Lecture 12: Hash Open Indexing

Data Structures and Algorithms

3 Minutes

Linear Probing

Insert the following values into the Hash Table using a hashFunction of % table size and linear probing to resolve collisions
38, 19, 8, 109, 10

8

0	1	2	3	4	5	6	7	8	9
10								38	109

Problem:

- Linear probing causes clustering
- Clustering causes more looping when probing

Primary Clustering

When probing causes long chains of occupied slots within a hash table

Administrivia

HW 4 Partner Form due tonight 11:59pm

HW 3 code modeling got more details

Midterm Friday

- Debugging
- More office hours today
- Review session on Wednesday 4pm – 5:50pm PAA A102

2 Minutes

Runtime

When is runtime good?

Empty table

When is runtime bad?

Table nearly full

When we hit a “cluster”

Maximum Load Factor?

λ at most 1.0

When do we resize the array?

$\lambda \approx ½$

Can we do better?

Clusters are caused by picking new space near natural index

Solution 2: Open Addressing

Type 2: Quadratic Probing

If we collide instead try the next i^2 space

```
public int hashFunction(String s)
    int naturalHash = this.getHash(s);
    if(natural hash in use) {
        int i = 1;
        while (index in use) {
            try (naturalHash + i * i);
            i++;
```

Quadratic Probing

Insert the following values into the Hash Table using a hashFunction of % table size and quadratic probing to resolve collisions
89, 18, 49, 58, 79

0	1	2	3	4	5	6	7	8	9
		58	79					18	89 49

(49 % 10 + 0 * 0) % 10 = 9
(49 % 10 + 1 * 1) % 10 = 0

(58 % 10 + 0 * 0) % 10 = 8
(58 % 10 + 1 * 1) % 10 = 9
(58 % 10 + 2 * 2) % 10 = 2

(79 % 10 + 0 * 0) % 10 = 9
(79 % 10 + 1 * 1) % 10 = 0
(79 % 10 + 2 * 2) % 10 = 3

Problems:

If $\lambda \geq \frac{1}{2}$ we might never find an empty spot

  Infinite loop!

Can still get clusters

Secondary Clustering

3 Minutes

Insert the following values into the Hash Table using a hashFunction of % table size and quadratic probing to resolve collisions
19, 39, 29, 9

0	1	2	3	4	5	6	7	8	9
39			29					9	19

Secondary Clustering

When using quadratic probing sometimes need to probe the same sequence of table cells, not necessarily next to one another

Probing

- h(k) = the natural hash
- h'(k, i) = resulting hash after probing
- i = iteration of the probe
- T = table size

Linear Probing:

$h'(k, i) = (h(k) + i) \% T$

Quadratic Probing

$h'(k, i) = (h(k) + i^2) \% T$

For both types there are only O(T) probes available

- Can we do better?

Double Hashing

Probing causes us to check the same indices over and over- can we check different ones instead?

Use a second hash function!

h'(k, i) = (h(k) + i * g(k)) % T

<- Most effective if g(k) returns value prime to table size

```
public int hashFunction(String s)
    int naturalHash = this.getHash(s);
    if(natural hash in use) {
        int i = 1;
        while (index in use) {
            try (naturalHash + i * jump_Hash(key));
            i++;
```

Second Hash Function

Effective if g(k) returns a value that is *relatively prime* to table size

- If T is a power of 2, make g(k) return an odd integer
- If T is a prime, make g(k) return any smaller, non-zero integer
  - g(k) = 1 + (k % T(-1))

How many different probes are there?

- T different starting positions
- T – 1 jump intervals
- $O(T^2)$ different probe sequences
  - Linear and quadratic only offer O(T) sequences

Resizing

How do we resize?

- Remake the table
- Evaluate the hash function over again.
- Re-insert.

When to resize?

- Depending on our load factor λ
- Heuristic:
  - for separate chaining λ between 1 and 3 is a good time to resize.
  - For open addressing λ between 0.5 and 1 is a good time to resize.

Separate chaining: Running Times

What are the running times for:

`insert`

Best: $O(1)$

Worst: $O(n)$ (if insertions are always at the end of the linked list)

`find`

Best: $O(1)$

Worst: $O(n)$

`delete`

Best: $O(1)$
Worst: $O(n)$

Linear probing: Average-case insert

If $\lambda < 1$ we'll find a spot eventually.

