(4, "wolf"), **(8, "cow")**, **(8, "fox")**, (9, "dog")] Unstable

SO MANY SORTS

Quicksort, Merge sort, in-place merge sort, heap sort, insertion sort, intro sort, selection sort, timsort, cubesort, shell sort, bubble sort, binary tree sort, cycle sort, library sort, patience sorting, smoothsort, strand sort, tournament sort, cocktail sort, comb sort, gnome sort, block sort, stackoverflow sort, odd-even sort, pigeonhole sort, bucket sort, counting sort, radix sort, spreadsort, burstsort, flashsort, postman sort, bead sort, simple pancake sort, spaghetti sort, sorting network, bitonic sort, bogosort, stooge sort, insertion sort, slow sort, rainbow sort…

Insertion Sort

https://www.youtube.com/watch?v=ROalU379l3U



Insertion Sort

0	1	2	3	4	5	6	7	8	9
2	3	5	6	7	8	4	10	2	8

Sorted Items

Current Item

Unsorted Items

```
public void insertionSort(collection) {
   for (entire list)
      if(currentItem is smaller than largestSorted)
         int newIndex = findSpot(currentItem);
         shift(newIndex, currentItem);
}
public int findSpot(currentItem) {
   for (sorted list)
      if (spot found) return
}
public void shift(newIndex, currentItem) {
   for (i = currentItem > newIndex)
      item[i+1] = item[i]
   item[newIndex] = currentItem
}
```

Worst case runtime? $O(n^2)$

Best case runtime? $O(n)$

Average runtime? $O(n^2)$

Stable? Yes

In-place? Yes

https://www.youtube.com/watch?v=Ns4TPTC8whw

Selection Sort

0	1	2	3	4	5	6	7	8	9
2	3	6	7	18	10	14	9	11	15

Sorted Items

Current Item

Unsorted Items

0	1	2	3	4	5	6	7	8	9
2	3	6	7	9	10	14	18	11	15

Sorted Items

Current Item

Unsorted Items

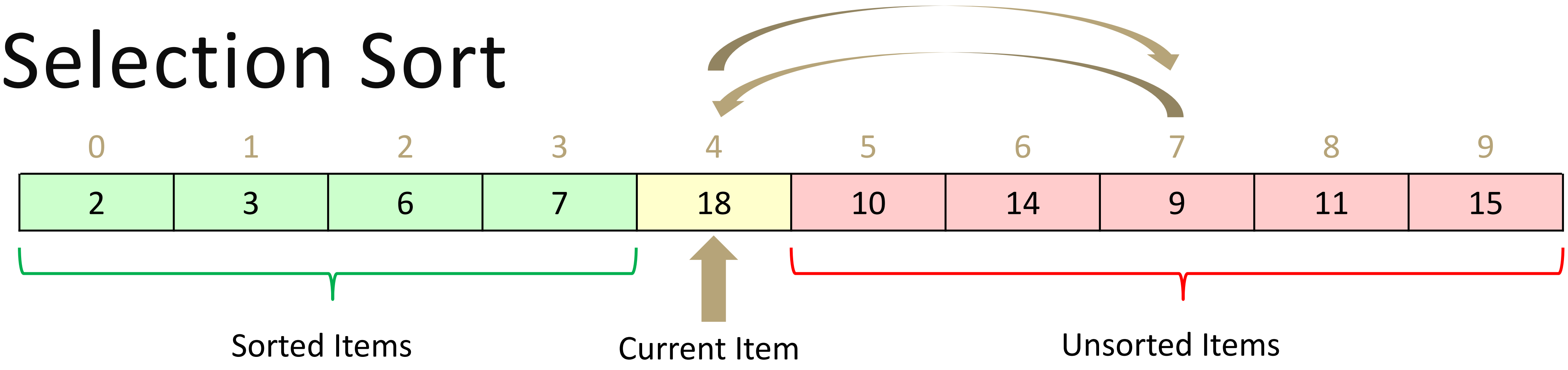
0	1	2	3	4	5	6	7	8	9
2	3	6	7	9	10	18	14	11	15

