Lecture 9: Intro to Trees

CSE 373: Data Structures and Algorithms

Warm Up

1. Write a recurrence for this piece of code (assume each node has exactly or 2 children)

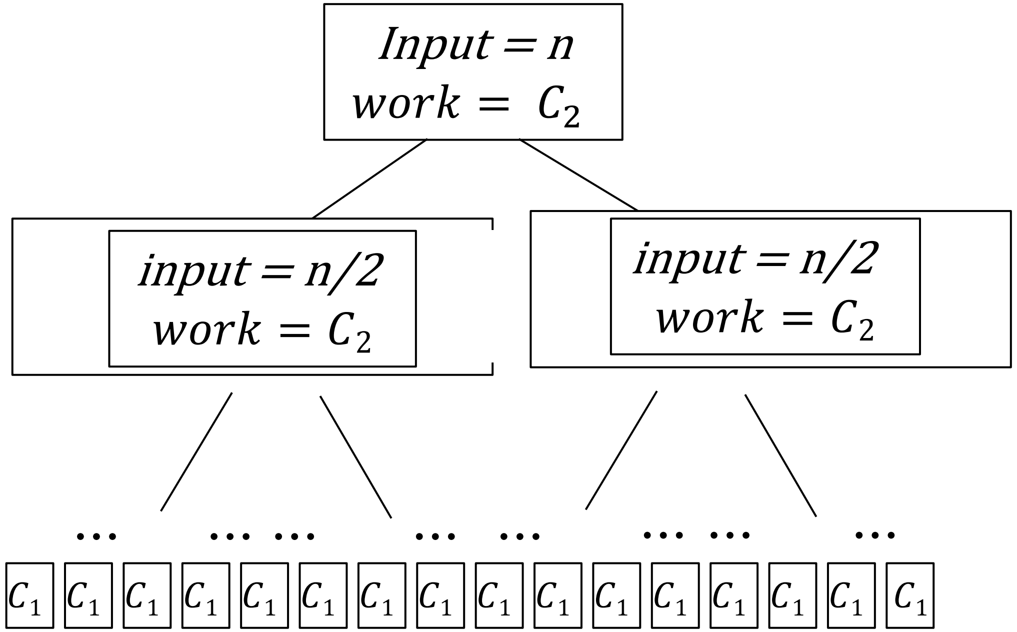
```
private IntTreeNode doublePositivesHelper(IntTreeNode node) {
    if (node != null) {
        if (node.data > 0) {
            node.data *= 2;                               + C
        }
        node.left = doublePositivesHelper(node.left);     T(n/2)
        node.right = doublePositivesHelper(node.right);   T(n/2)
        return node; + C
    } else {
        return null; + C
    }
}
```

$$\boxed{T(n) = \begin{cases} C_1 \; when \; n < 1 \\ C_2 + \; 2T\left(\frac{n}{2}\right) otherwise \end{cases}}$$

Extra Credit:
Go to PollEv.com/champk
Text CHAMPK to 22333 to join session, text "1" or "2" to select your answer

Warm Up Continued

$$T(n) = \begin{cases} C_1 \text{ when } n < 1 \\ C_2 + 2T\left(\frac{n}{2}\right) \text{ otherwise} \end{cases}$$



Level (i)	Number of Nodes	Input to Node	Work per Node
0			
1			
2			
3			
base			

Warm Up Continued

$$T(n) = \begin{cases} C_1 \text{ when } n < 1 \\ C_2 + 2T\left(\frac{n}{2}\right) \text{ otherwise} \end{cases}$$

1. How many nodes on each branch level?

   2^i

2. How much work for each branch node?

   c_2

3. How much work per branch level?

   $2^i C_2$

4. How many branch levels?

   $\frac{n}{2^i} < 1 \Rightarrow \mathrm{i} = \log_2 n$

5. How much work for each leaf node?

   C_1

6. How many leaf nodes?

   $2^{\log_2 n + 1} = 2n$

Level (i)	Number of Nodes	Input to Node	Work per Node
0	2^0=1	$\frac{n}{2^0} = n$	c_2
1	2^1=2	$\frac{n}{2^1} = \frac{n}{2}$	c_2
2	2^2=4	$\frac{n}{2^2} = \frac{n}{4}$	c_2
3	2^3=8	$\frac{n}{2^3} = \frac{n}{8}$	c_2
base	$\Rightarrow 2^{\log_2 n}$+1	0	C_1

Combining it all together…

$$T(n) = \sum_{i=0}^{\log_2 n} 2^i C_2 + 2nC_1$$

Trees

Storing Sorted Items in an Array

get() – O(logn)

put() – O(n)

remove() – O(n)

Can we do better with insertions and removals?

Review: Trees!

A **tree** is a collection of nodes
- Each node has at most 1 parent and 0 or more children

Root node: the single node with no parent, “top” of the tree

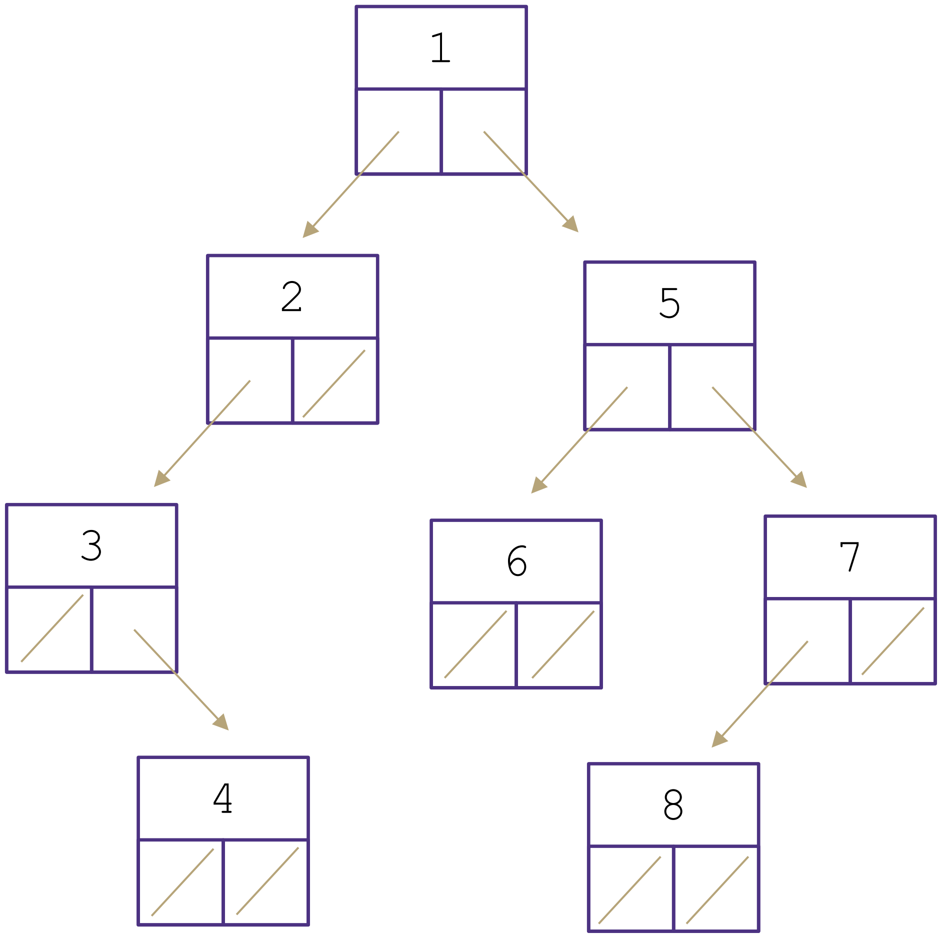
Branch node: a node with one or more children

Leaf node: a node with no children

Edge: a pointer from one node to another

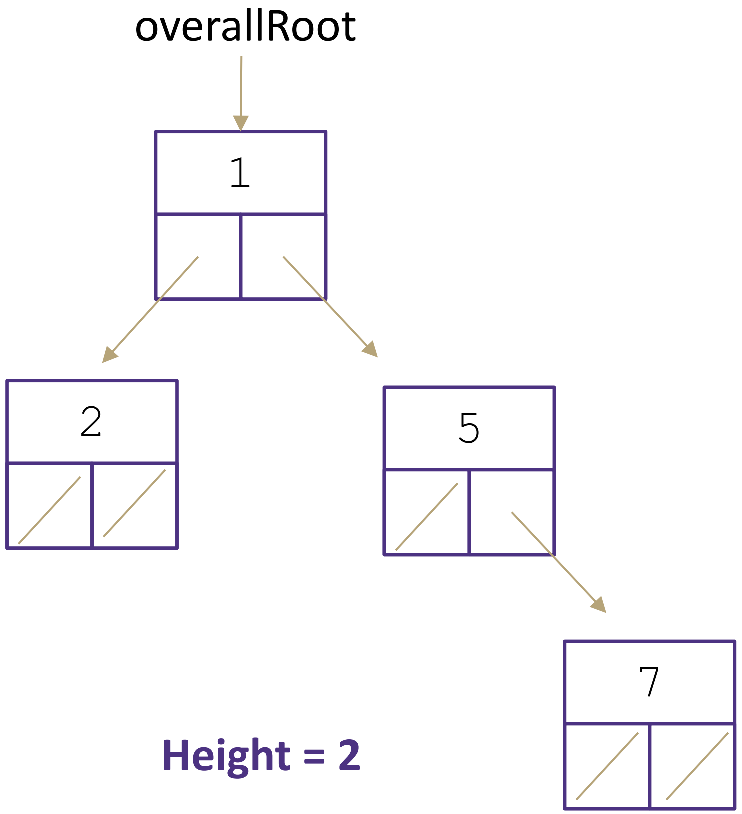
Subtree: a node and all it descendants

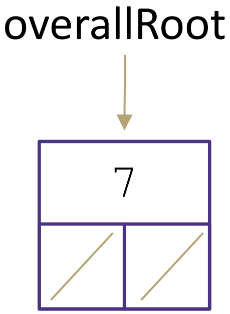
Height: the number of edges contained in the longest path from root node to some leaf node

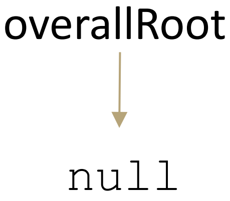


Tree Height

What is the height of the following trees?







