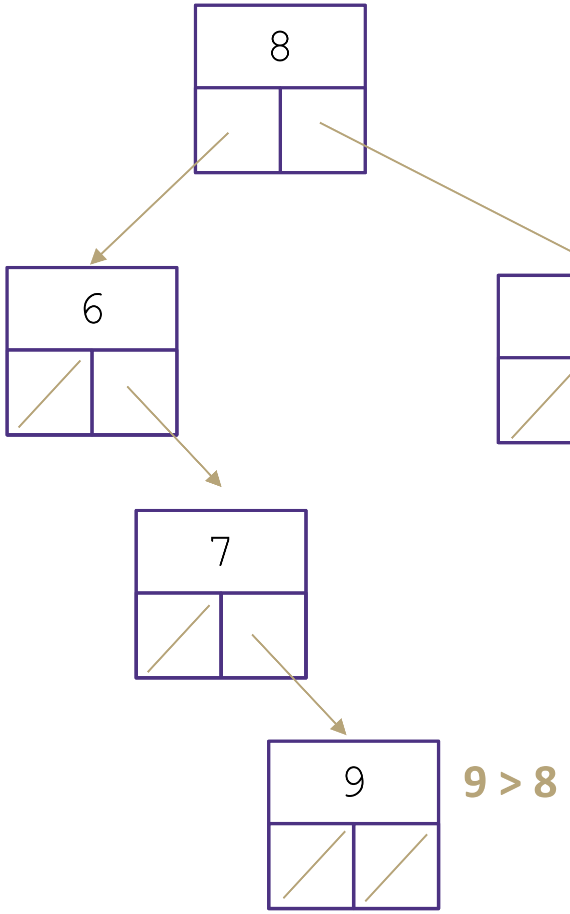
Lecture 10: BST and AVL Trees

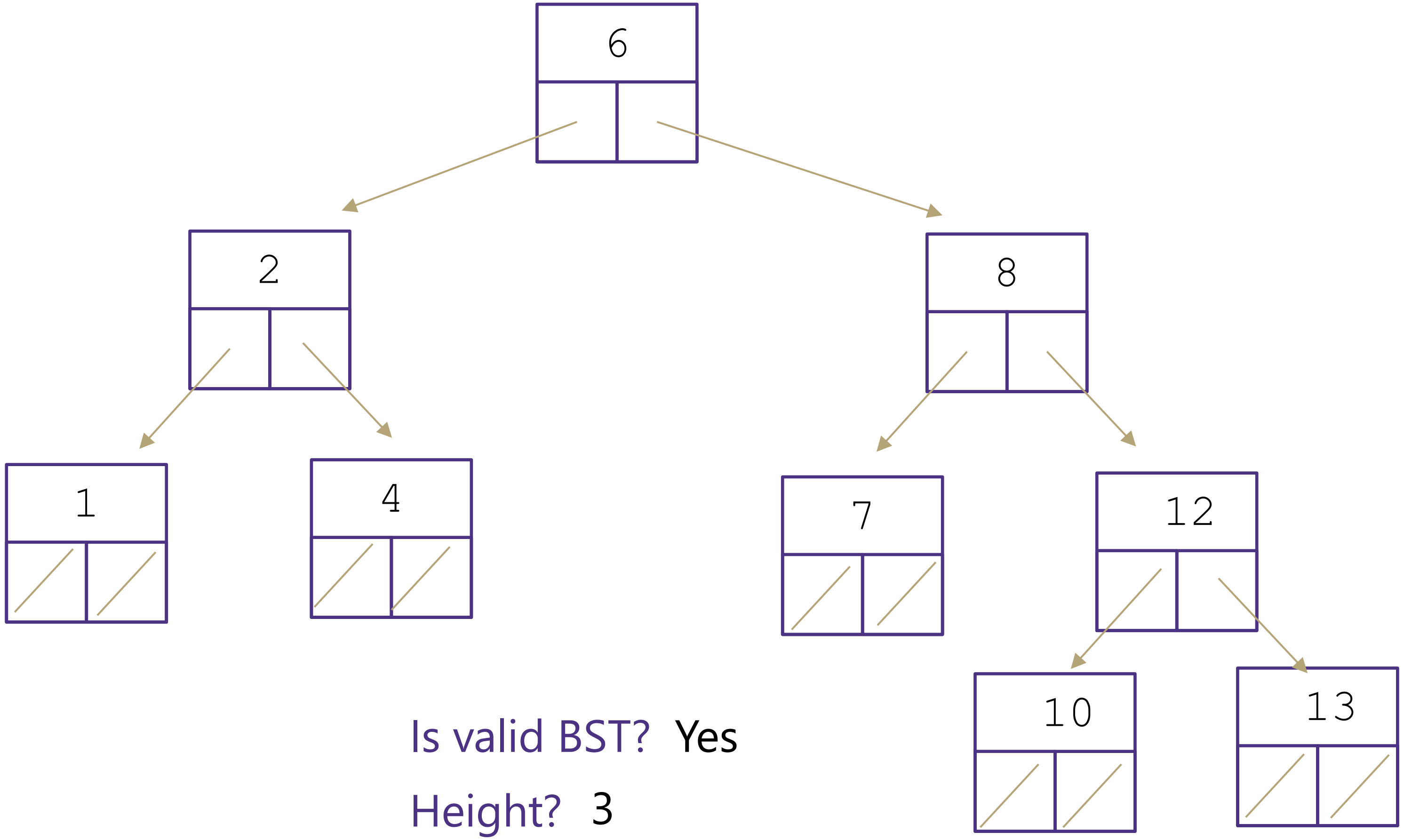
CSE 373: Data Structures and Algorithms

Warm Up

Is valid BST? No

Height? 2





Is valid BST? Yes

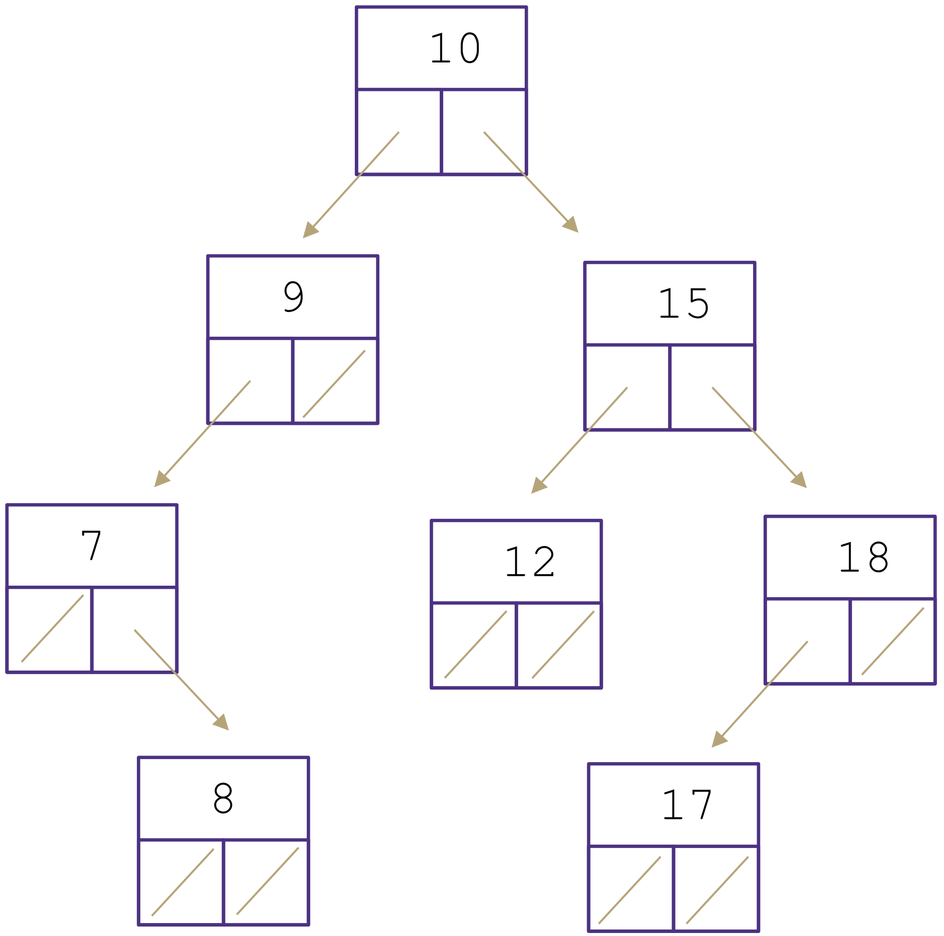
Height? 3

Administrivia

Trees

Binary Search Trees

A **binary search tree** is a binary tree that contains comparable items such that for every node, all children to the left contain smaller data and all children to the right contain larger data.