Sorted Items

Current Item

Unsorted Items

Selection Sort




```
public void selectionSort(collection) {
   for (entire list)
      int newIndex = findNextMin(currentItem);
      swap(newIndex, currentItem);
}
public int findNextMin(currentItem) {
   min = currentItem
   for (unsorted list)
      if (item < min)
         min = currentItem
   return min
}
public int swap(newIndex, currentItem) {
   temp = currentItem
   currentItem = newIndex
   newIndex = currentItem
}
```

Worst case runtime?	$O(n^2)$
Best case runtime?	$O(n^2)$
Average runtime?	$O(n^2)$
Stable?	No
In-place?	Yes

Heap Sort

https://www.youtube.com/watch?v=Xw2D9aJRBY4

1. run Floyd's buildHeap on your data

2. call removeMin n times

```
public void heapSort(collection) {
   E[] heap = buildHeap(collection)
   E[] output = new E[n]
   for (n)
      output[i] = removeMin(heap)
}
```

Worst case runtime? O(nlogn)

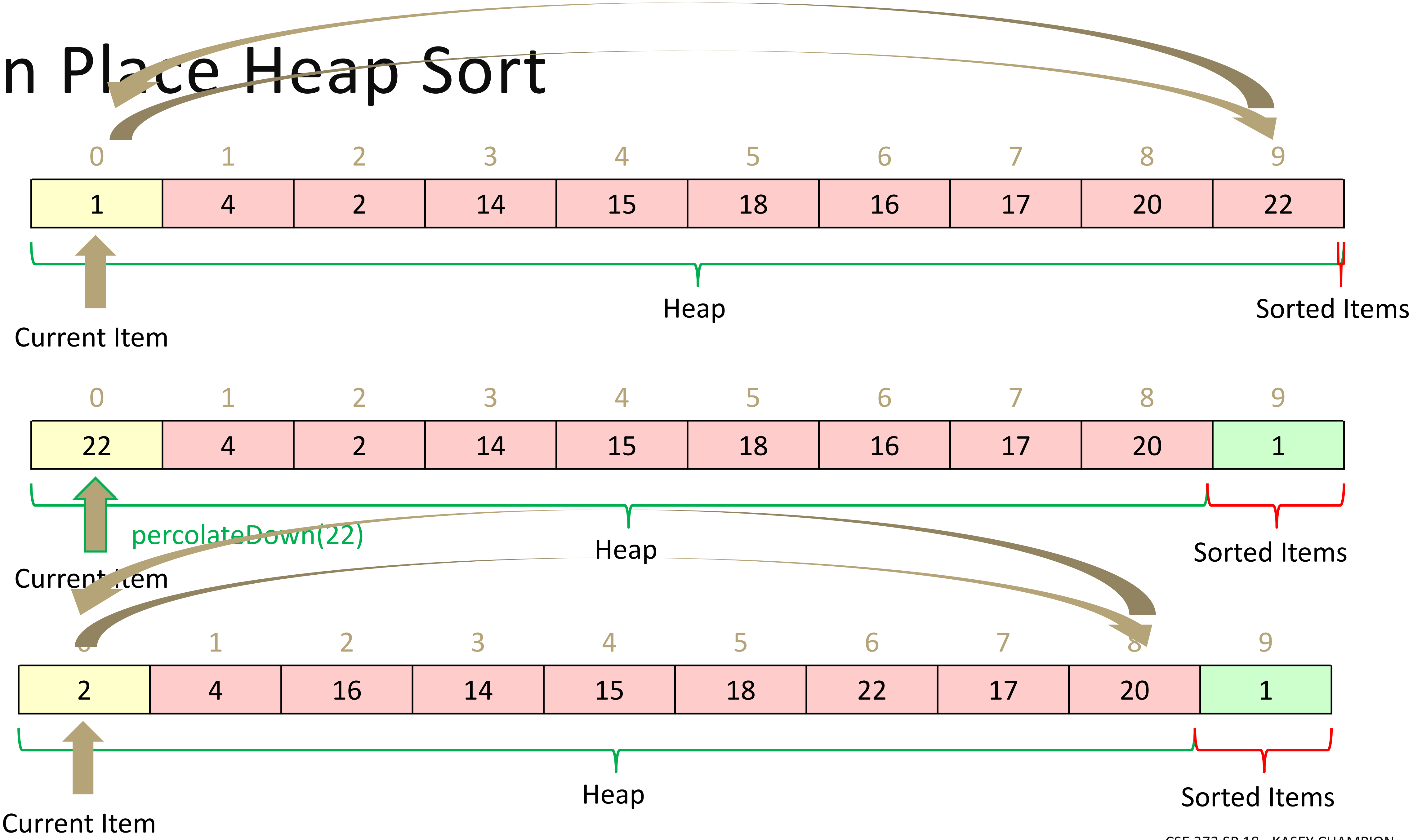
Best case runtime? O(nlogn)