Height = 2

Height = 0

Height = -1 or NA

Traversals

traversal: An examination of the elements of a tree.

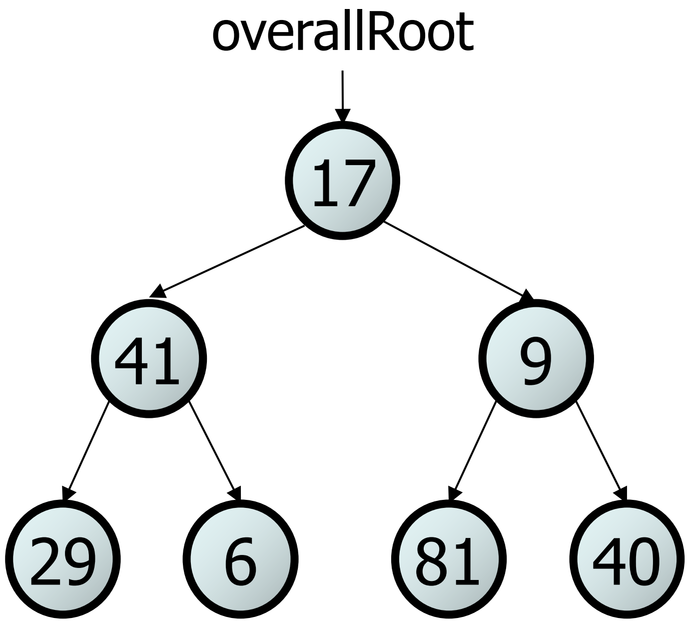
- A pattern used in many tree algorithms and methods

Common orderings for traversals:

- **pre-order**: process root node, then its left/right subtrees
  - `17 41 29 6 9 81 40`
- **in-order**: process left subtree, then root node, then right
  - `29 41 6 17 81 9 40`
- **post-order**: process left/right subtrees, then root node
  - `29 6 41 81 40 9 17`

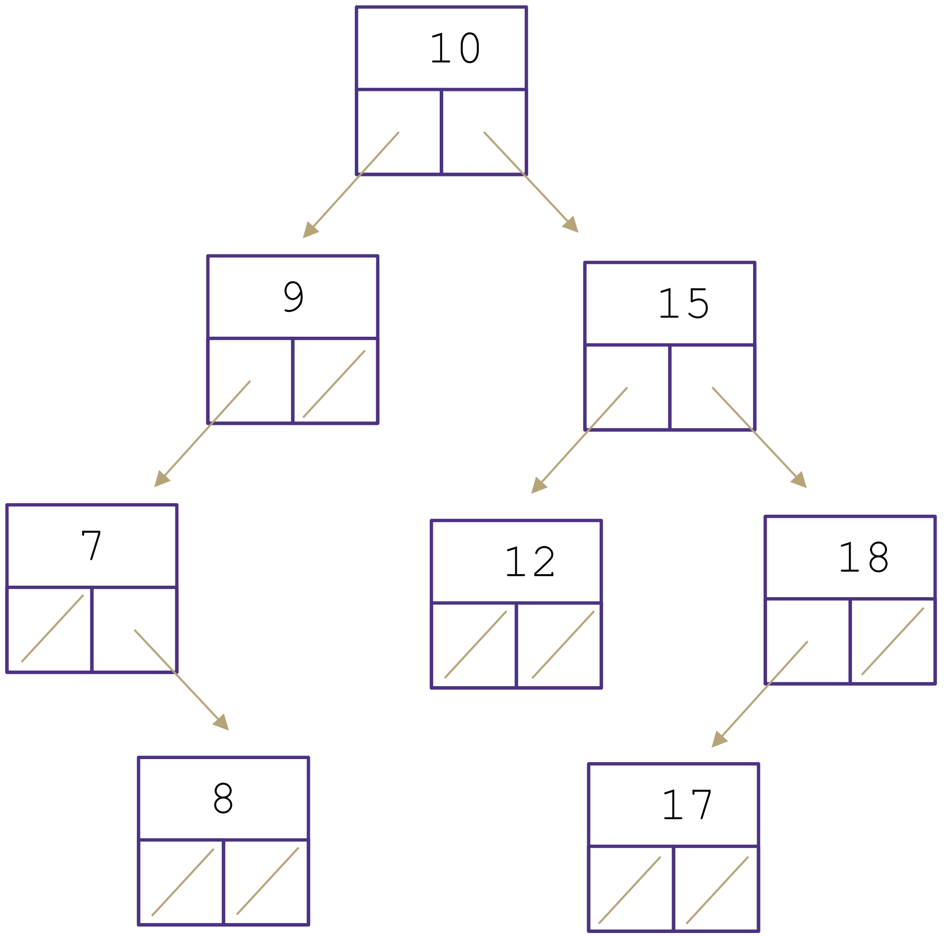
Traversal Trick: Sailboat method

- Trace a path around the tree.
- As you pass a node on the
  proper side, process it.
  - pre-order: left side
  - in-order: bottom
  - post-order: right side



Binary Search Trees

A **binary search tree** is a binary tree that contains comparable items such that for every node, all children to the left contain smaller data and all children to the right contain larger data.



Implement Dictionary

Binary Search Trees allow us to:

- quickly find what we're looking for
- add and remove values easily

Dictionary Operations:

Runtime in terms of height, "h"

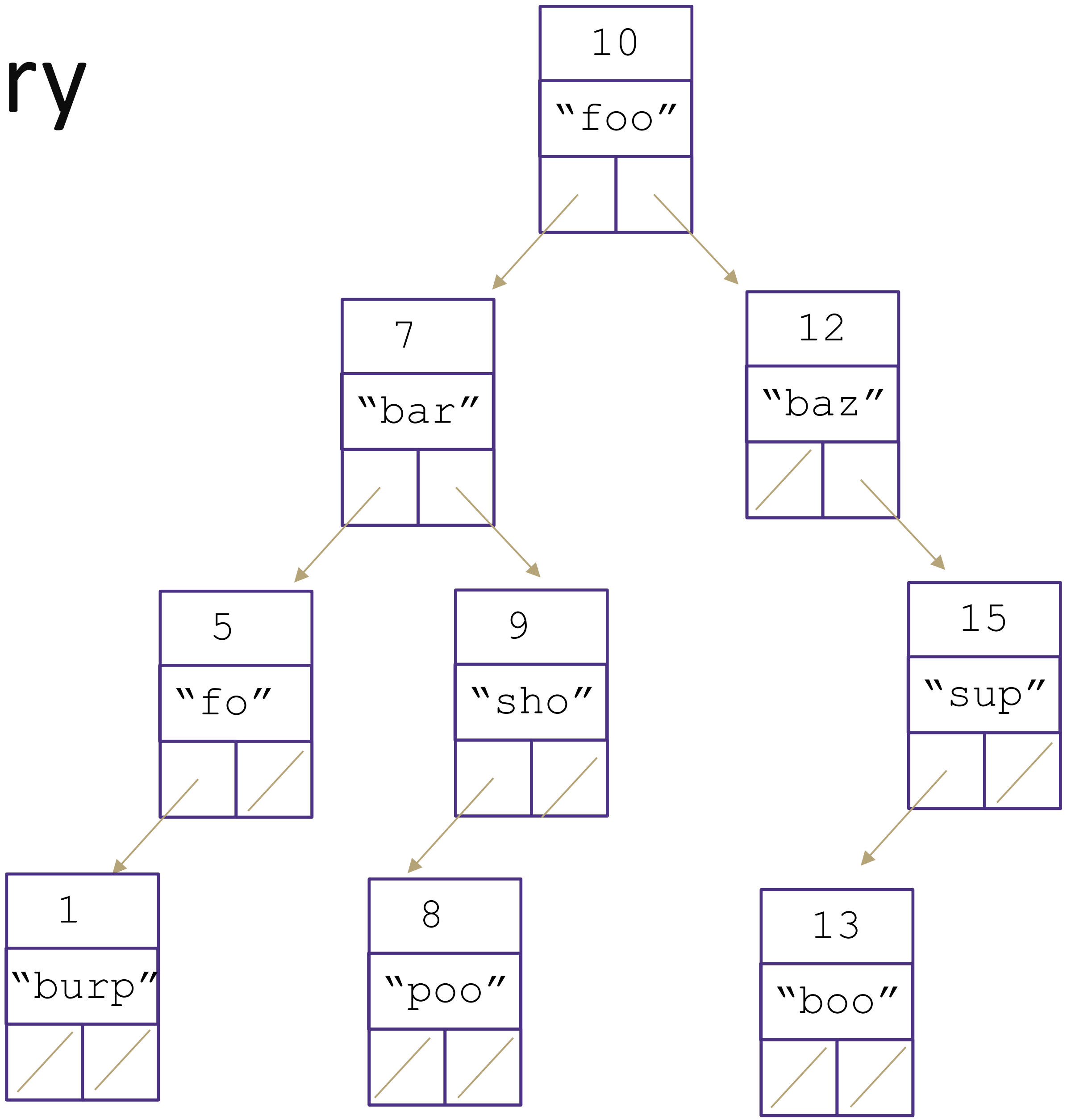
get() – O(h)

put() – O(h)

remove() – O(h)

What do you replace the node with?

Largest in left sub tree or smallest in right sub tree



Height in terms of Nodes

For “balanced” trees $h \approx \log_c(n)$ where c is the maximum number of children

Balanced binary trees $h \approx \log_2(n)$

Balanced trinary tree $h \approx \log_3(n)$

Thus for balanced trees operations take $\Theta(\log_c(n))$

Unbalanced Trees

Is this a valid Binary Search Tree?