Implement Dictionary

Binary Search Trees allow us to:

- quickly find what we're looking for
- add and remove values easily

Dictionary Operations:

Runtime in terms of height, "h"

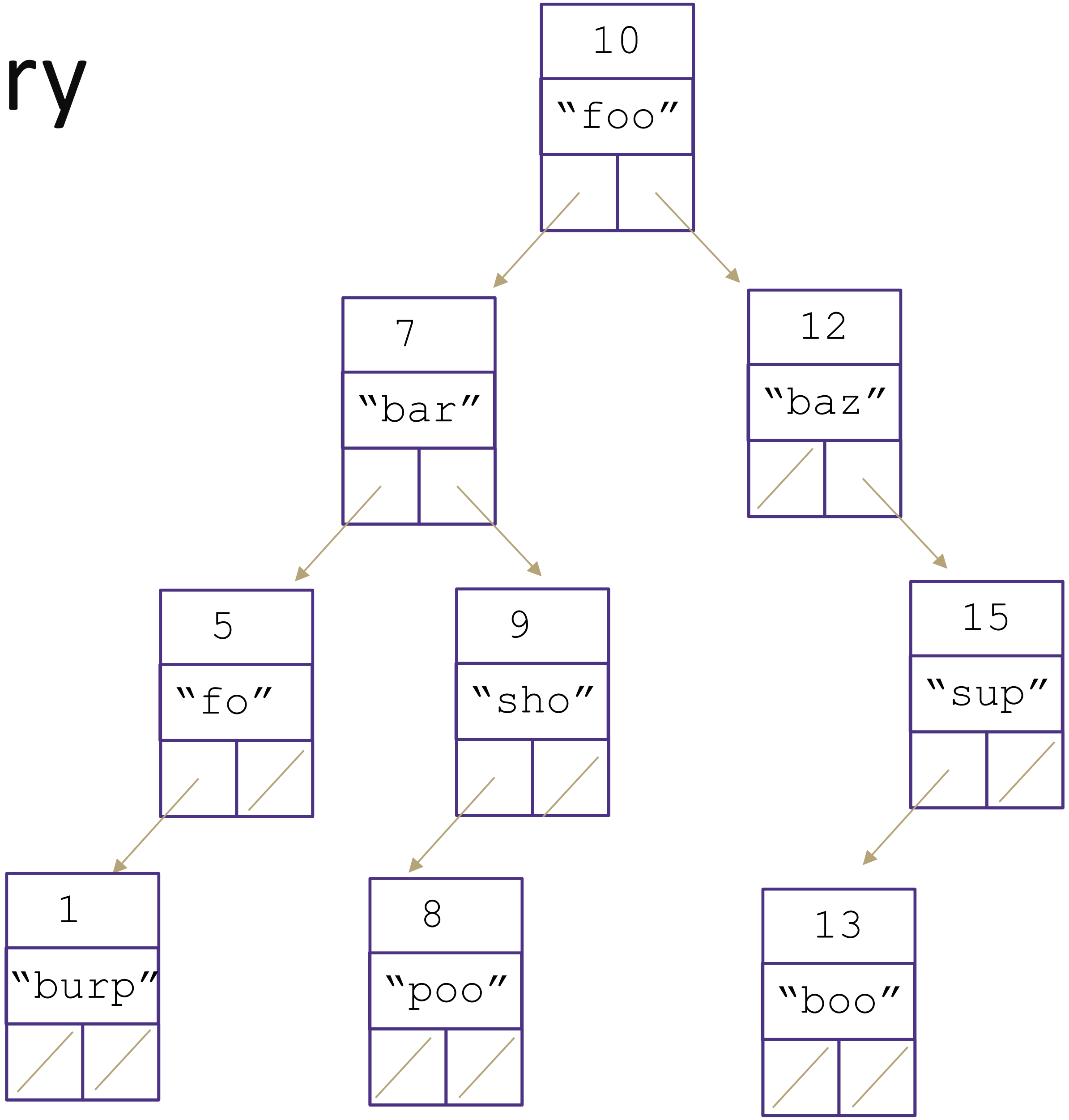
get() – O(h)

put() – O(h)

remove() – O(h)

What do you replace the node with?

Largest in left sub tree or smallest in right sub tree



Height in terms of Nodes

For “balanced” trees $h \approx \log_c(n)$ where c is the maximum number of children

Balanced binary trees $h \approx \log_2(n)$

Balanced trinary tree $h \approx \log_3(n)$

Thus for balanced trees operations take $\Theta(\log_c(n))$

Unbalanced Trees

Is this a valid Binary Search Tree?

Yes, but…

We call this a **degenerate tree**

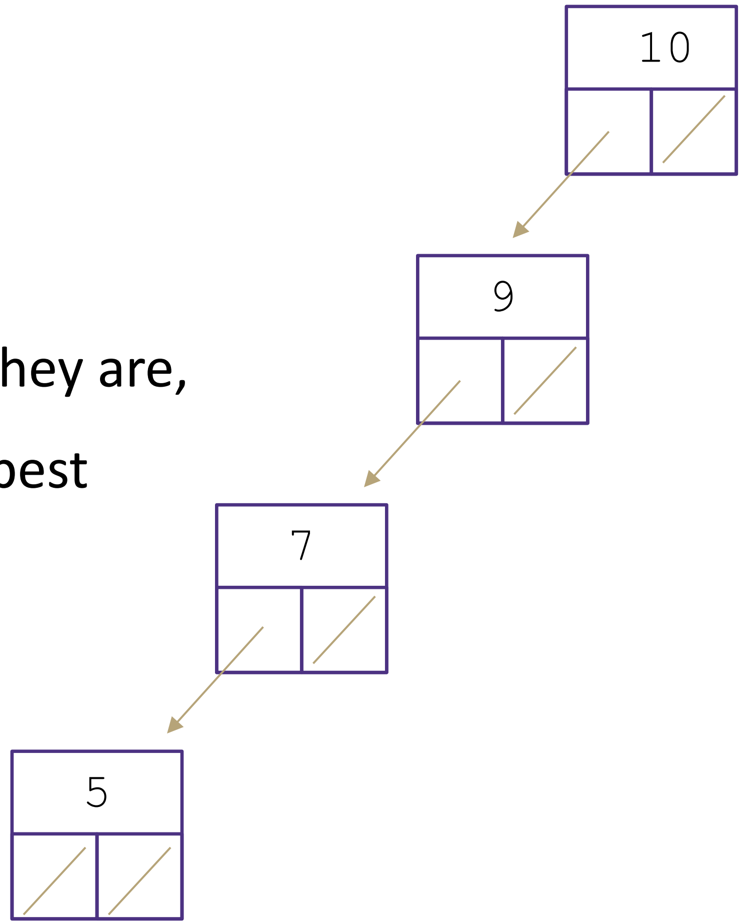
For trees, depending on how balanced they are,

Operations at worst can be O(n) and at best

can be O(logn)

How are degenerate trees formed?