What's the average running time?

Uniform Hashing Assumption
for any pair of elements x, y the probability that h(x) = h(y) is $\frac{1}{TableSize}$

If find is unsuccessful: $\frac{1}{2}\left(1 + \frac{1}{(1-\lambda)^2}\right)$

If find is successful: $\frac{1}{2}\left(1 + \frac{1}{(1-\lambda)}\right)$

We won't ask you to prove these

Summary

1. Pick a hash function to:
   - Avoid collisions
   - Uniformly distribute data
   - Reduce hash computational costs

2. Pick a collision strategy
   - Chaining
     - LinkedList
     - AVL Tree
   - Probing
     - Linear
     - Quadratic
   - Double Hashing

No clustering
Potentially more “compact” (λ can be higher)

Managing clustering can be tricky
Less compact (keep $\lambda < ½$)
Array lookups tend to be a constant factor faster than traversing pointers

Summary

Separate Chaining

- Easy to implement
- Running times $O(1+\lambda)$

Open Addressing

- Uses less memory.
- Various schemes:
- Linear Probing – easiest, but need to resize most frequently
- Quadratic Probing – middle ground
- Double Hashing – need a whole new hash function, but low chance of clustering.

Which you use depends on your application and what you're worried about.

Other applications of hashing

- Cryptographic hash functions: Hash functions with some additional properties
  - Commonly used in practice: SHA-1, SHA-265
  - To verify file integrity. When you share a large file with someone, how do you know that the other person got the exact same file? Just compare hash of the file on both ends. Used by file sharing services (Google Drive, Dropbox)
  - For password verification: Storing passwords in plaintext is insecure. So your passwords are stored as a hash.
  - For Digital signature
  - Lots of other crypto applications

- Finding similar records: Records with similar but not identical keys
  - Spelling suggestion/corrector applications
  - Audio/video fingerprinting
  - Clustering

- Finding similar substrings in a large collection of strings
  - Genomic databases
  - Detecting plagiarism

- Geometric hashing: Widely used in computer graphics and computational geometry

Wrap Up

Hash Tables:

- Efficient find, insert, delete **on average, under some assumptions**
- Items not in sorted order
- Tons of real world uses
- …and really popular in tech interview questions.

Need to pick a good hash function.

- Have someone else do this if possible.
- Balance getting a good distribution and speed of calculation.

Resizing:

- Always make the table size a prime number.
- λ determines when to resize, but depends on collision resolution strategy.

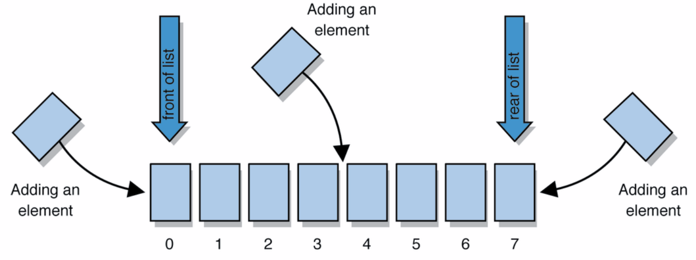
ADT Review

List ADT

List ADT

state
Set of ordered items
Count of items

behavior
get(index) return item at index
set(item, index) replace item at index
append(item) add item to end of list
insert(item, index) add item at index
delete(index) delete item at index
size() count of items

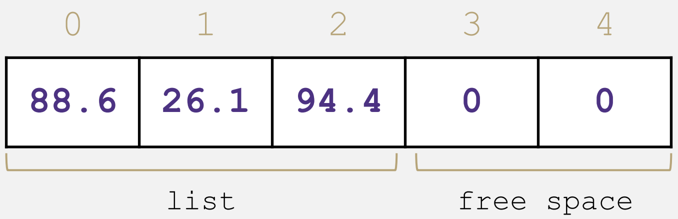


ArrayList
uses an Array as underlying storage

ArrayList<E>

state
data[]
size

behavior
get return data[index]
set data[index] = value
append data[size] = value, if out of space grow data
insert shift values to make hole at index, data[index] = value, if out of space grow data
delete shift following values forward
size return size