Yes, but…

We call this a **degenerate tree**

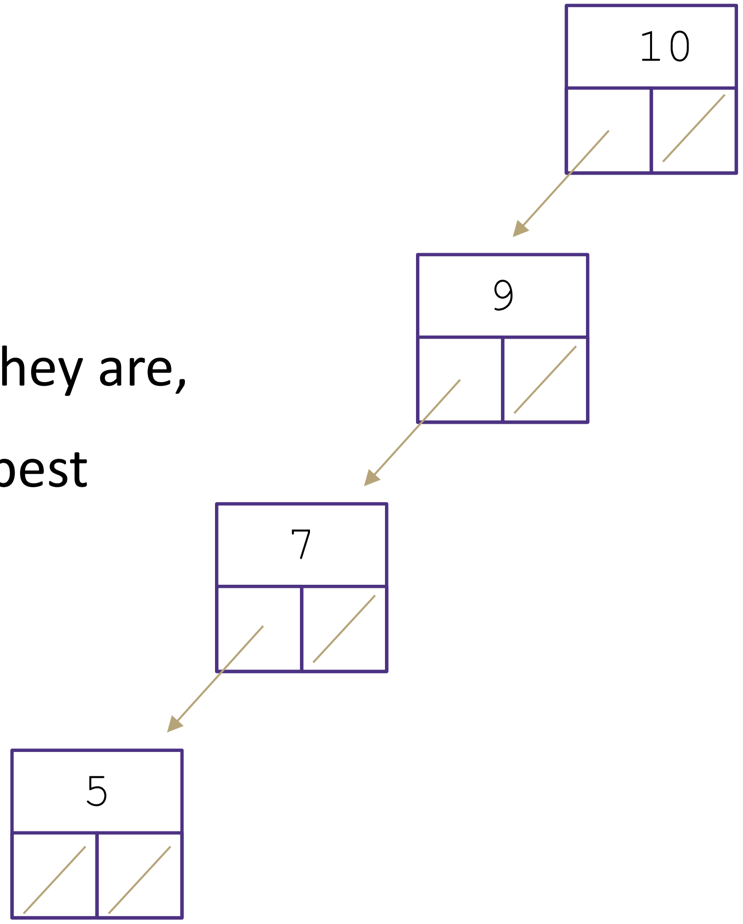
For trees, depending on how balanced they are,

Operations at worst can be O(n) and at best

can be O(logn)

How are degenerate trees formed?

- insert(10)
- insert(9)
- insert(7)
- insert(5)



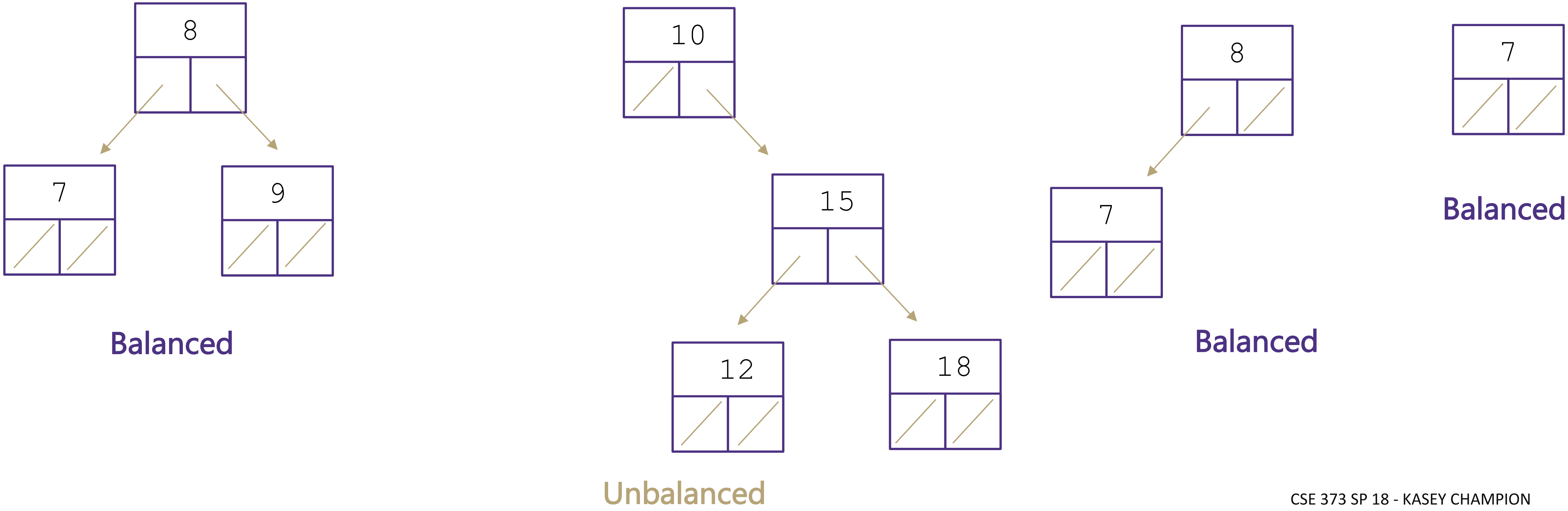
2 Minutes

Measuring Balance

Measuring balance:

For each node, compare the heights of its two sub trees

Balanced when the difference in height between sub trees is no greater than 1



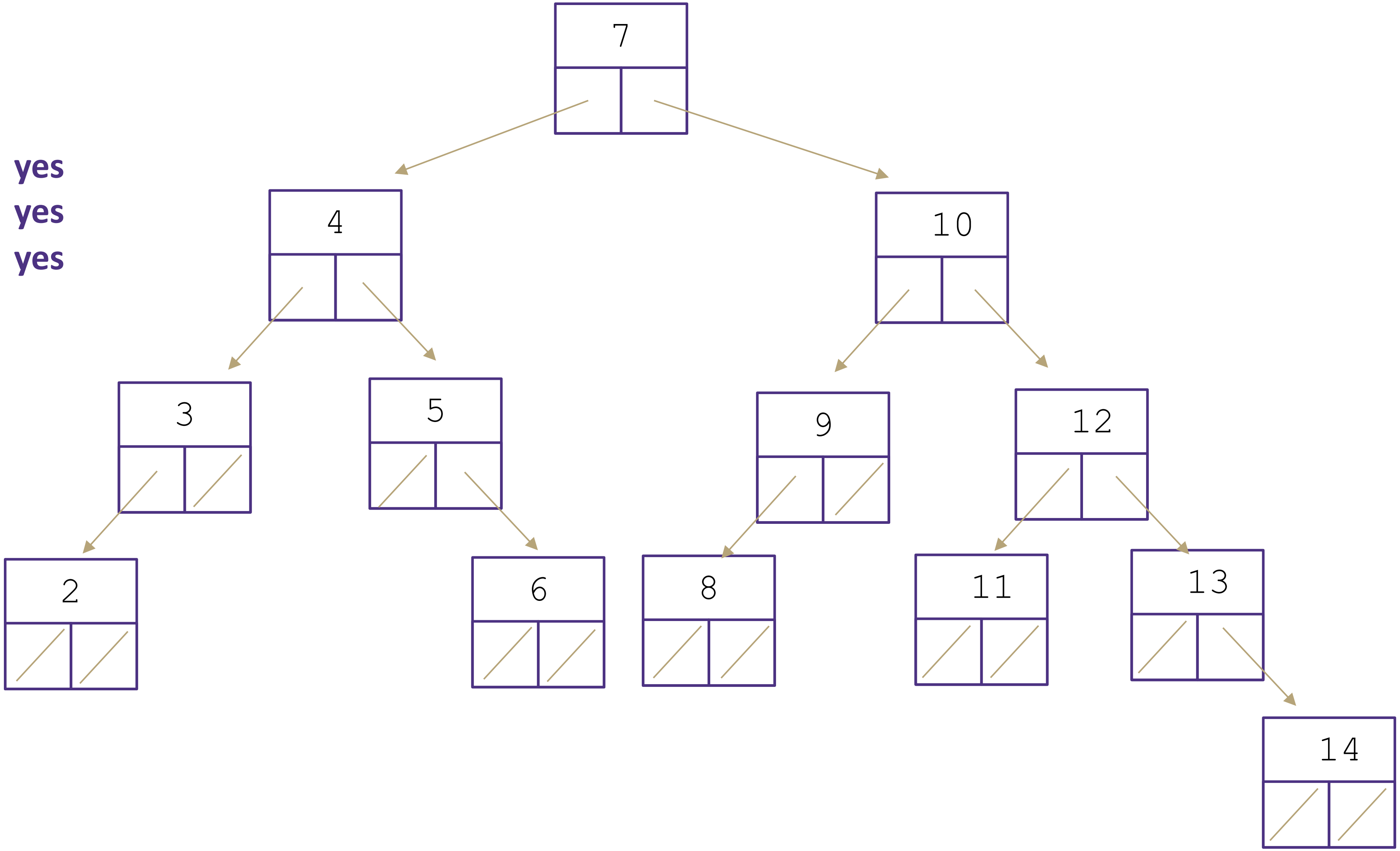
Meet AVL Trees

AVL Trees must satisfy the following properties:

- binary trees: all nodes must have between 0 and 2 children
- binary search tree: for all nodes, all keys in the left subtree must be smaller and all keys in the right subtree must be larger than the root node
- balanced: for all nodes, there can be no more than a difference of 1 in the height of the left subtree from the right. Math.abs(height(left subtree) – height(right subtree)) ≤ 1

AVL stands for **A**delson-**V**elsky and **L**andis (the inventors of the data structure)

Is this a valid AVL tree?



Is it…
- Binary **yes**
- BST **yes**
- Balanced? **yes**

Is this a valid AVL tree?