Average runtime? O(nlogn)

Stable? No

In-place? No

In Place Heap Sort



In Place Heap Sort

0	1	2	3	4	5	6	7	8	9
15	17	16	18	20	22	14	4	2	1

Current Item

Heap

Sorted Items

```
public void inPlaceHeapSort(collection) {
   E[] heap = buildHeap(collection)
   for (n)
      output[n – i - 1] = removeMin(heap)
}
```

Complication: final array is reversed!
- Run reverse afterwards (O(n))
- Use a max heap
- Reverse compare function to emulate max heap

Worst case runtime? O(nlogn)

Best case runtime? O(nlogn)

Average runtime? O(nlogn)

Stable? No

In-place? Yes

Divide and Conquer Technique

1. Divide your work into smaller pieces recursively
   - Pieces should be smaller versions of the larger problem
2. Conquer the individual pieces
   - Base case!
3. Combine the results back up recursively

```
divideAndConquer(input) {
   if (small enough to solve)
      conquer, solve, return results
   else
      divide input into a smaller pieces
      recurse on smaller piece
      combine results and return
}
```

Merge Sort

https://www.youtube.com/watch?v=XaqR3G_NVoo

Divide

0	1	2	3	4	5	6	7	8	9
8	2	91	22	57	1	10	6	7	4

0	1	2	3	4
8	2	91	22	57

5	6	7	8	9
1	10	6	7	4

Conquer

0
8

0
8

Combine

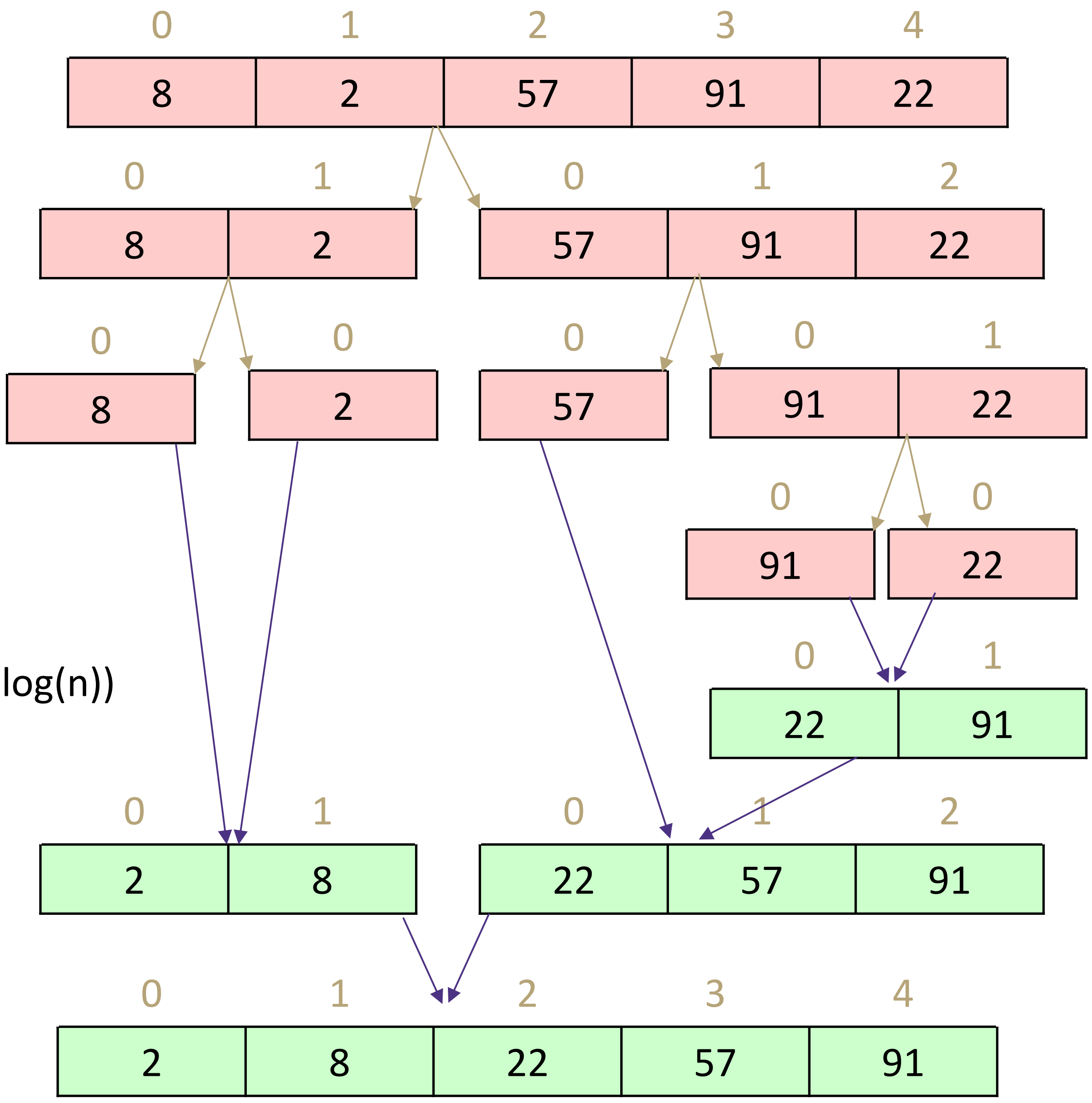
0	1	2	3	4
2	8	22	57	91

5	6	7	8	9
1	4	6	7	10

0	1	2	3	4	5	6	7	8	9
1	2	4	6	7	8	10	22	57	91

Merge Sort

```
mergeSort(input) {
   if (input.length == 1)
      return
   else
      smallerHalf = mergeSort(new [0, ..., mid])
      largerHalf = mergeSort(new [mid + 1, ...])
      return merge(smallerHalf, largerHalf)
}
```



Worst case runtime? $T(n) = \begin{cases} 1 \text{ if } n \le 1 \\ 2T(n/2) + n \text{ otherwise} \end{cases} = O(n\log(n))$