- insert(10)
- insert(9)
- insert(7)
- insert(5)



2 Minutes

Implementing Dictionary with BST

```
public boolean contains(K key, BSTNode node) {
    if (node == null) {
        return false;
```

$+C_1$

```
    }
    int compareResult = compareTo(key, node.data);
```

$+C_2$

```
    if (compareResult < 0) {
        returns contains(key, node.left);
```

$+T(n/2)$ best
$+ T(n-1)$ worst

```
    } else if (compareResult > 0) {
        returns contains(key, node.right);
```

$+T(n/2)$ best
$+ T(n-1)$ worst

```
    } else {
        returns true;
```

$+C_3$

```
    }
}
```

Best Case (assuming key is at the bottom)

$$\boldsymbol{T(n) = \begin{cases} C \text{ when } n < 0 \text{ or key found} \\ T\left(\frac{n}{2}\right) + C \text{ otherwise} \end{cases}}$$

Worst Case (assuming key is at the bottom)

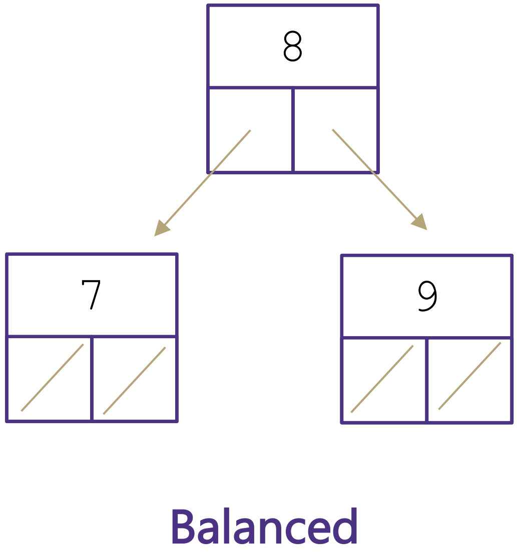
$$\boldsymbol{T(n) = \begin{cases} C \text{ when } n < 0 \text{ or key found} \\ T(n-1) + C \text{ otherwise} \end{cases}}$$

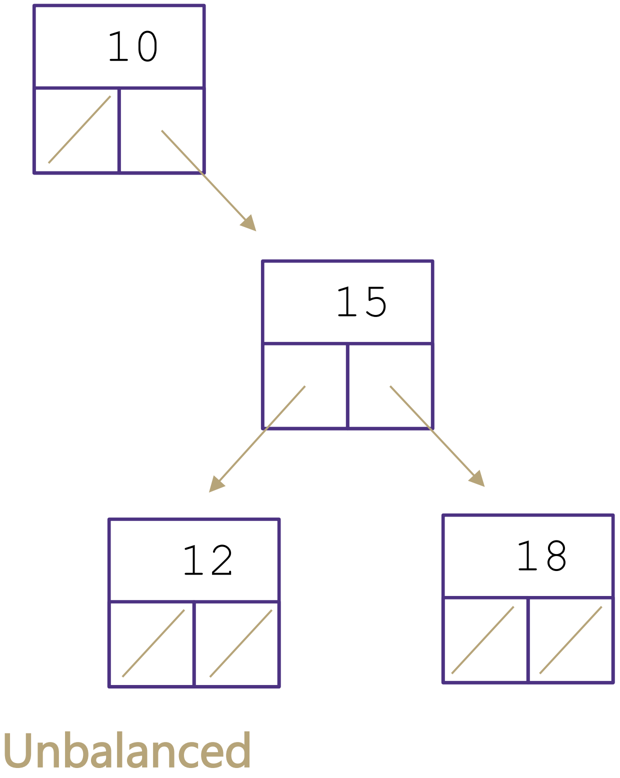
Measuring Balance

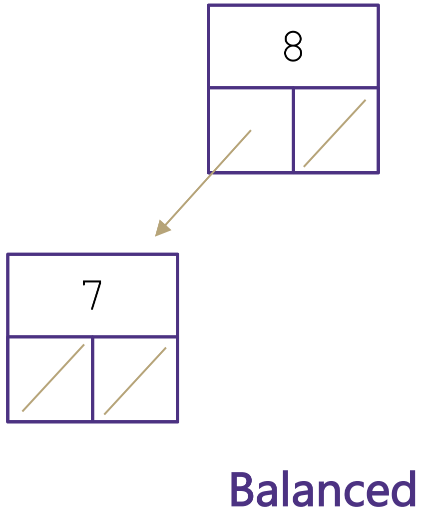
Measuring balance:

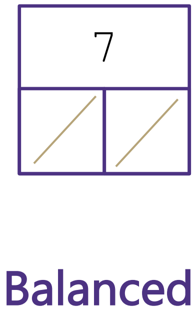
For each node, compare the heights of its two sub trees

Balanced when the difference in height between sub trees is no greater than 1









Meet AVL Trees

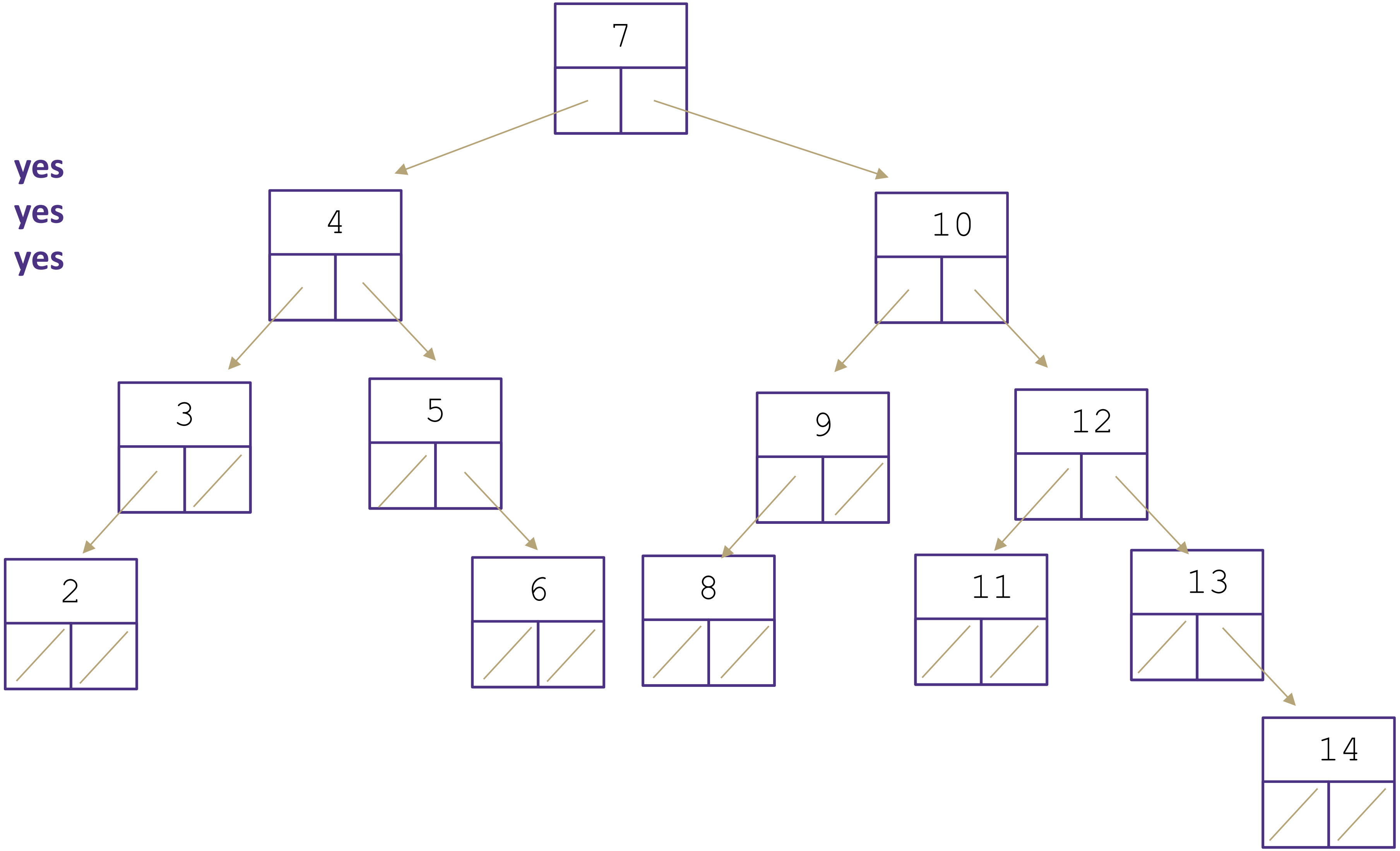
AVL Trees must satisfy the following properties:

- binary trees: all nodes must have between 0 and 2 children
- binary search tree: for all nodes, all keys in the left subtree must be smaller and all keys in the right subtree must be larger than the root node
- balanced: for all nodes, there can be no more than a difference of 1 in the height of the left subtree from the right. Math.abs(height(left subtree) – height(right subtree)) ≤ 1

AVL stands for **A**delson-**V**elsky and **L**andis (the inventors of the data structure)

Is this a valid AVL tree?