Is it…
- Binary **yes**
- BST **yes**
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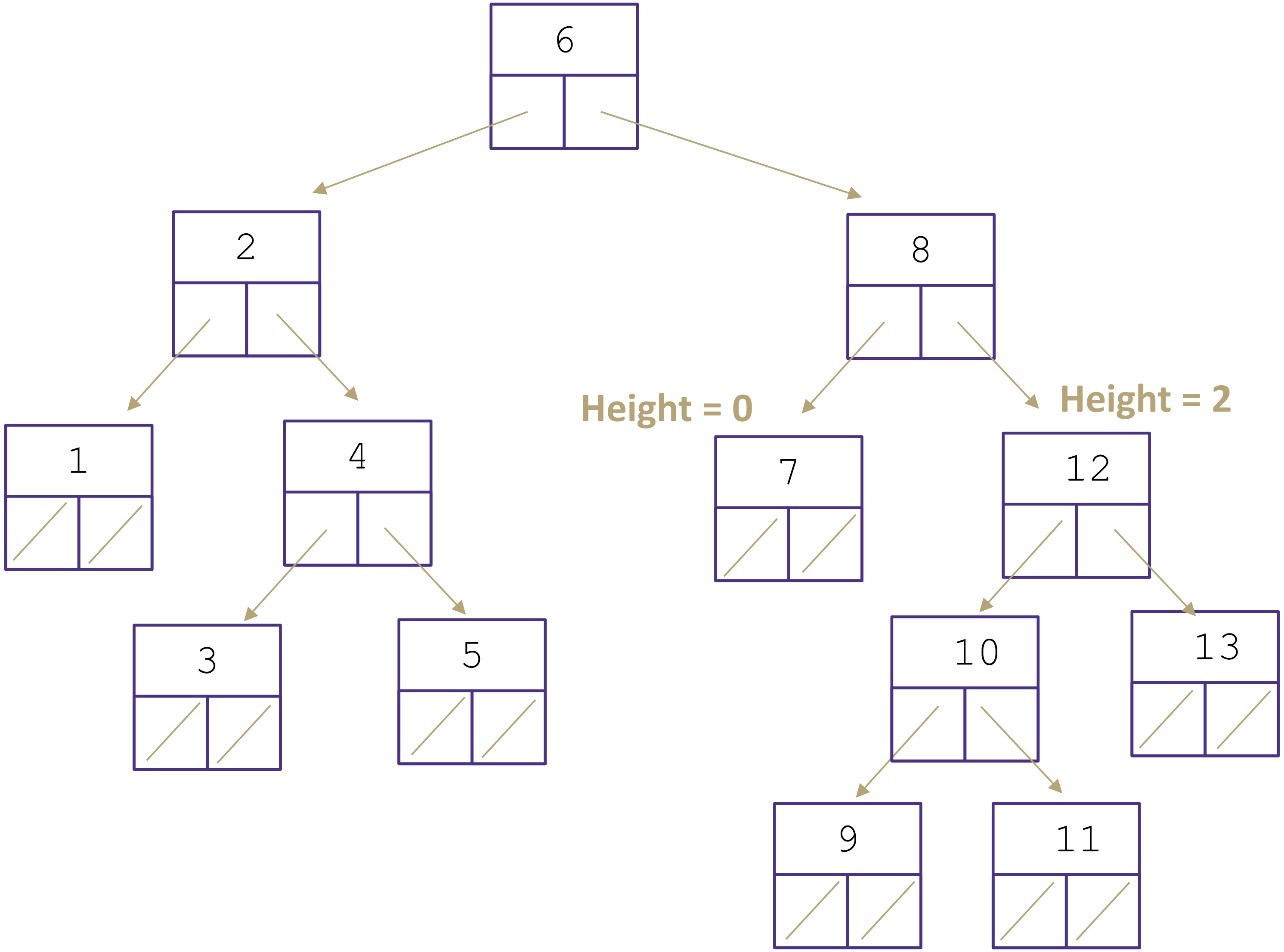

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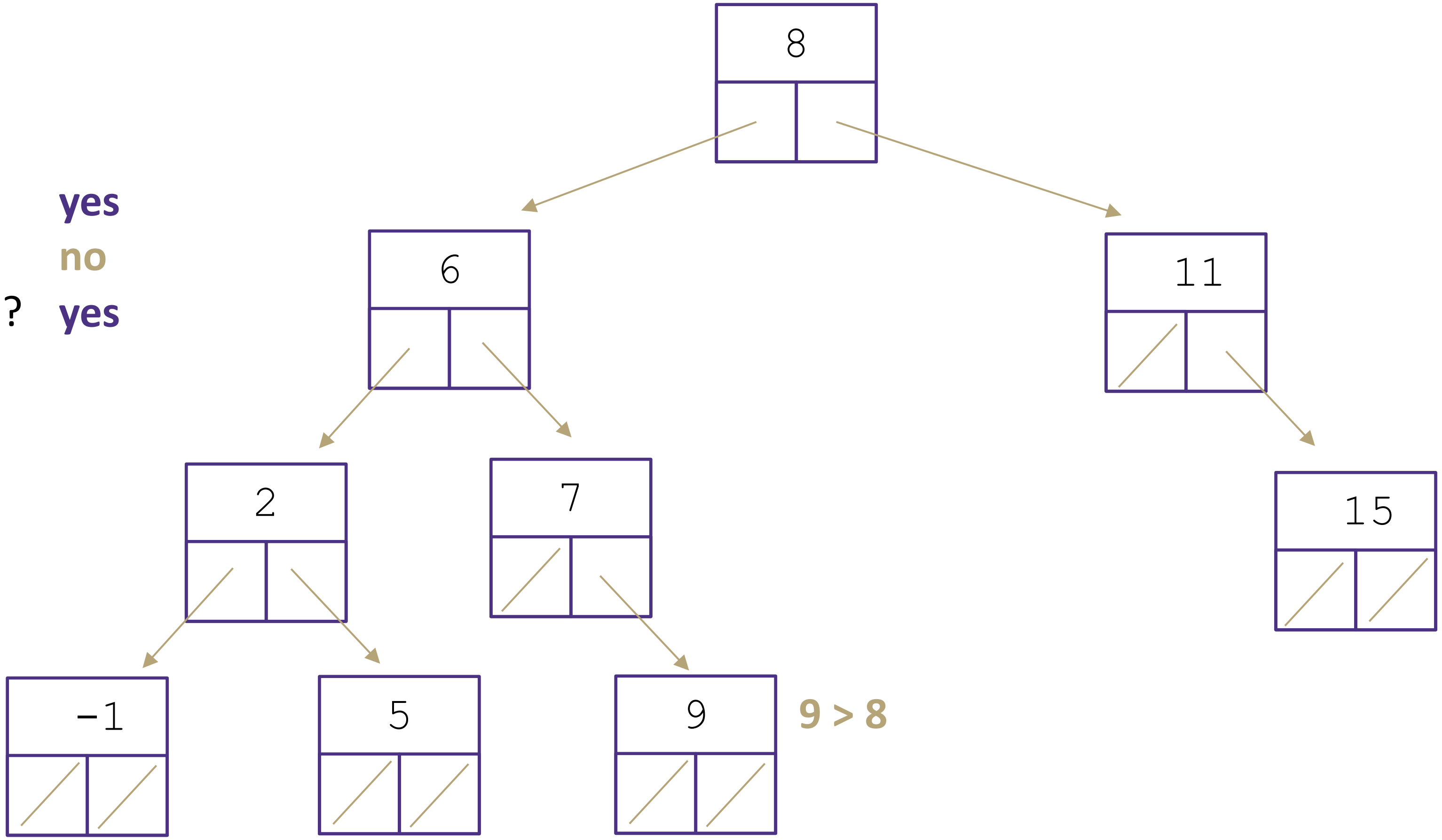


9 > 8

Implementing an AVL tree dictionary

Dictionary Operations:

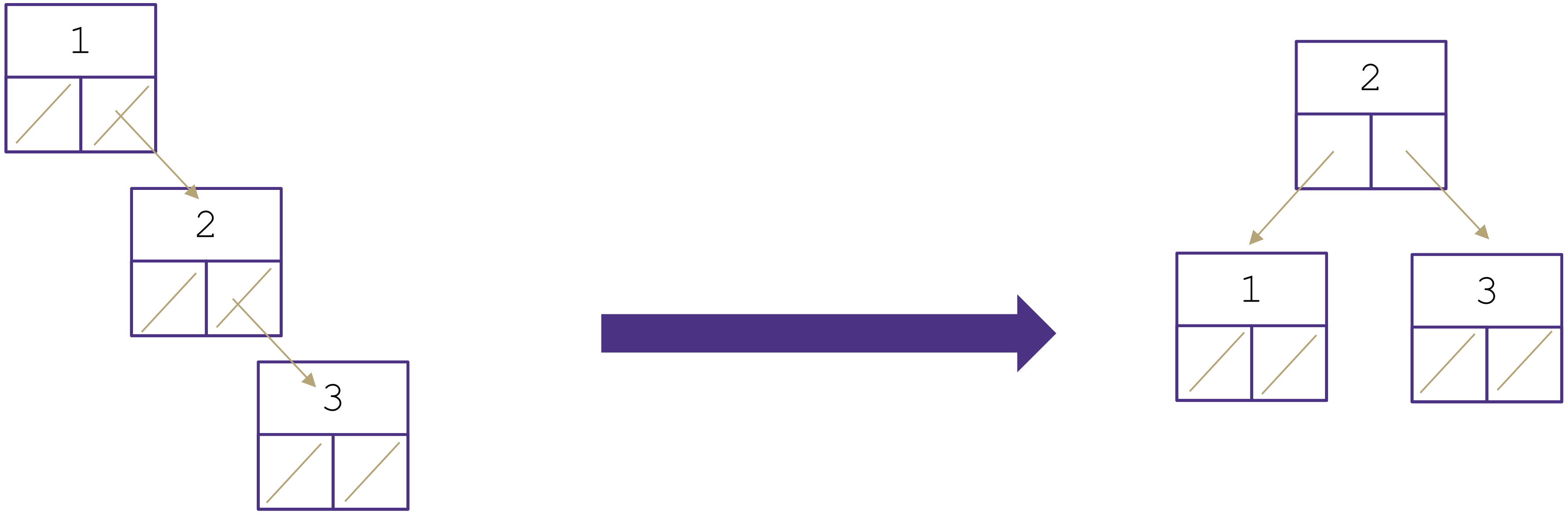
get() – same as BST

containsKey() – same as BST

put() - Add the node to keep BST, fix AVL property if necessary

remove() - Replace the node to keep BST, fix AVL property if necessary

Unbalanced!



