

1. Applying Algorithms

- (a) Given a directed graph G and a node v in the graph, devise an algorithm that returns whether or not v is part of a cycle in G .

- (b) If **True**, give a (short) explanation why. If **False**, give a counterexample:

Assume a connected graph G that we have run Dijkstra on, with start s and goal g . This means each node has its predecessor set correctly.

After this, I increase the weight of some edge $u \rightarrow v$ on the shortest path between s and g (you don't know by how much). Although my shortest path may no longer be valid, I can look at all of v 's incoming edges, select the cheapest one, and set v 's new predecessor to be the other endpoint of that cheapest edge.

This sets all of the predecessors correctly, and I have repaired my shortest path despite the edge change.

4. Runtime Analysis

Consider this recursive code:

```
1 // The Node class
2 public Node {
3     public boolean visited; // True if this node has been visited
4     public final Node[] children; // The children of this Node
5     public final value; // The value stored in this node
6 }
7
8 // The recursive method
9 public static boolean mystery(Node node, int val) {
10     node.visited = true;
11     if (node.value == val) {
12         return true;
13     }
14     Node[] children = node.children;
15     boolean b;
16     for (int i = 0; i < children.length; i++) {
17         if (children[i].visited != true && mystery(children[i], val)) {
18             return true;
19         }
20     }
21     return false;
22 }
```

- (a) How is this graph represented? What advantages/disadvantages does it have to using a Dictionary implementation?

- (b) What is the recursive method doing?

- (c) Which graph algorithm is this most similar to and why?

- (d) What is the worst-case and its runtime?

- (e) What is the best-case and its runtime?

- (f) Is this version more or less efficient (time and/or memory) compared to a non-recursive version using the Dictionary implementation of graphs? Why? (Hint: how would you implement this method if this graph was a Dictionary?)