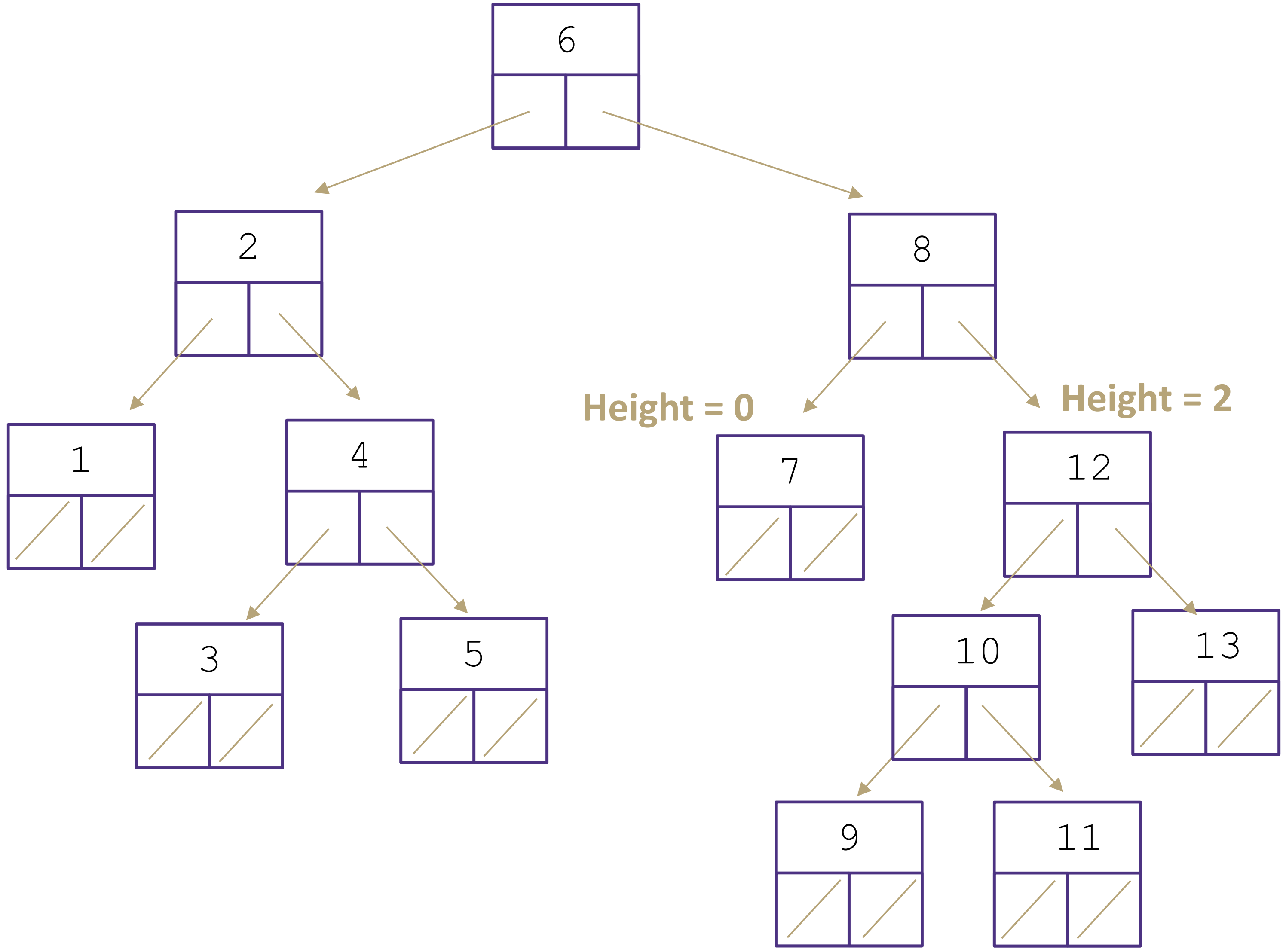
Is it…
- Binary **yes**
- BST **yes**
- Balanced? **yes**



Is this a valid AVL tree?

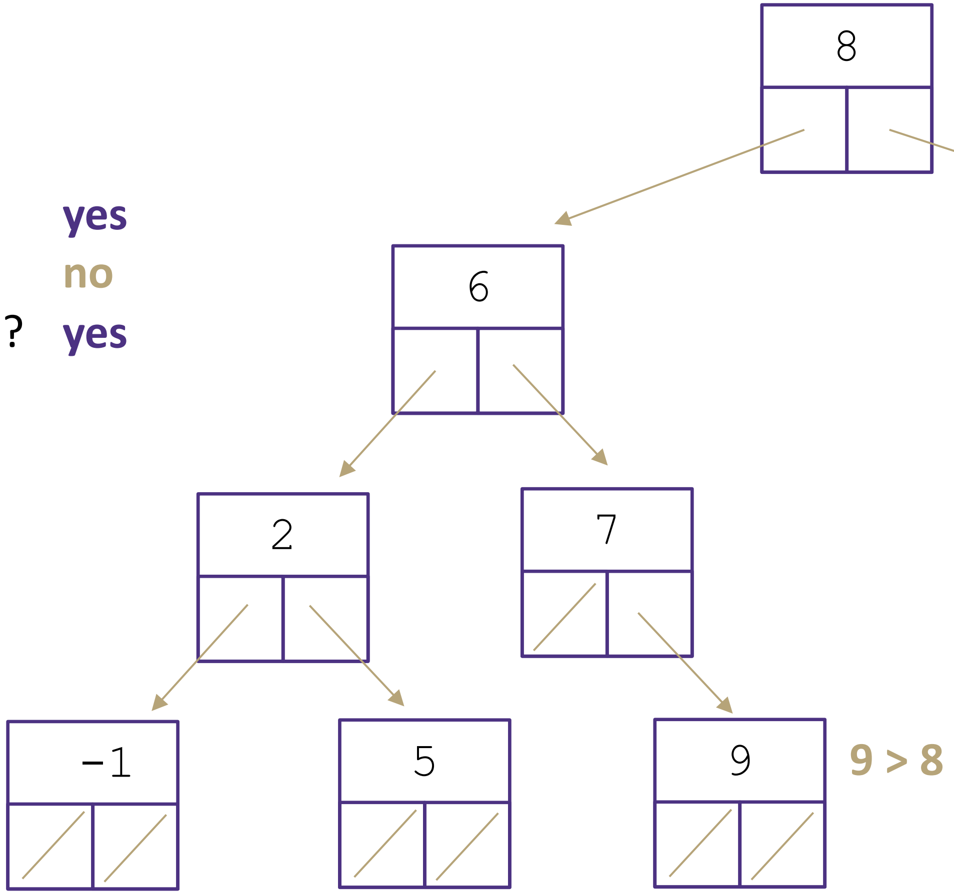
2 Minutes

Is it…
- Binary yes
- BST yes
- Balanced? no



Is this a valid AVL tree?