Warm Up

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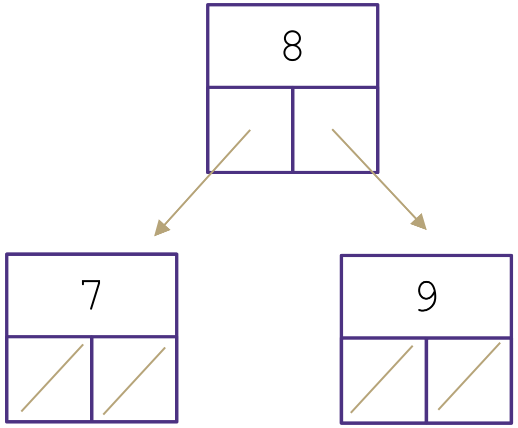
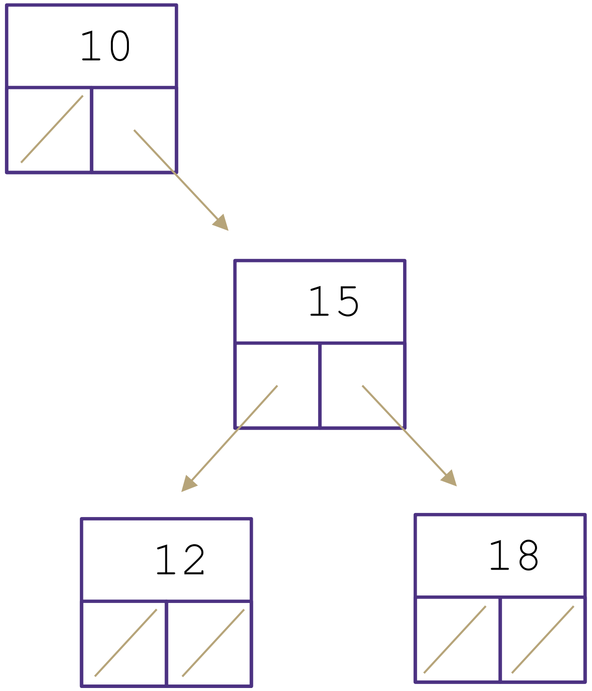
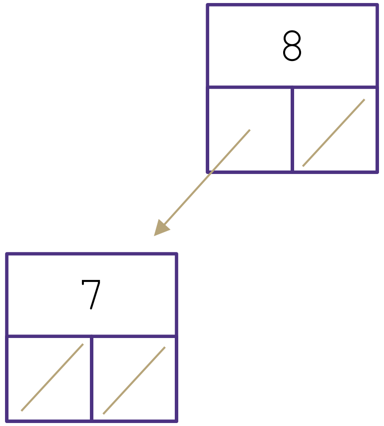


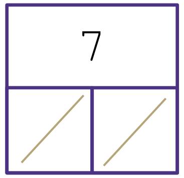
Balanced



Unbalanced

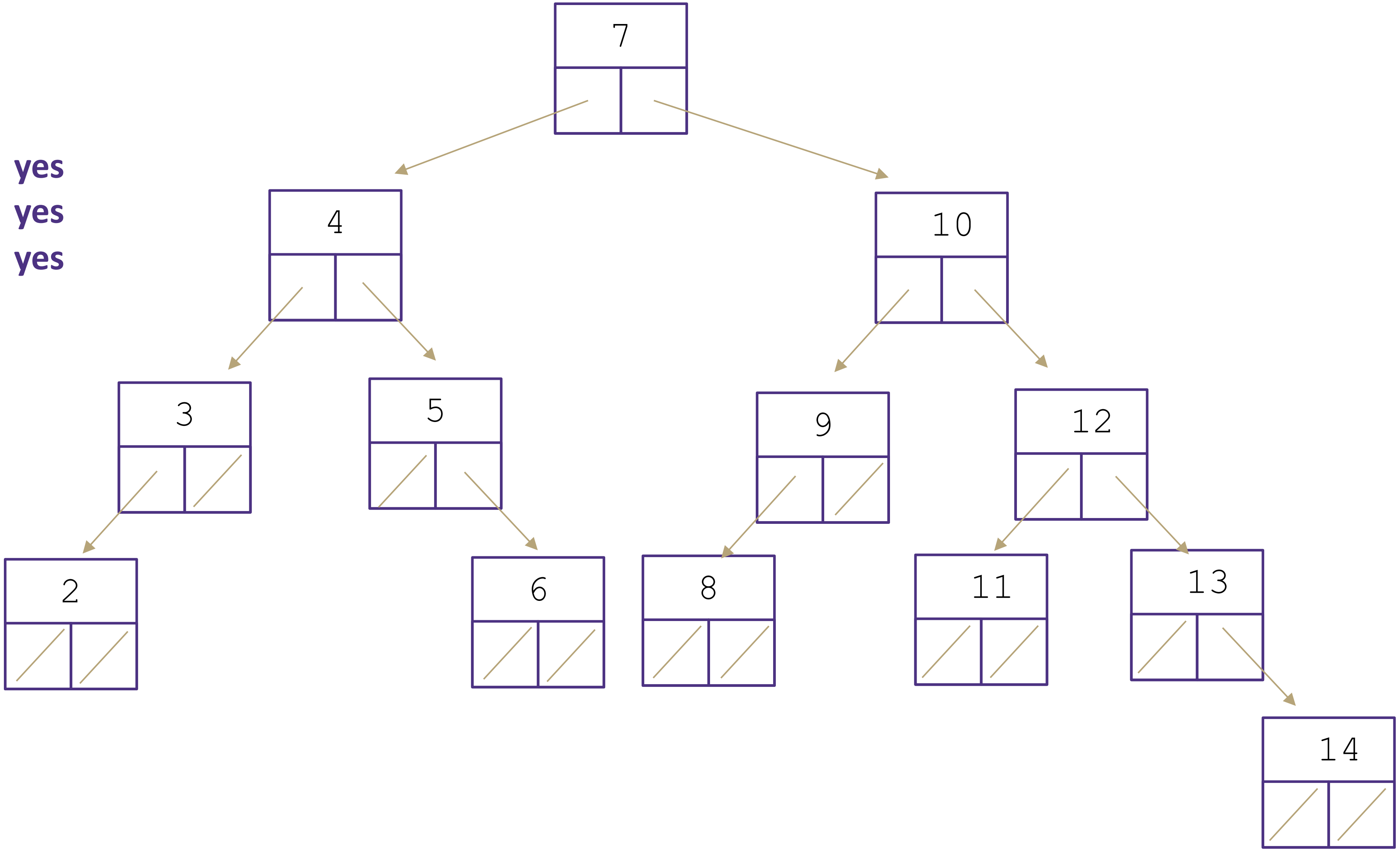


Balanced



Balanced

Is this a valid AVL tree?



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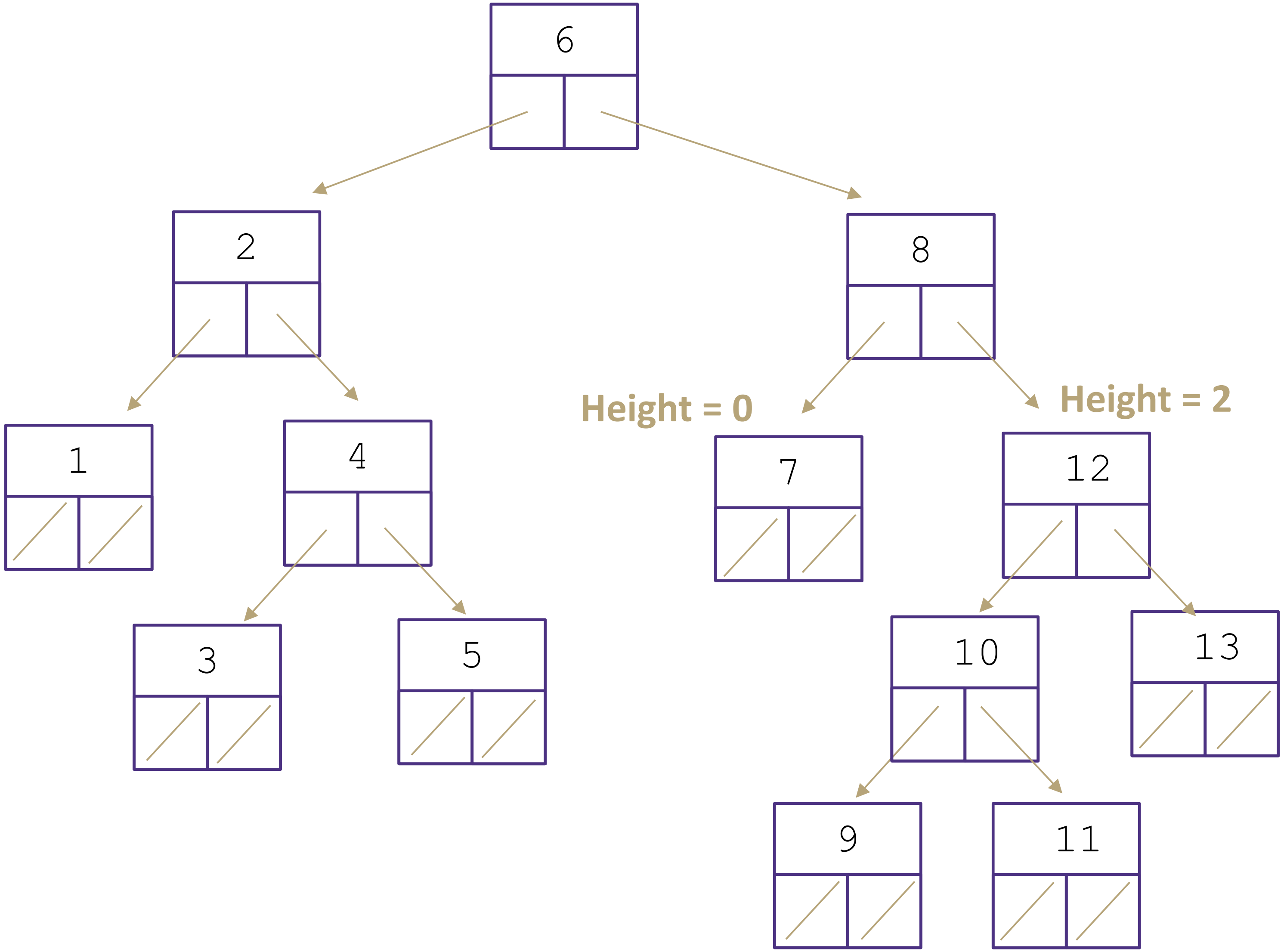