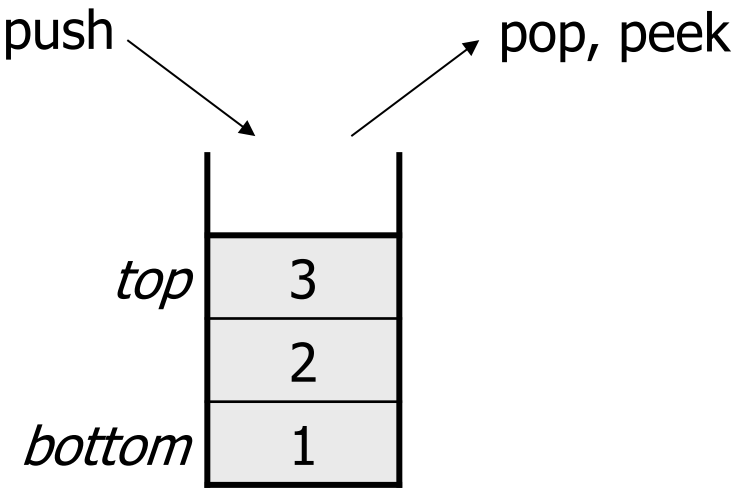
LinkedList
uses nodes as underlying storage

LinkedList<E>

state
Node front
size

behavior
get loop until index, return node's value
set loop until index, update node's value
append create new node, update next of last node
insert create new node, loop until index, update next fields
delete loop until index, skip node
size return size

Stack ADT



stack: A collection based on the principle of adding elements and retrieving them in the opposite order.

- Last-In, First-Out ("LIFO")
- Elements are stored in order of insertion.
  - We do not think of them as having indexes.
- Client can only add/remove/examine the last element added (the "top").

Stack ADT

state

Set of ordered items
Number of items

behavior

push(item) add item to top
pop() return and remove item at top
peek() look at item at top
size() count of items
isEmpty() count of items is 0?

ArrayStack<E>

state

```
data[]
size
```

behavior

```
push data[size] = value, if out of room grow data
pop return data[size - 1], size-1
peek return data[size - 1]
size return size
isEmpty return size == 0
```

LinkedStack<E>

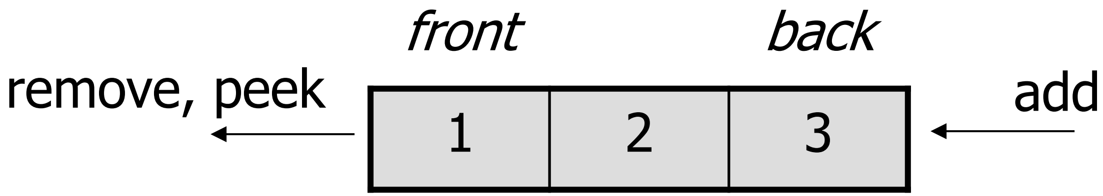
state

```
Node top
size
```

behavior

```
push add new node at top
pop return and remove node at top
peek return node at top
size return size
isEmpty return size == 0
```

Queue ADT



queue: Retrieves elements in the order they were added.

- First-In, First-Out ("FIFO")
- Elements are stored in order of insertion but don't have indexes.
- Client can only add to the end of the queue, and can only examine/remove the front of the queue.

Queue ADT

state

Set of ordered items
Number of items

behavior

add(item) add item to back
remove() remove and return item at front
peek() return item at front
size() count of items
isEmpty() count of items is 0?

ArrayQueue<E>

state

```
data[]
Size
front index
back index
```

behavior

```
add - data[size] = value, if out of room grow data
remove - return data[size - 1], size-1
peek - return data[size - 1]
size - return size
isEmpty - return size == 0
```