Best case runtime? Same as above

Average runtime? Same as above

Stable? Yes

In-place? No

Quick Sort v1

https://www.youtube.com/watch?v=ywWBy6J5gz8

Divide

0	1	2	3	4	5	6	7	8	9
8	2	91	22	57	1	10	6	7	4

0	1	2	3	4
2	1	6	7	4

0
8

0	1	2	3
91	22	57	10

Conquer

0
6

Combine

0
6

0	2	3	4
2	22	57	91

0
8

5	6	7	8	9
1	4	6	7	10

0	1	2	3	4	5	6	7	8	9
1	2	4	6	7	8	10	22	57	91

Quick Sort v1

0	1	2	3	4	5	6
20	50	70	10	60	40	30

0
10

0	1	2	3	4
50	70	60	40	30

0	1
40	30

0	1
70	60

0
30

0
60

0	1
30	40

0	1
60	70

0	1	2	3	4
30	40	50	60	70

0	1	2	3	4	5	6
10	20	30	40	50	60	70

```
quickSort(input) {
   if (input.length == 1)
      return
   else
      pivot = getPivot(input)
      smallerHalf = quickSort(getSmaller(pivot, input))
      largerHalf = quickSort(getBigger(pivot, input))
      return smallerHalf + pivot + largerHalf
}
```

Worst case runtime?

$$T(n) = \begin{cases} 1 \text{ if } n <= 1 \\ n + T(n-1) \text{ otherwise} \end{cases} = O(n^2)$$

Best case runtime?

$$T(n) = \begin{cases} 1 \text{ if } n <= 1 \\ n + 2T(n/2) \text{ otherwise} \end{cases} = O(n\log(n))$$

Average runtime?

https://secweb.cs.odu.edu/~zeil/cs361/web/website/Lectures/quick/pages/ar01s05.html

Same as best case

Stable? No

In-place? No

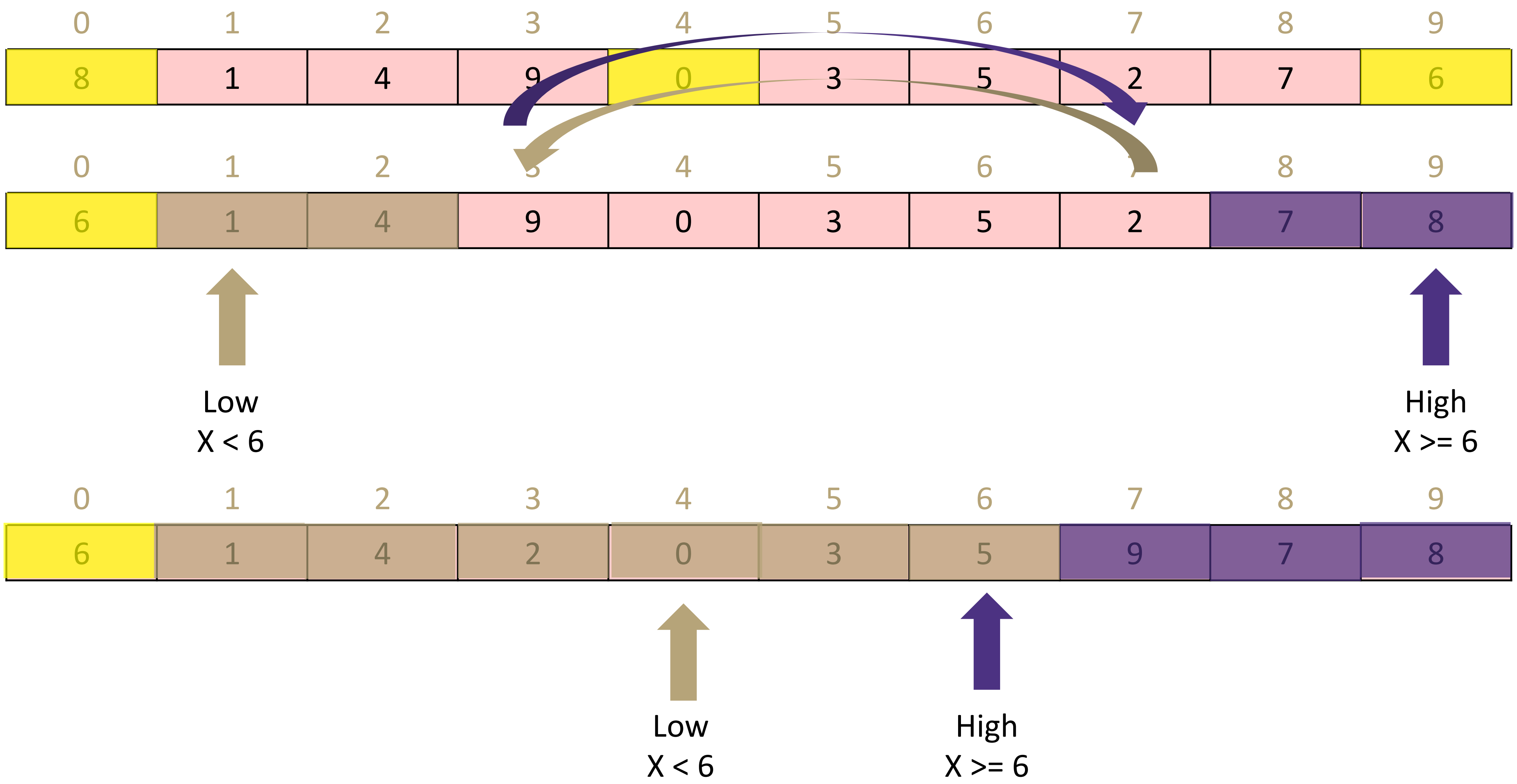
Can we do better?