Is it…
- Binary **yes**
- BST **no**
- Balanced? **yes**



Implementing an AVL tree dictionary

Dictionary Operations:

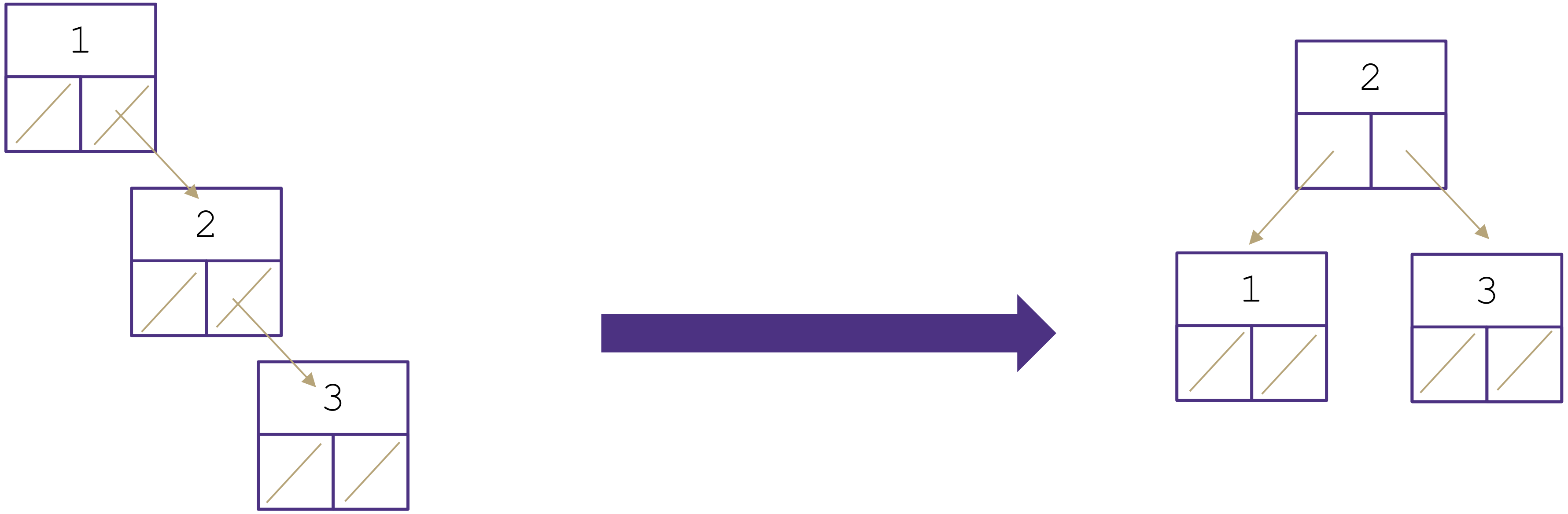
get() – same as BST

containsKey() – same as BST

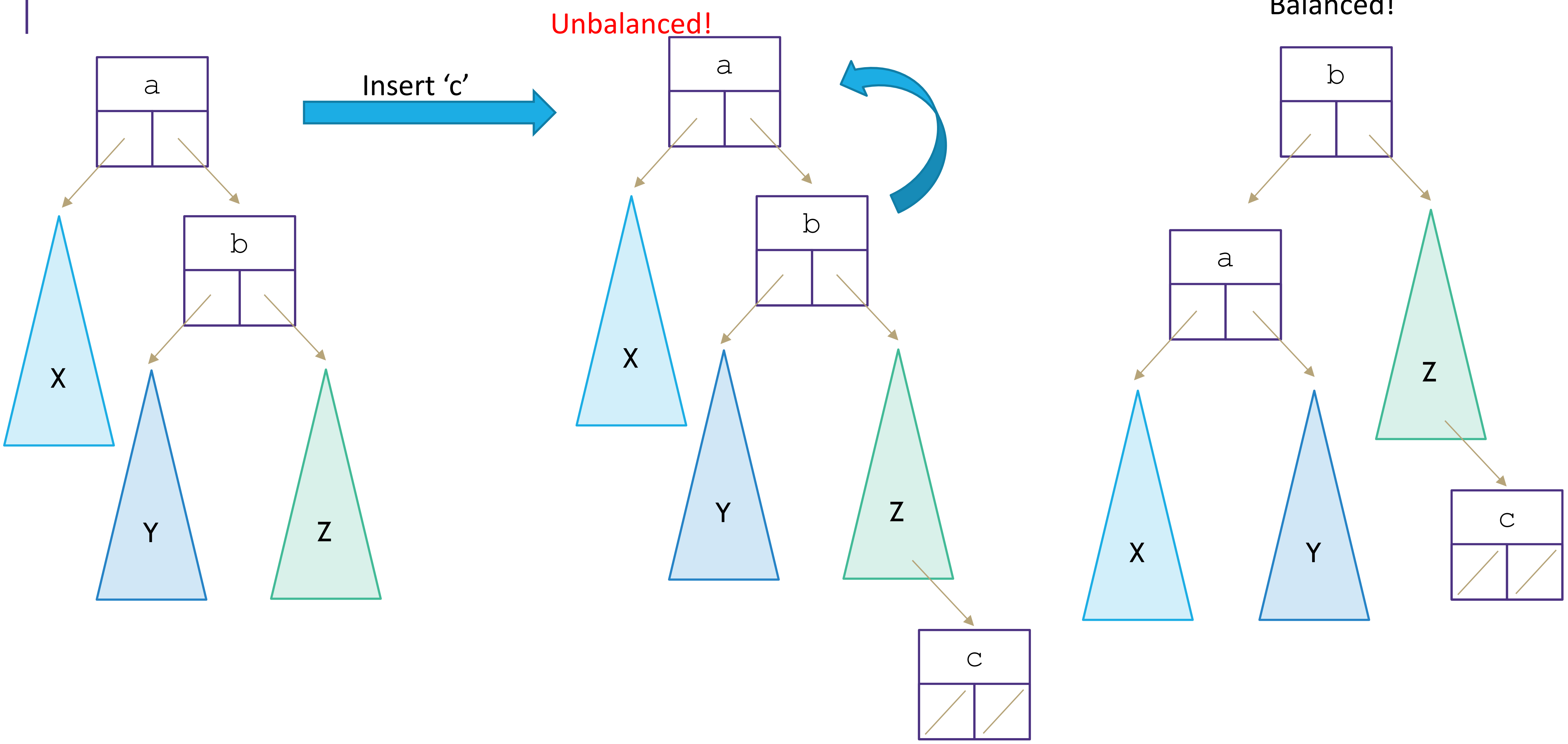
put() - Add the node to keep BST, fix AVL property if necessary

remove() - Replace the node to keep BST, fix AVL property if necessary

Unbalanced!