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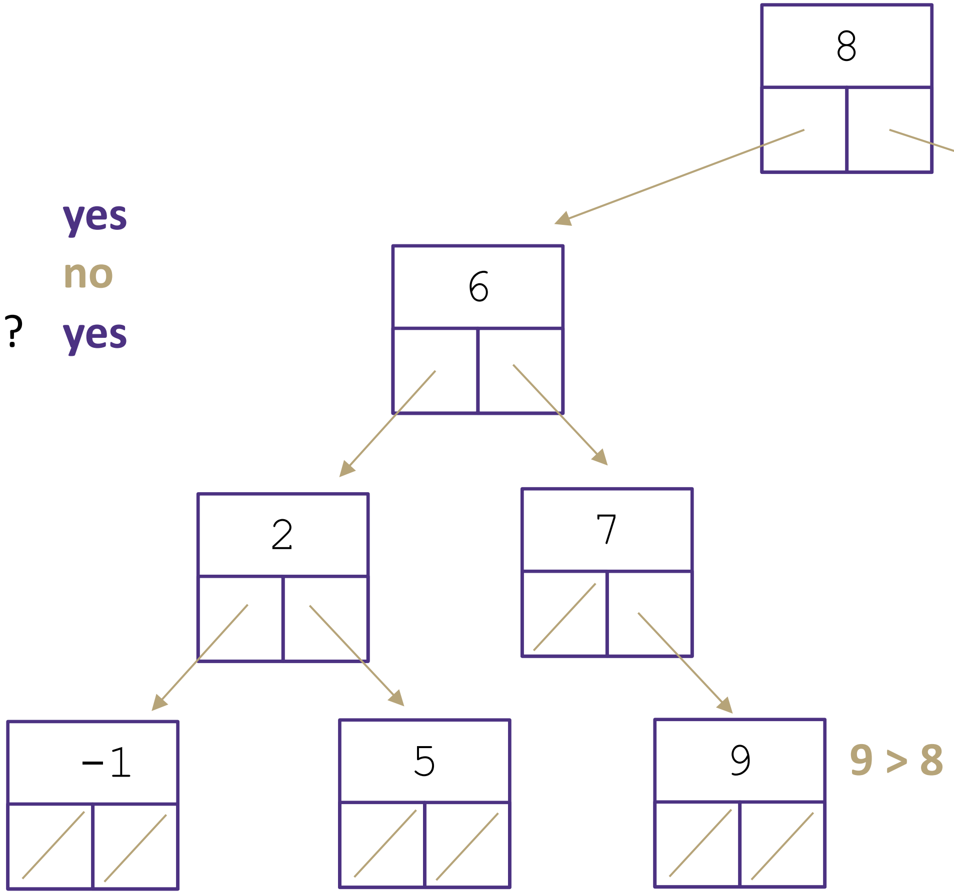

Is it…

- Binary **yes**
- BST **no**
- Balanced? **yes**

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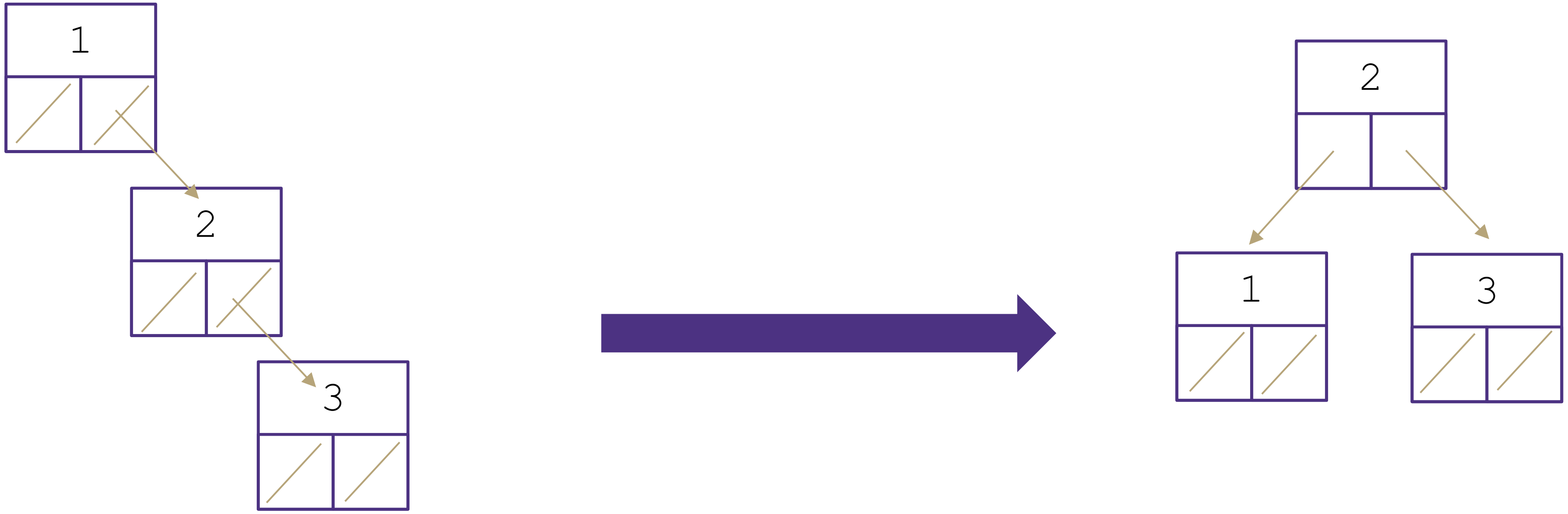
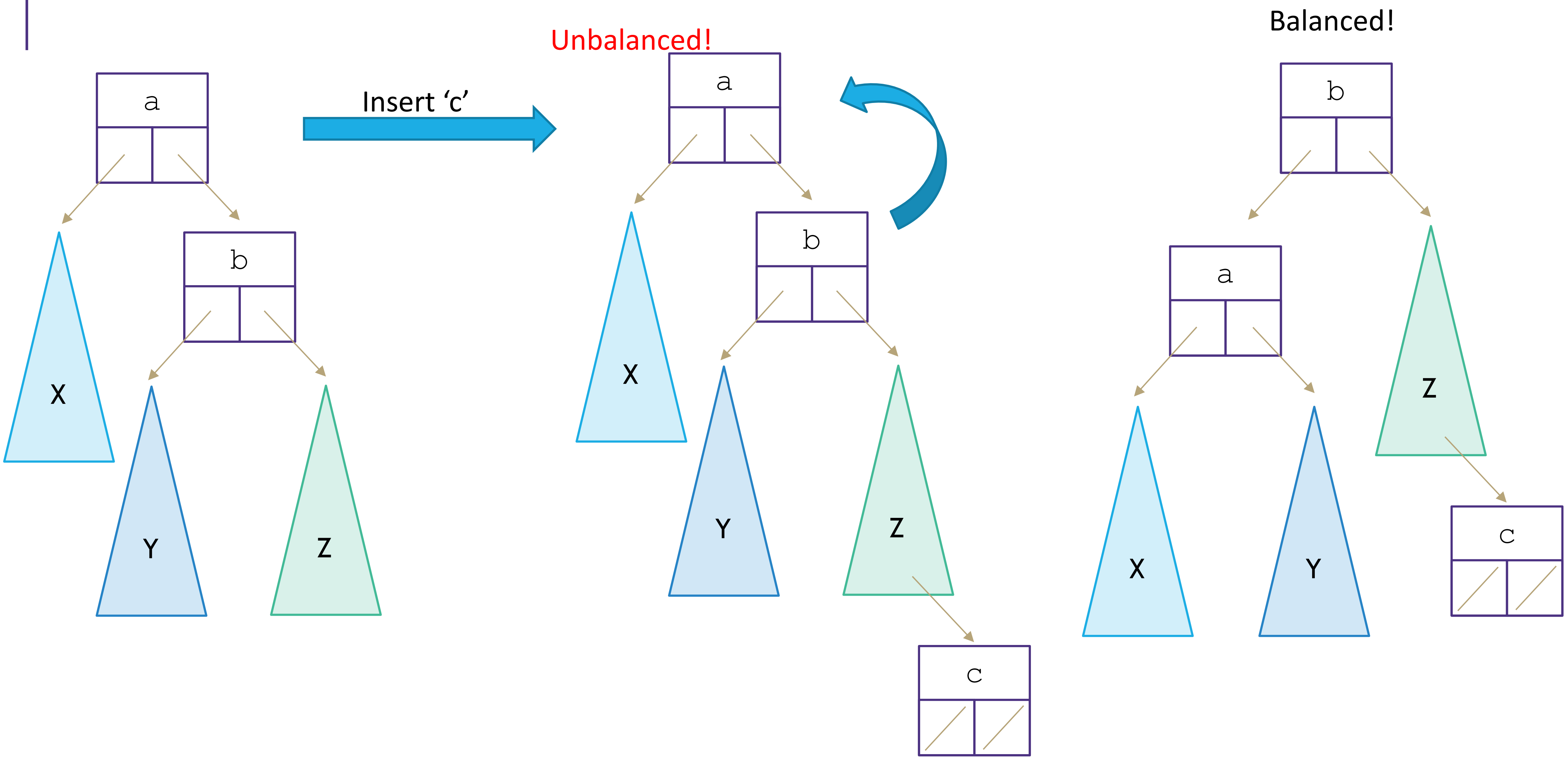
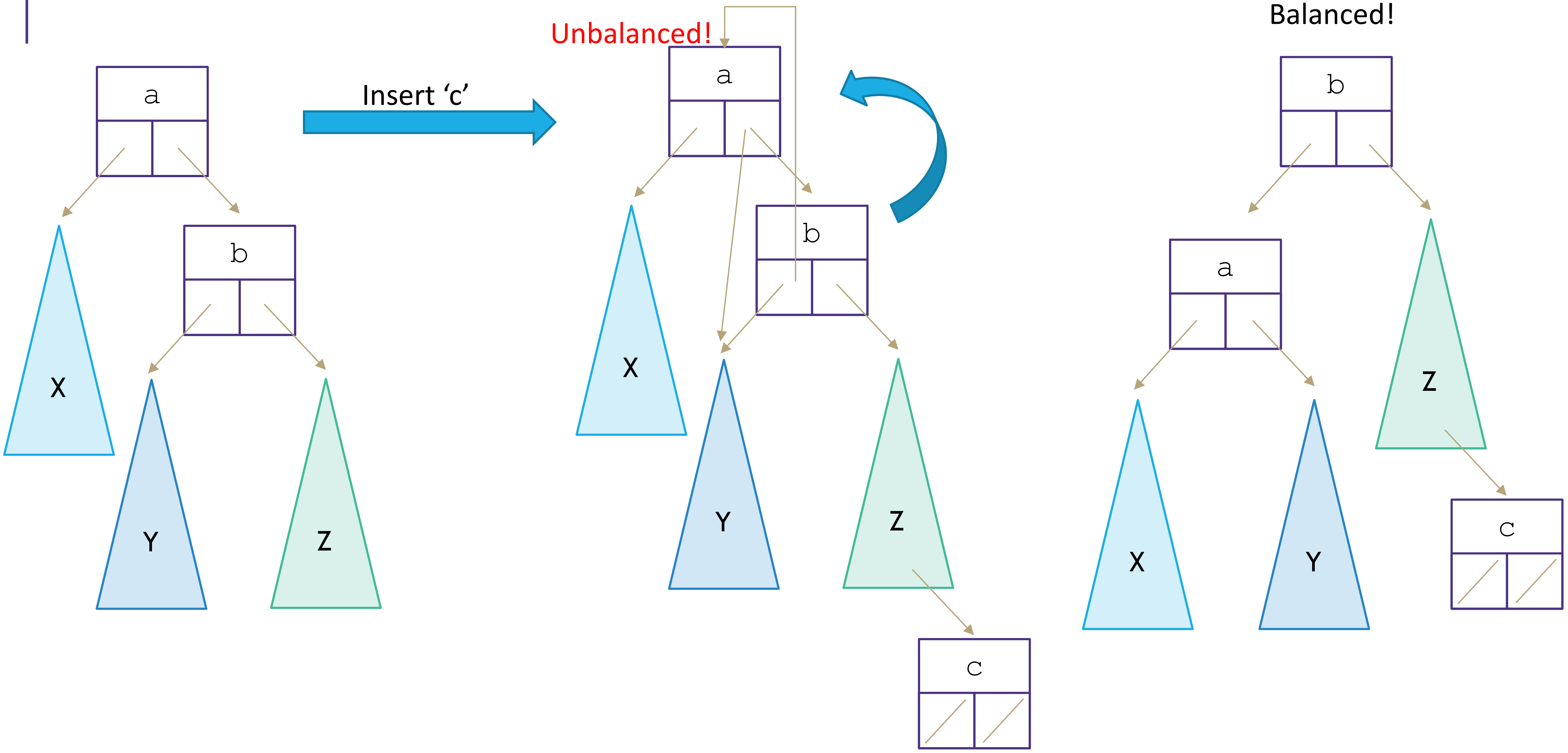