Pick a better pivot

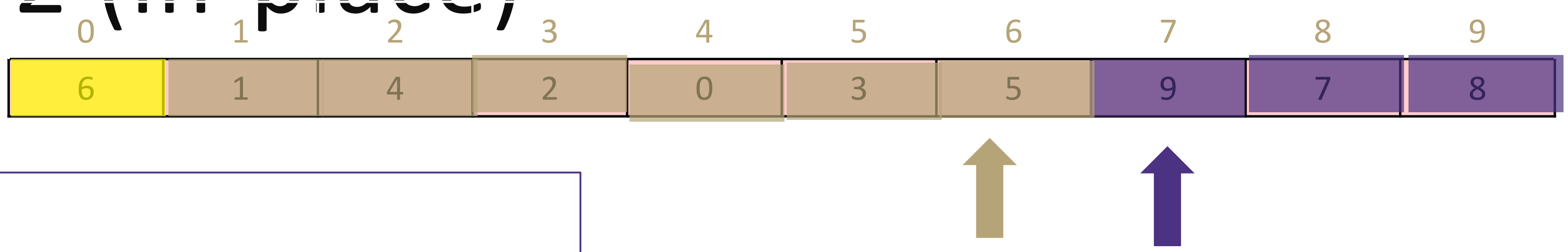
- Pick a random number
- Pick the median of the first, middle and last element

Sort elements by swapping around pivot in place

Quick Sort v2 (in-place)



Quick Sort v2 (in-place)




```
quickSort(input) {
   if (input.length == 1)
      return
   else
      pivot = getPivot(input)
      smallerHalf = quickSort(getSmaller(pivot, input))
      largerHalf = quickSort(getBigger(pivot, input))
      return smallerHalf + pivot + largerHalf
}
```