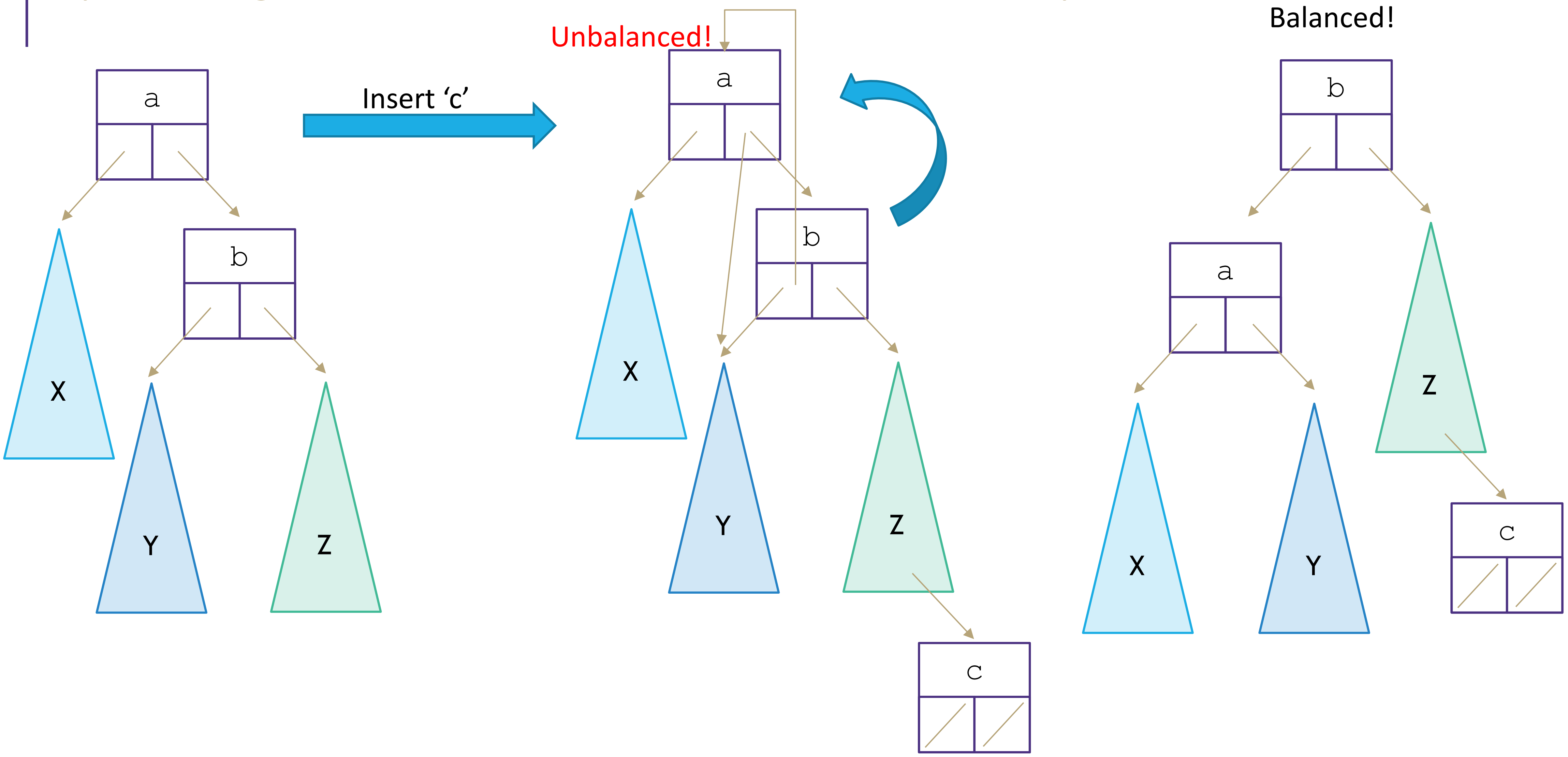


Rotations!



Rotate Left

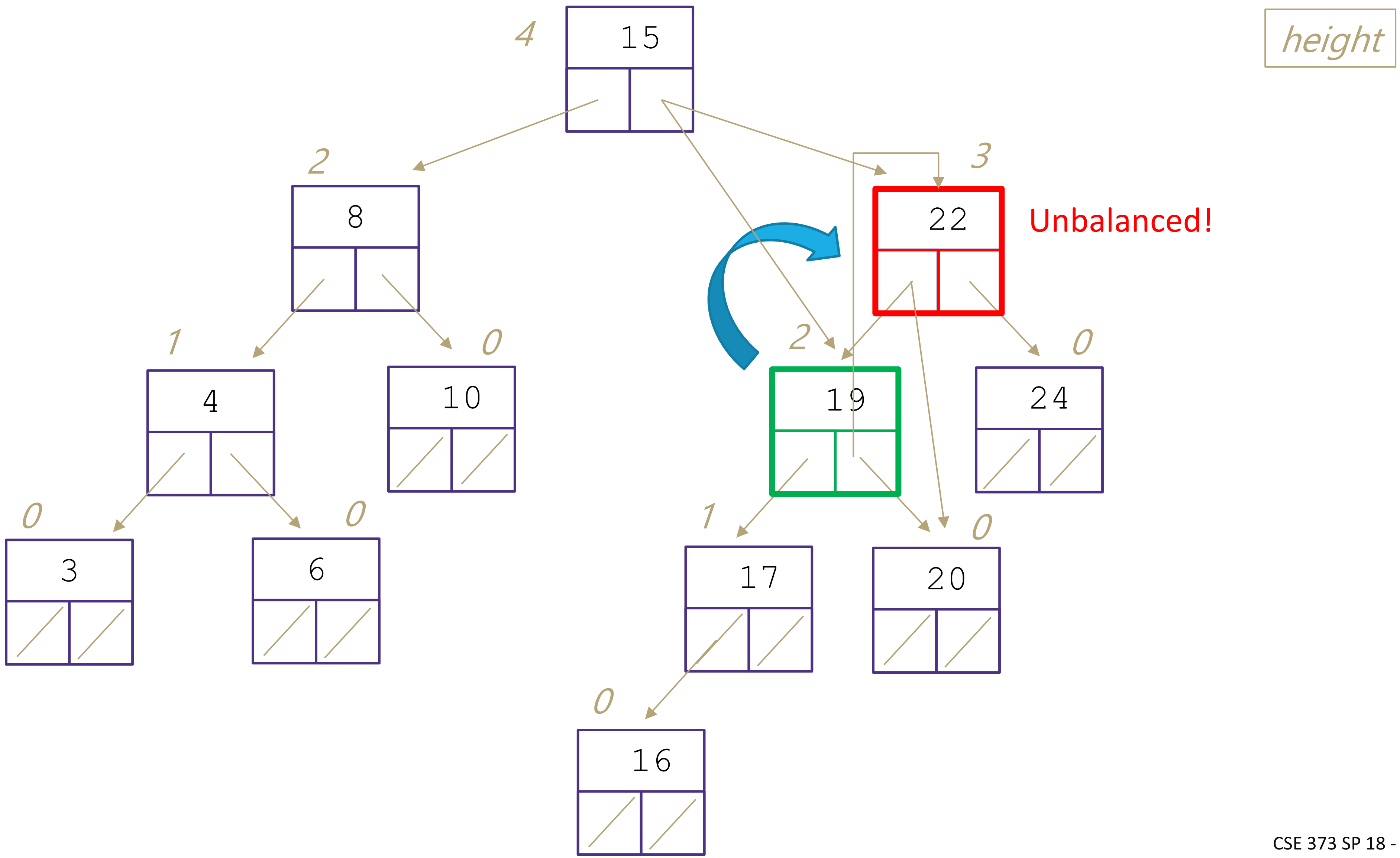
parent's right becomes child's left, child's left becomes its parent



Rotate Right

parent's left becomes child's right, child's right becomes its parent

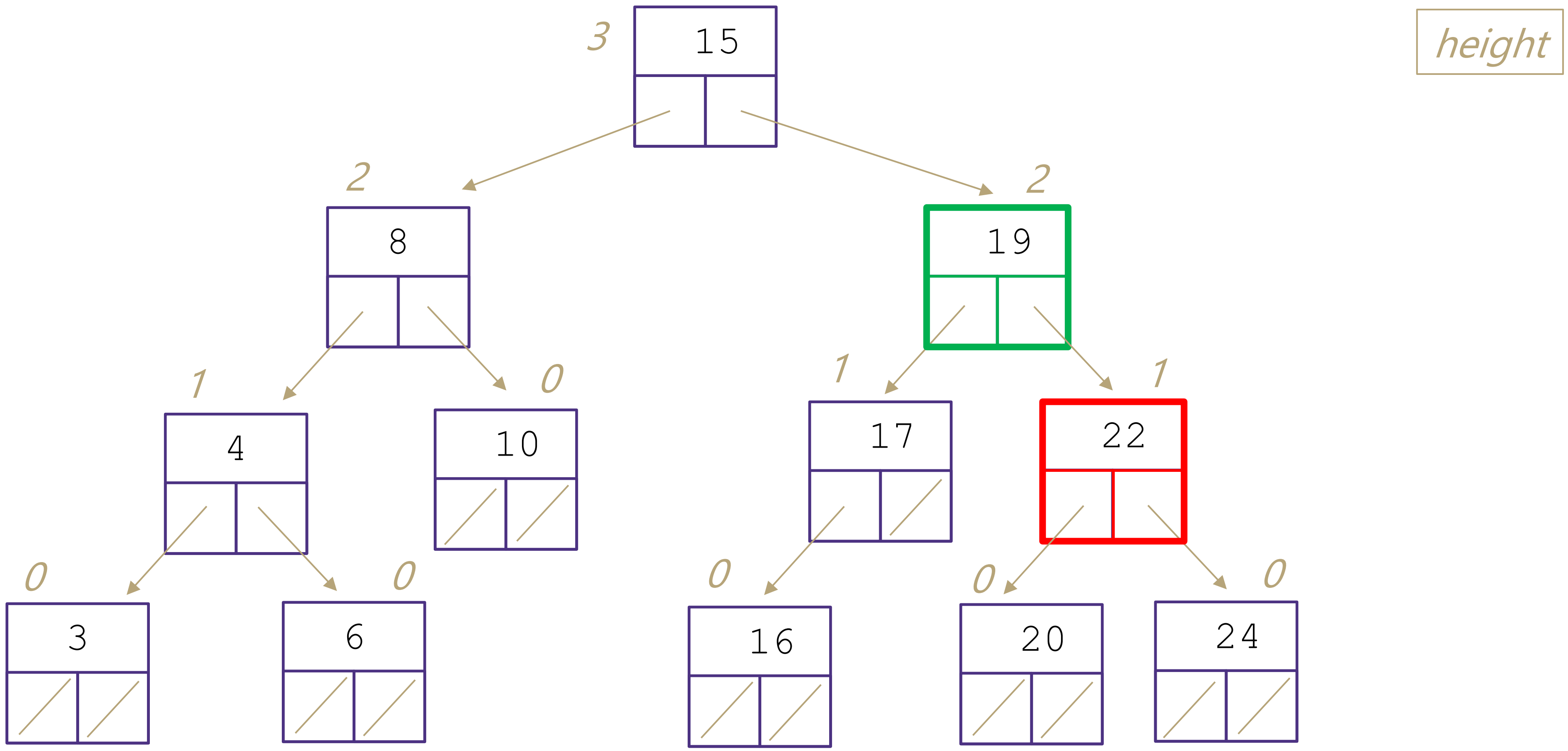
put(16);