Rotations!



Rotate Left

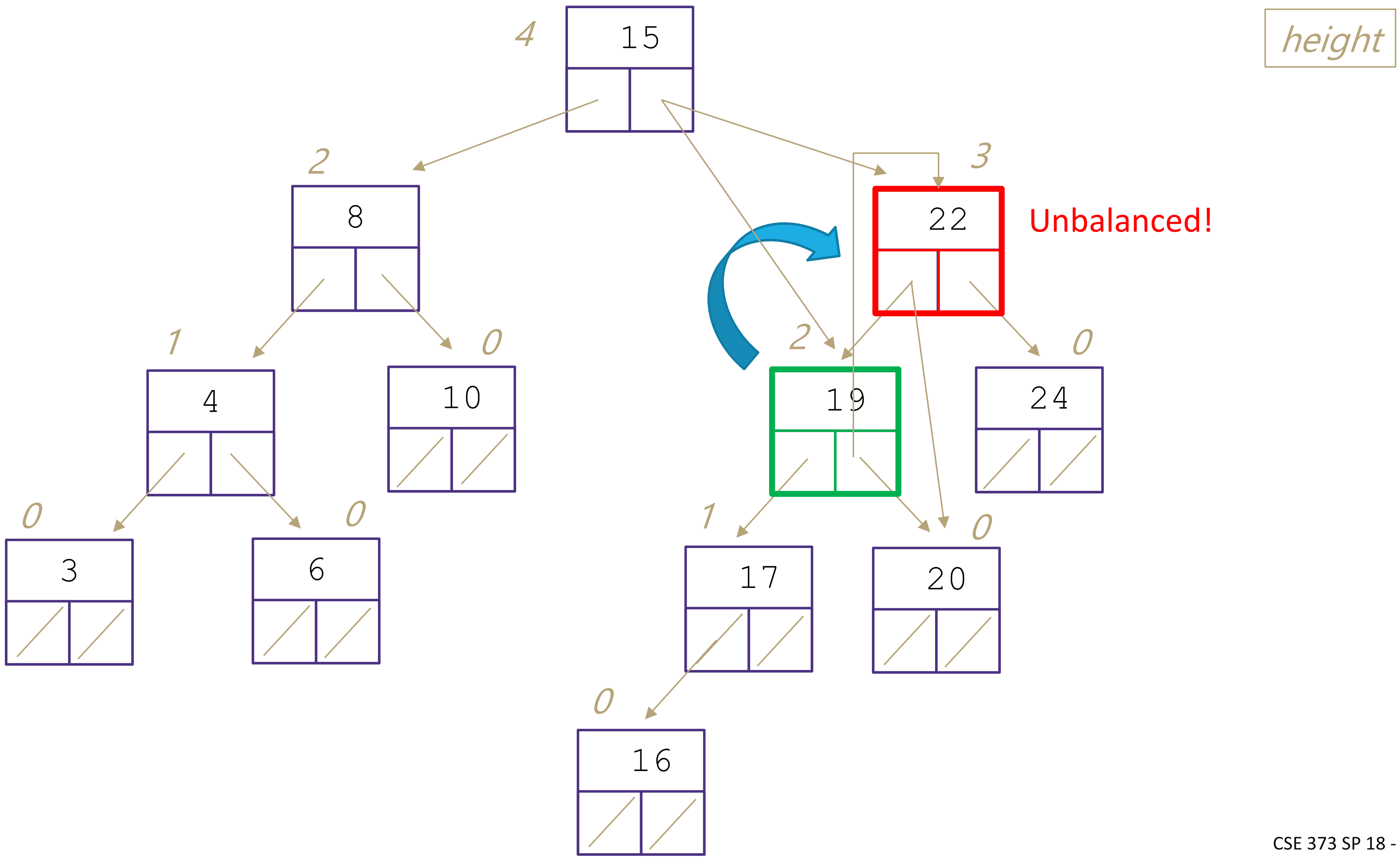
parent's right becomes child's left, child's left becomes its parent



Rotate Right

parent's left becomes child's right, child's right becomes its parent

put(16);



Rotate Right

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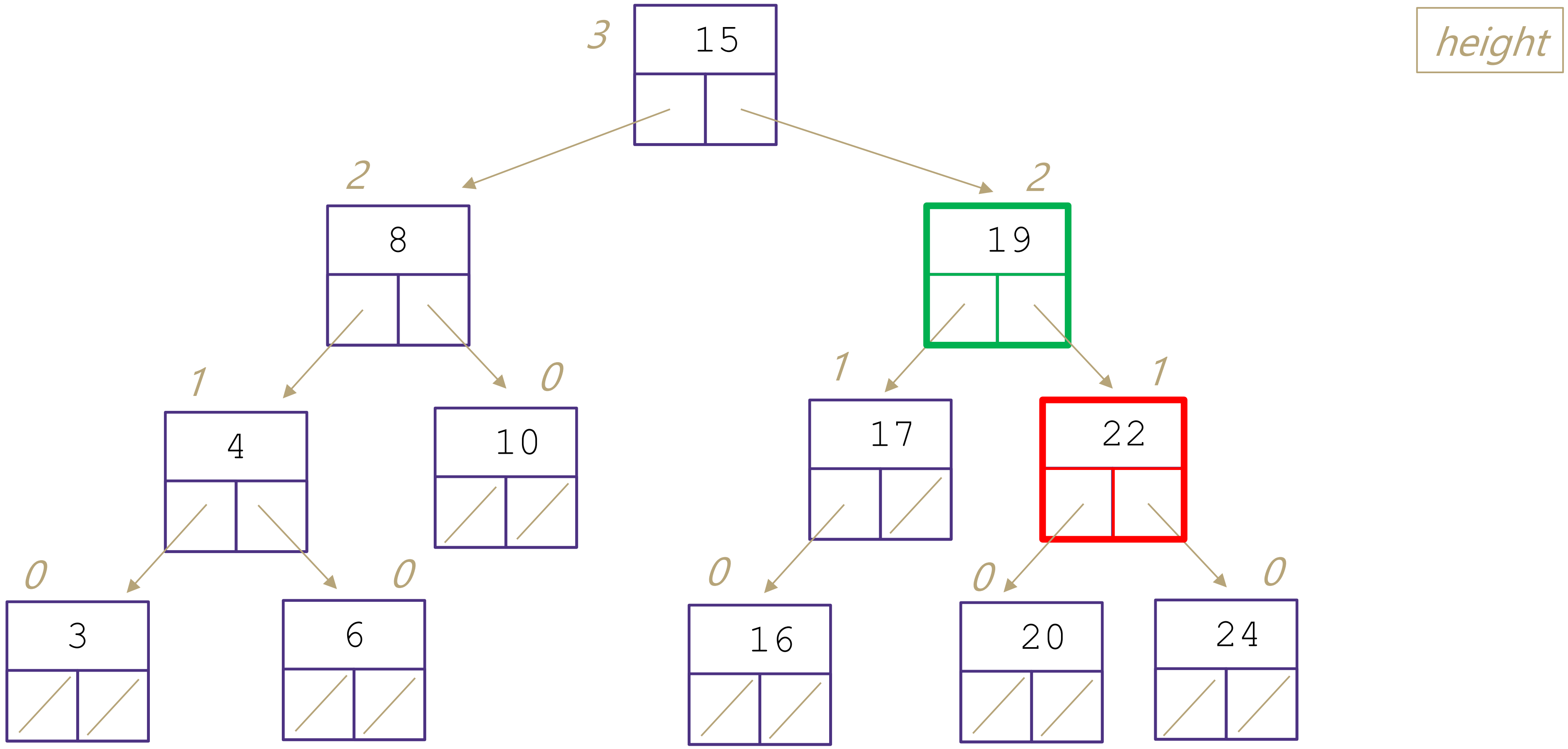
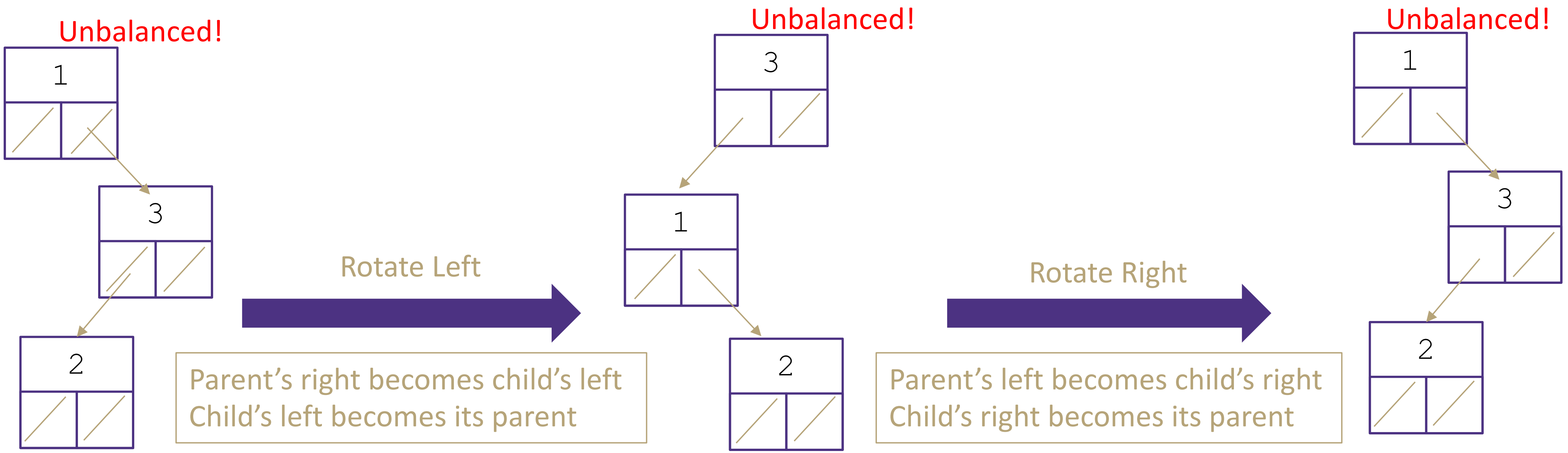