LinkedQueue<E>

state

```
Node front
Node back
size
```

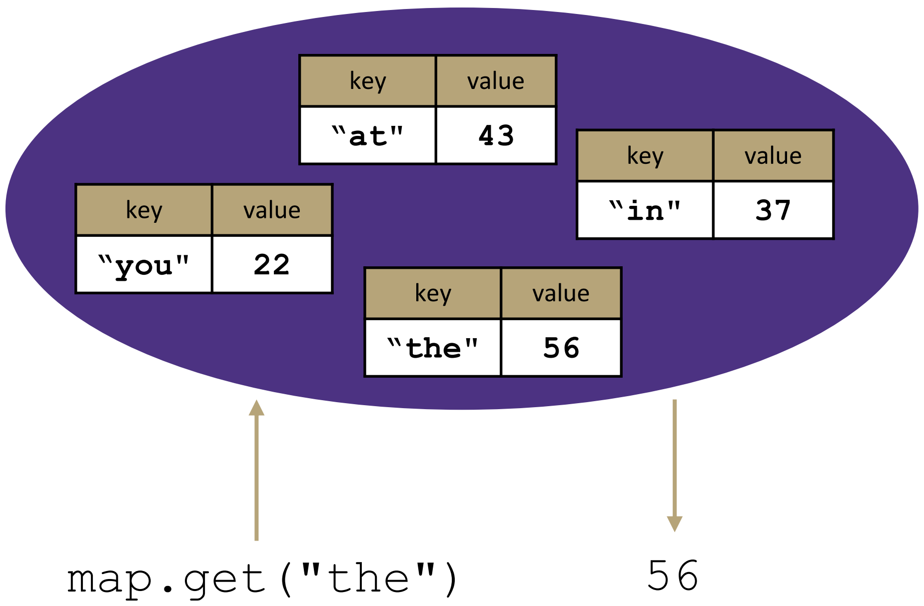
behavior

```
add - add node to back
remove - return and remove node at front
peek - return node at front
size - return size
isEmpty - return size == 0
```

Map/Dictionary ADT

dictionary: Holds a set of unique *keys* and a collection of *values*, where each key is associated with one value.

- a.k.a. "dictionary", "associative array", "hash"



Dictionary ADT

state

Set of items & keys
Count of items

behavior

put(key, item) add item to collection indexed with key
get(key) return item associated with key
containsKey(key) return if key already in use
remove(key) remove item and associated key
size() return count of items

ArrayDictionary<K, V>

state

`Pair<K, V>[] data`

behavior

`put` create new pair, add to next available spot, grow array if necessary
`get` scan all pairs looking for given key, return associated item if found
`containsKey` scan all pairs, return if key is found
`remove` scan all pairs, replace pair to be removed with last pair in collection
`size` return count of items in dictionary

LinkedDictionary<K, V>

state

`front`
`size`

behavior

`put` if key is unused, create new with pair, add to front of list, else replace with new value
`get` scan all pairs looking for given key, return associated item if found
`containsKey` scan all pairs, return if key is found
`remove` scan all pairs, skip pair to be removed
`size` return count of items in dictionary

Binary Search Trees

A **binary search tree** is a binary tree that contains comparable items such that for every node, all children to the left contain smaller data and all children to the right contain larger data.

Binary Search Trees allow us to:

- quickly find what we're looking for
- add and remove values easily

Dictionary Operations:

Runtime in terms of height, "h"

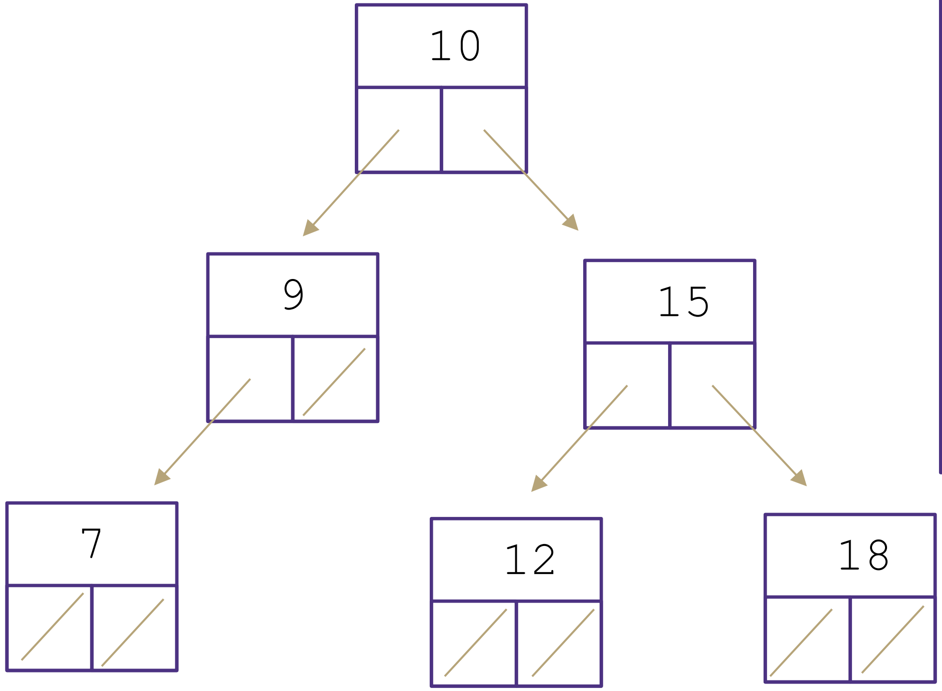
get() – O(h)