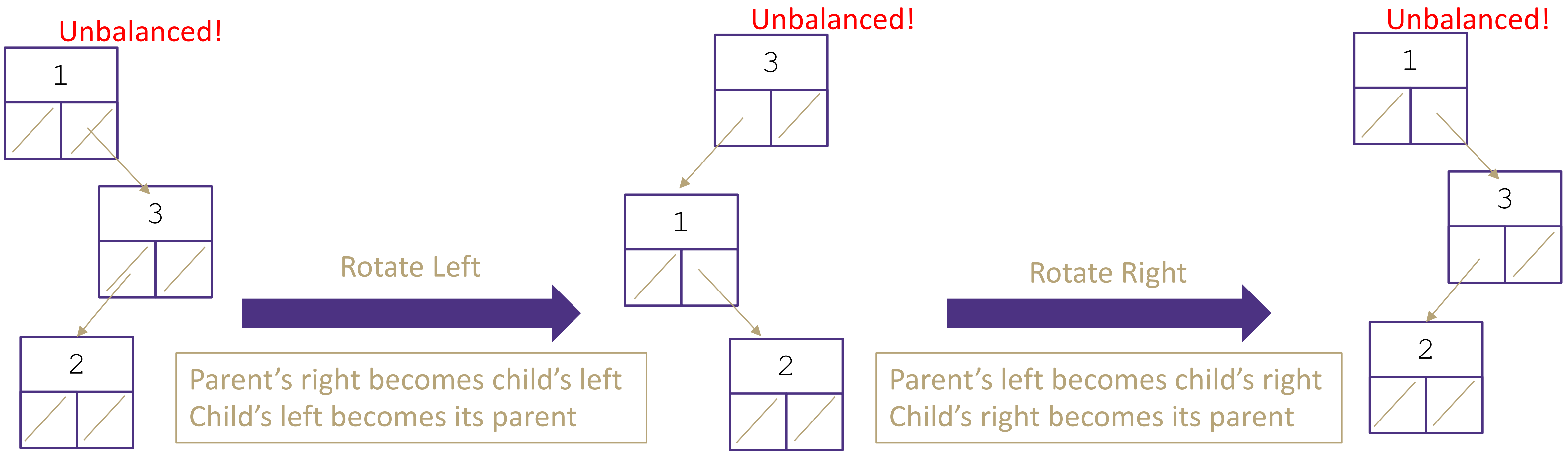
Rotate Right

parent's left becomes child's right, child's right becomes its parent

put(16);

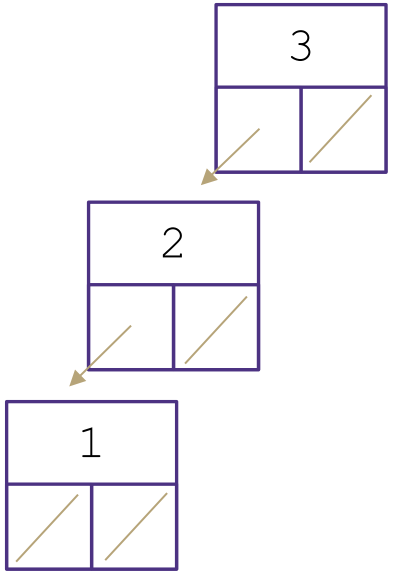


So much can go wrong

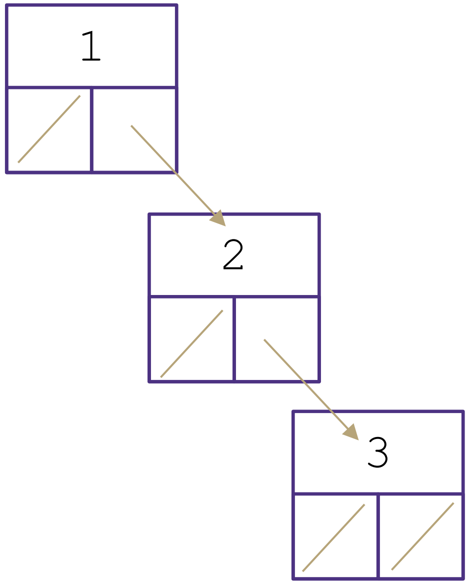


Two AVL Cases

Line Case
Solve with **1** rotation

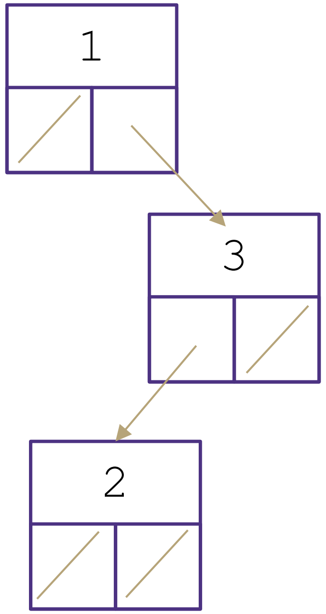


Rotate Right
Parent's left becomes child's right
Child's right becomes its parent