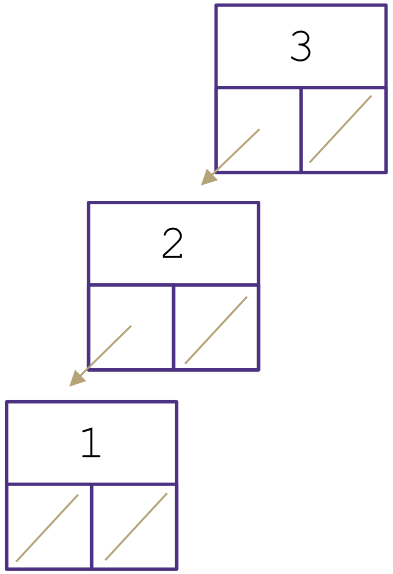
So much can go wrong

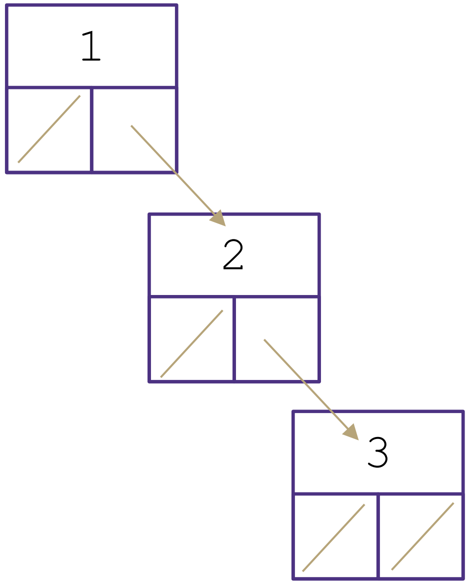


Two AVL Cases

Line Case

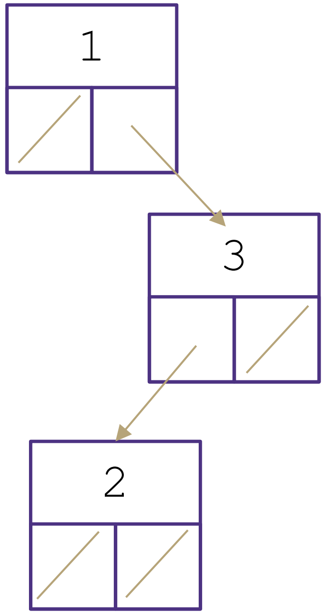
Solve with **1** rotation

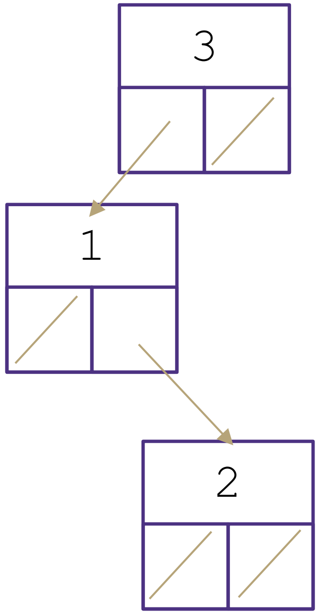




Kink Case

Solve with **2** rotations





Rotate Right

Parent's left becomes child's right

Child's right becomes its parent

Rotate Left

Parent's right becomes child's left

Child's left becomes its parent

Right Kink Resolution

Rotate subtree left

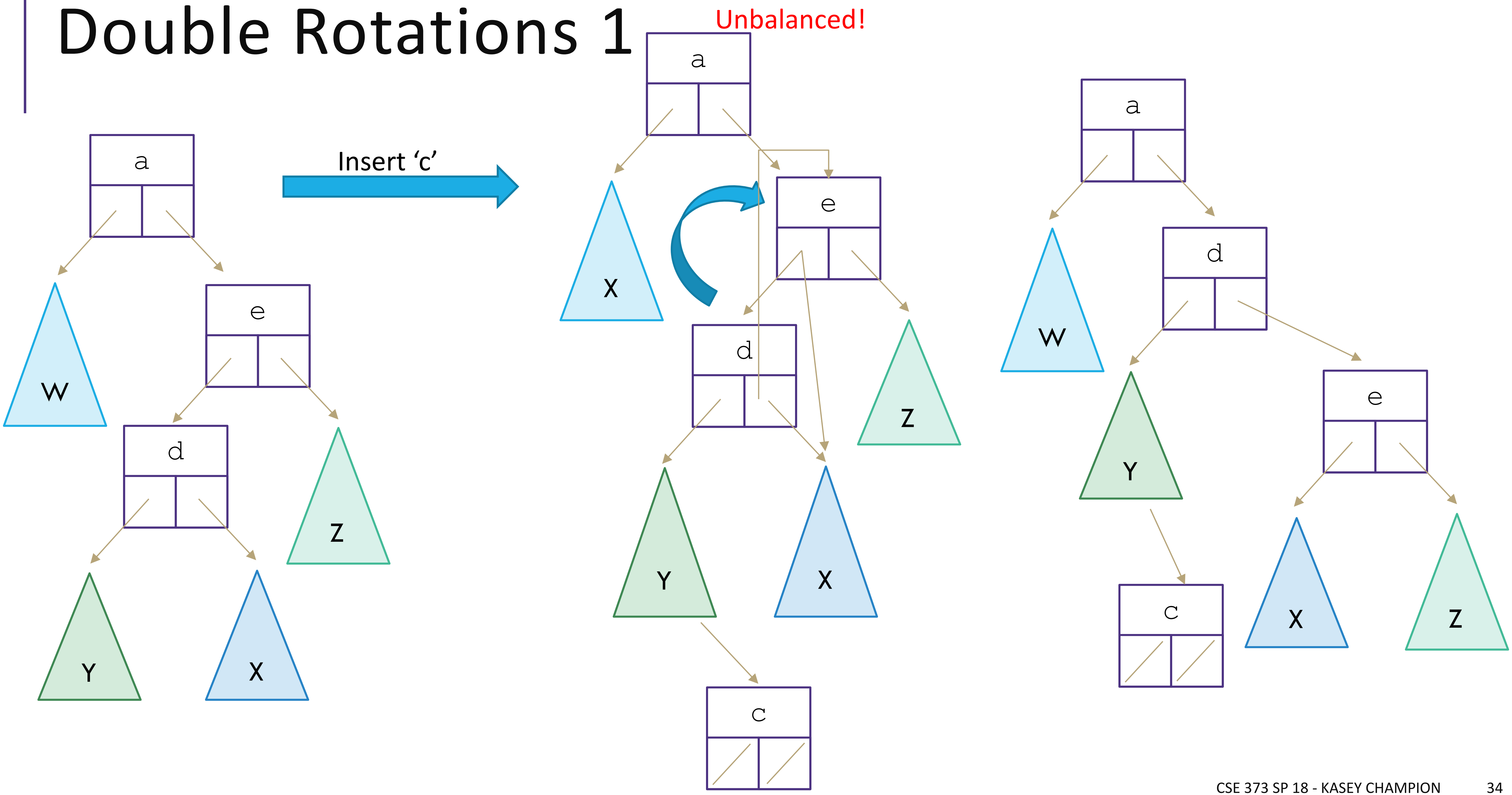
Rotate root tree right

Left Kink Resolution

Rotate subtree right

Rotate root tree left

Double Rotations 1



Double Rotations 2