## 373 18au Midterm practice

October 31, 2018

> Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

| Question | Points | Score |
| :---: | :---: | :---: |
| Design | 0 |  |
| Binary Trees | 0 |  |
| AVL Trees | 0 |  |
| Hash | 0 |  |
| Heap | 0 |  |
| Runtime Analysis | 0 |  |
| Total: | 0 |  |

## Note:

This is a practice exam. It hasn't been reviewed thoroughly, so it's very likely there are typos and other mistakes. If something does not make sense, ask.

1. Design choices: For the following, choose the most appropriate data structure to solve the problem and briefly (less than 1-2 sentences) justify your answer.

Stack, AVL tree, Binary search tree, Heap, Hash table, Linked List
If there are multiple equally good candidates, list them all.
(a) To store keys and associated values in a system such the actual key values can be hidden from the implementor of the data structure.
(b) Sorting an almost sorted list: An almost sorted list is a list where elements are "almost" sorted. This means that elements in the list can be in a position that is at most $k$ places away from their sorted place. Assume that the list is too large to load into RAM to sort the entire list. Which data structure would you use to efficiently read this list and print a sorted list?
(c) Near homework deadlines gitlab pipelines get stuck. Gitlab pipelines execute jobs in a queue, on a first come first serve basis. This means that if one student submit too many jobs continuously, all other jobs will be held up. One solution is to implement a queue system that instills somewhat fairness is where subsequent jobs from the same student get lower priority. So if one student submits 10 jobs, his/her first job has the highest priority and the $10^{t h}$ job has the lowest priority. Which data structure(s) would be appropriate to implement this? Why?


Figure 1: A binary tree
2. (a) What is the minimum number of nodes in a binary tree of height $h$ : $\qquad$
(b) What is the maximum number of nodes in a binary tree of height $h$ : $\qquad$
(c) What is the maximum number of leaf nodes in a binary tree of height $h$ can have:
(d) Print pre-order of the tree in Figure 1: $\qquad$


## 3. AVL Trees

(a) Insert the following sequence of values into an empty AVL tree in the given order.

$$
10,20,-1,0,5,14,1,6,5,8,1
$$

You do not have to show the intermediary steps, but no work and a wrong answer will result in no credit. Draw your final tree in the following figure, Figure 2. Clearly, there are more nodes in the figure than in your final tree. Leave the unused nodes empty. You may use the space below Figure 2 to draw your intermediary trees or for scratch work.


Figure 2: Fill this tree with your final answer. Leave unused nodes empty.
(b) Consider a set $S=1,2,3,4,5,6,7,8,9,10$. With four keys from this set create a 4 -node AVL tree such that any fifth insertion done with any remaining key in the set will break the AVL balance property of the tree. Draw your 4-node AVL tree in the following figure. Leave unused nodes empty.

4. Consider inserting data with integer keys $41,94,95,56,87,65,5,25,18,74,62,22,25,66,7$ in that order into a hash table of size 20 where the hashing function is $h($ key $)=k e y+3$.
Show an open addressing that uses double hashing after doing the insertions. The second hash function is $g(k e y)=$ key $\% 23$. Do not worry about resizing.

| 0 | 1 | 2 | 3 | 4 | 5 |  | 6 | 7 | 8 |  |  | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

5. Insert the following sequence of values in an empty min-heap in the given order.

$$
99,77,92,97,35,42,51,14,10,80,29,15,53,
$$

Fill Figure 3 with your final answer. You do not have to show the intermediary steps, but no work and a wrong answer will result in no credit. Drawing intermediate trees may result in partial credit. You can use the space below Figure 3 or the back of this page to draw your intermediary steps.


Figure 3: Fill this tree with your final answer. Leave unused nodes empty.
6. For each of the following, describe the worst-case running time with respect to variable $n$ (and any additional specified variables). Please give a tight big- $O$ bound and in the most simplified form. You do not need to justify your answer.
(a) Give a tight big- $O$ for function $f(n) . f(n)=100 n^{3}+4 n-100 n^{3}$ : : $\qquad$
(b) Give a tight big- $O$ for function $f(n) . f(n)=25 n \log n+50 n+2 n^{2}$ : : $\qquad$
(c) Find recurrence $T(n)$ for the following snippet of the code

```
public int bar(int n) {
    if (n <= 10) {
            return 0;
    }
    for (int i = 0; i < n; i++) {
                System.out.println("!");
    }
    int a = 2 + bar(n/2);
    int b = 1 + bar(n/2)
    return a + b;
}
```

$\square$
(d) Demonstrate that $4+100 n$ is dominated by $n$ by finding a $c$ and $n_{0}$. Show your work.
(e) Simplify the following summation to a closed form

$$
\sum_{i=20}^{n} i
$$