put() – O(h)

remove() – O(h)

What do you replace the node with?

Largest in left sub tree or smallest in right sub tree



TreeDictionary<K, V>

state

overallRoot
size

behavior

put if key is unused, create new pair, place in BST order, rotate to maintain balance

get traverse through tree using BST property, return item if found

containsKey traverse through tree using BST property, return if key is found

remove traverse through tree using BST property, replace or nullify as appropriate

size return count of items in dictionary

Meet AVL Trees

AVL Trees must satisfy the following properties:

- binary trees: all nodes must have between 0 and 2 children
- binary search tree: for all nodes, all keys in the left subtree must be smaller and all keys in the right subtree must be larger than the root node
- balanced: for all nodes, there can be no more than a difference of 1 in the height of the left subtree from the right. Math.abs(height(left subtree) – height(right subtree)) ≤ 1

AVL stands for **A**delson-**V**elsky and **L**andis (the inventors of the data structure)

Dictionary Operations:

get() – same as BST

containsKey() – same as BST

put() – same as BST + rebalance

remove() – same as BST + rebalance

How long does AVL insert take?

AVL insert time = BST insert time + time it takes to rebalance the tree

= O(log n) + time it takes to rebalance the tree

How long does rebalancing take?

- Assume we store in each node the height of its subtree.
- How long to find an unbalanced node:
  - Just go back up the tree from where we inserted. ← O(log n)
- How many rotations might we have to do?
  - Just a single or double rotation on the lowest unbalanced node. ← O(1)

AVL insert time = O(log n)+ O(log n) + O(1) = O(log n)

Review: Dictionaries

Dictionary ADT

state

Set of items & keys
Count of items

behavior

put(key, item) add item to collection indexed with key
get(key) return item associated with key
containsKey(key) return if key already in use
remove(key) remove item and associated key
size() return count of items

Why are we so obsessed with Dictionaries? **It's all about data baby!**

When dealing with data:

- Adding data to your collection
- Getting data out of your collection
- Rearranging data in your collection

SUPER common in comp sci
- Databases
- Network router tables
- Compilers and Interpreters

Operation		ArrayList	LinkedList	BST	AVLTree
put(key,value)	best	O(1)	O(1)	O(1)	O(1)
	average	O(n)	O(n)	O(logn)	O(logn)
	worst	O(n)	O(n)	O(n)	O(logn)
get(key)	best	O(1)	O(1)	O(1)	O(1)
	average	O(n)	O(n)	O(logn)	O(logn)
	worst	O(n)	O(n)	O(n)	O(logn)
remove(key)	best	O(1)	O(1)	O(logn)	O(logn)
	average	O(n)	O(n)	O(logn)	O(logn)
	worst	O(n)	O(n)	O(n)	O(logn)

Hashing

HashMap<Integer>

state
`Data[]`
`size`
behavior
`put` mod key by table size, put item at result
`get` mod key by table size, get item at result
`containsKey` mod key by table size, return data[result] == null `remove` mod key by table size, nullify element at result
`size` return count of items in dictionary

Operation		Separate Chaining	Probing
put(key,value)	best	O(1)	O(1)
	average	$O(1 + \lambda)$	$O(1 + \lambda)$
	worst	O(n)	O(n)
get(key)	best	O(1)	O(1)
	average	$O(1 + \lambda)$	$O(1 + \lambda)$
	worst	O(n)	O(n)
remove(key)	best	O(1)	O(1)
	average	$O(1 + \lambda)$	$O(1 + \lambda)$
	worst	O(n)	O(n)