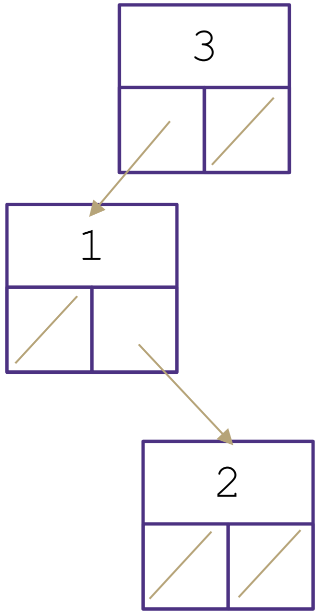


Rotate Left
Parent's right becomes child's left
Child's left becomes its parent

Kink Case
Solve with **2** rotations

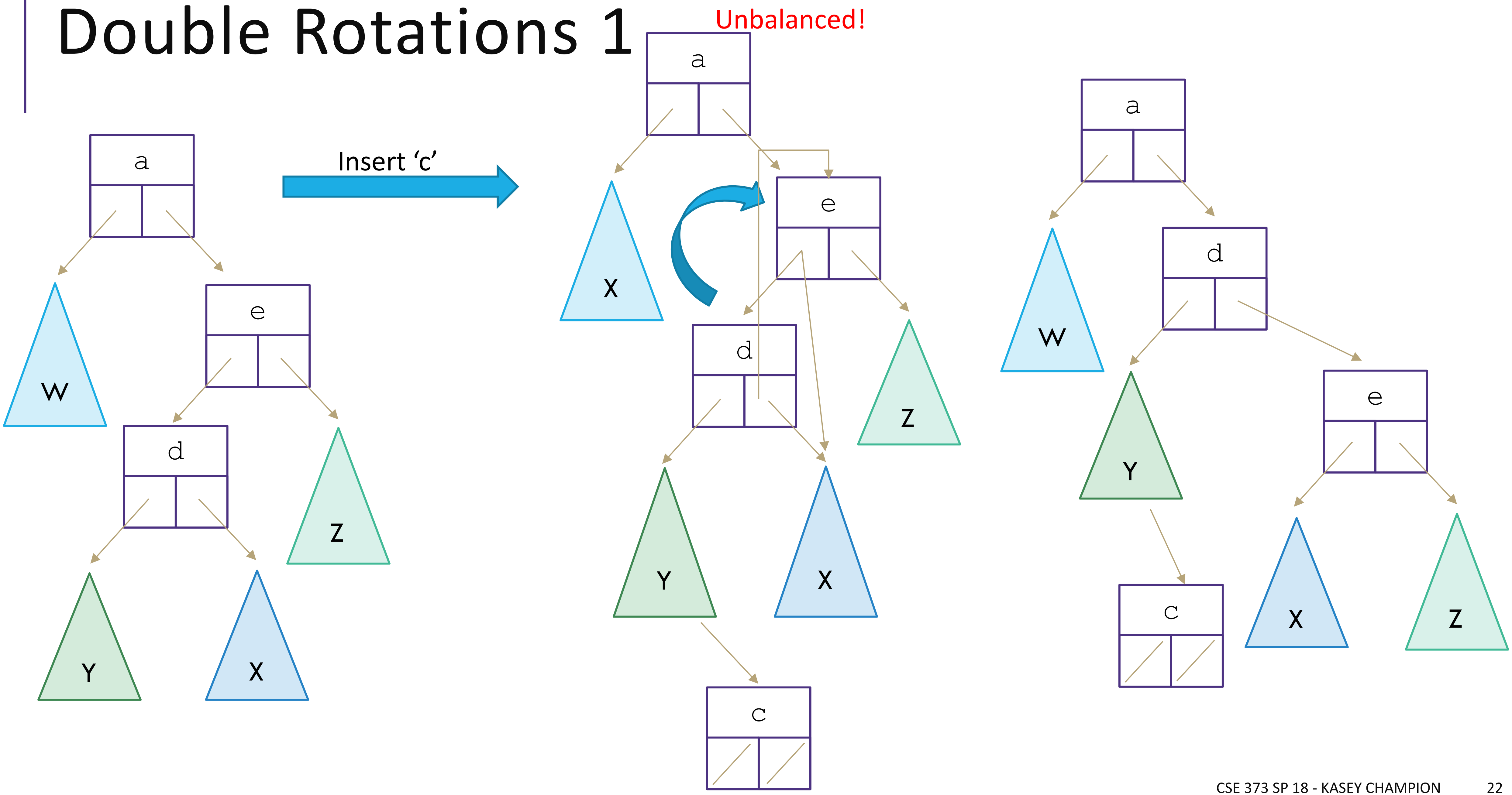


Right Kink Resolution
Rotate subtree left
Rotate root tree right

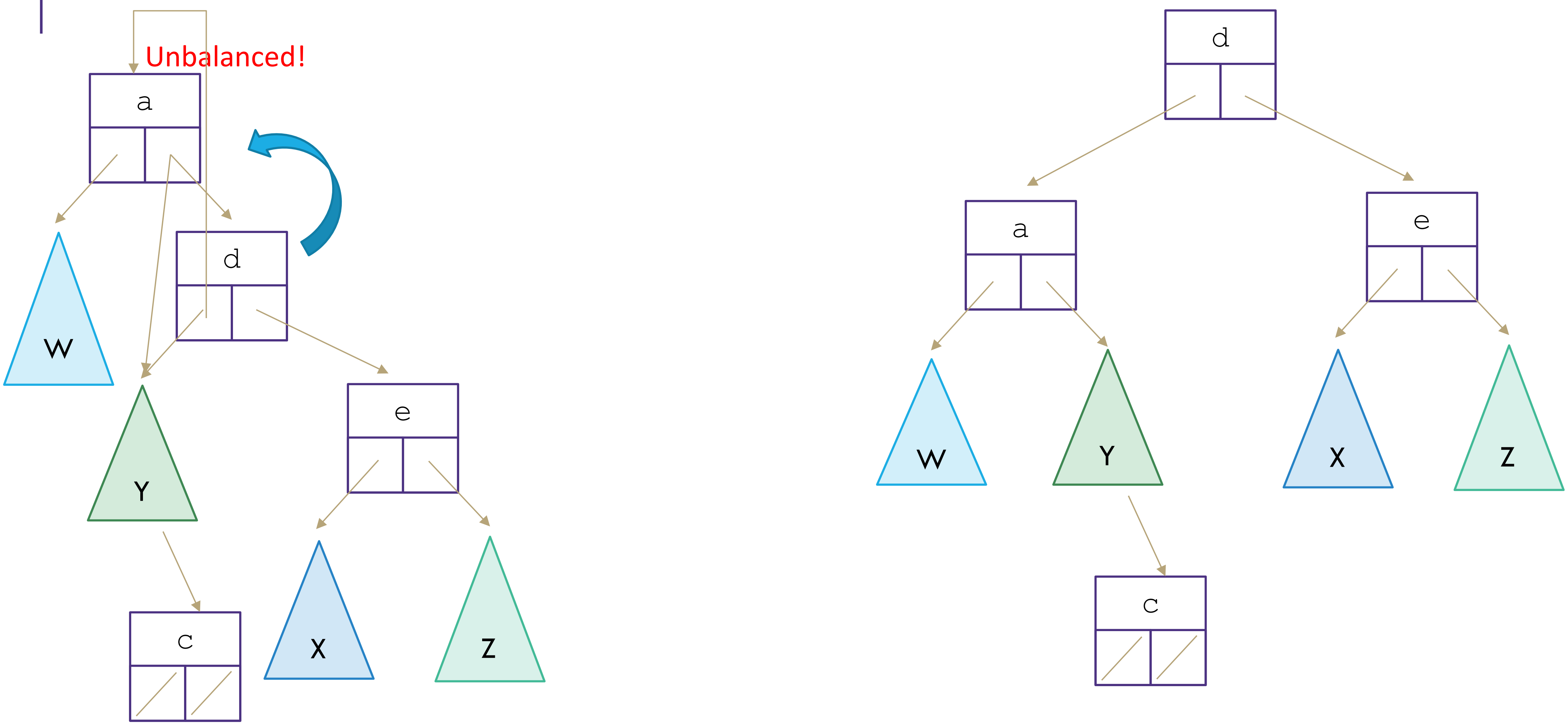


Left Kink Resolution
Rotate subtree right
Rotate root tree left

Double Rotations 1



Double Rotations 2



Implementing Dictionary with AVL

```
public boolean contains(K key, AVLNode node) {
    if (node == null) {                          +C1
        return false;
    }
    int compareResult = compareTo(key, node.data); +C2
    if (compareResult < 0) {                     +T(n/2)
        returns contains(key, node.left);
    } else if (compareResult > 0) {              +T(n/2)
        returns contains(key, node.right);
    } else {                                     +C3
        returns true;
    }
}
```

Worst Case Improvement Guaranteed with AVL

$$\boldsymbol{T(n)} = \begin{cases} \boldsymbol{C \text{ when } n < 0 \text{ or key found}} \\ \boldsymbol{T\left(\frac{n}{2}\right) + C \text{ otherwise}} \end{cases}$$

How long does AVL insert take?

AVL insert time = BST insert time + time it takes to rebalance the tree

= O(log n) + time it takes to rebalance the tree

How long does rebalancing take?

- Assume we store in each node the height of its subtree.
- How long to find an unbalanced node:
  - Just go back up the tree from where we inserted. ← O(log n)
- How many rotations might we have to do?
  - Just a single or double rotation on the lowest unbalanced node. ← O(1)

AVL insert time = O(log n)+ O(log n) + O(1) = O(log n)

AVL wrap up

Pros:

- $O(\log n)$ worst case for find, insert, and delete operations.
- Reliable running times than regular BSTs (because trees are balanced)

Cons:

- Difficult to program & debug [but done once in a library!]
- (Slightly) more space than BSTs to store node heights.

Lots of cool Self-Balancing BSTs out there!

Popular self-balancing BSTs include:

[AVL tree]

[Splay tree]

[2-3 tree]

[AA tree]

[Red-black tree]

[Scapegoat tree]

[Treap]

(Not covered in this class, but several are in the textbook and all of them are online!)

(From https://en.wikipedia.org/wiki/Self-balancing_binary_search_tree#Implementations)