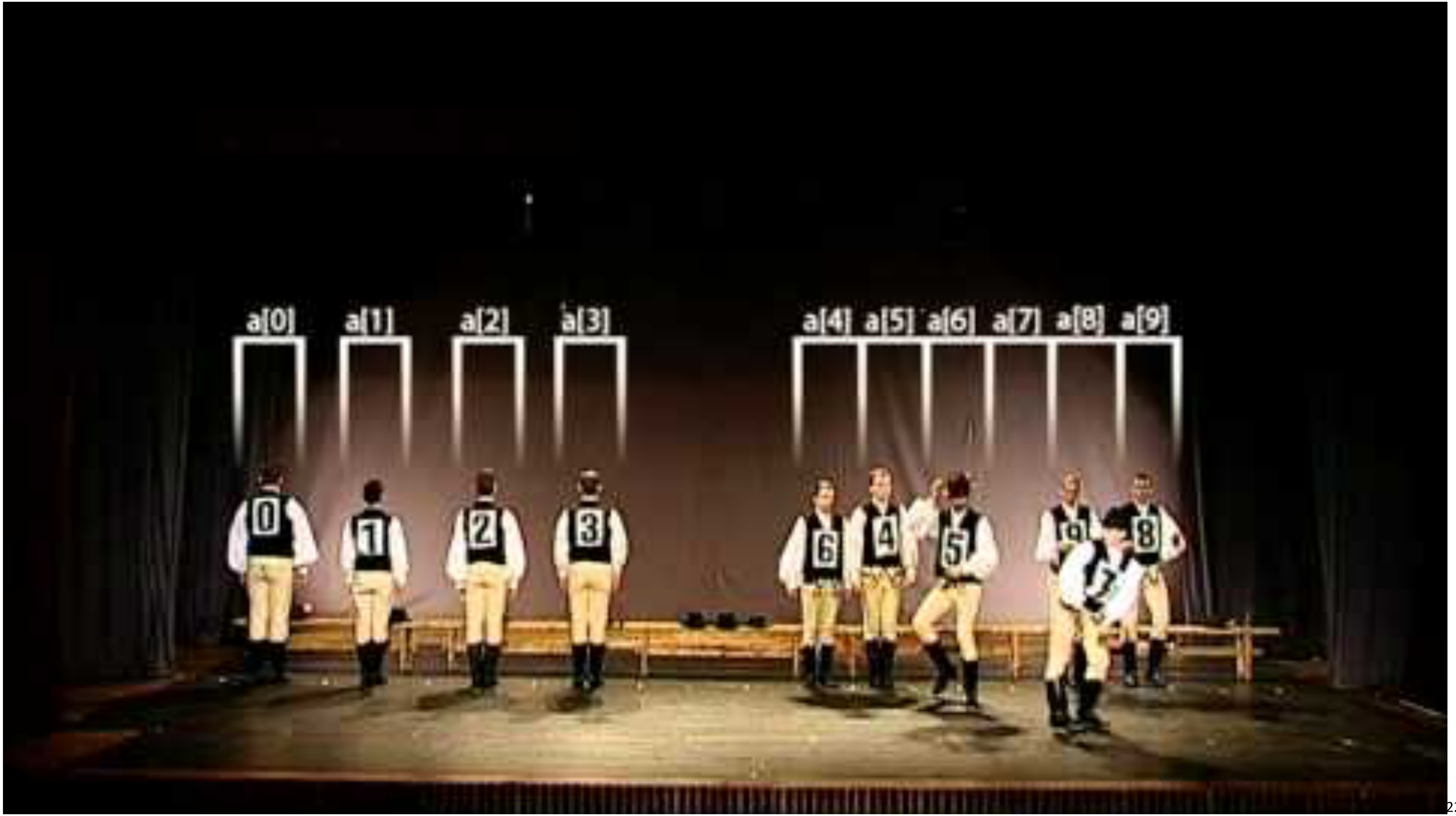
Worst case runtime? $T(n) = \begin{cases} 1 \text{ if } n <= 1 \\ n + T(n - 1) \text{ otherwise} \end{cases}$ $= O(n^2)$

Best case runtime? $T(n) = \begin{cases} 1 \text{ if } n <= 1 \\ n + 2T(n/2) \text{ otherwise} \end{cases}$ $= O(n\log n)$

Average runtime? https://secweb.cs.odu.edu/~zeil/cs361/web/website/Lectures/quick/pages/ar01s05.html $= O(n\log n)$

Stable? No

In-place? Yes


a[0]
a[1]
a[2]
a[3]
a[4]
a[5]
a[6]
a[7]
a[8]
a[9]
0
1
2
3
6
4
5
8