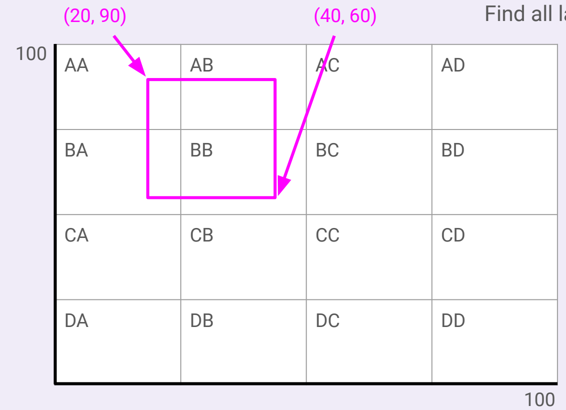
Goal: Fill in the findAnswer Method

```
public class Rasterer {
  private static final double MAX_X = 100;
  private static final double MAX_Y = 100;

  public static List<String> findAnswer(double x1, double y1,
                                        double x2, double y2) {
    // TODO
    return null;
  }
}
```

What are some potentially useful helper methods?

Q Arbitrary Challenging Problem of the Day



Find all labels that overlap the query.

findAnswer(20, 90, 40, 60)

[“AA”, “AB”, “BA”, “BB”]

Problem Decomposition in Software Engineering

Decomposition. Taking a complex task and breaking it into smaller parts. This is the heart of computer science. Using appropriate abstractions makes problem solving vastly easier.

Perspective 1: Software engineering.

Eliminating special cases in k-d tree nearest made code simpler and more obvious.

Modularization is decomposition for managing software complexity at a project level.

- **Autocomplete**. Efficient search bar prefix queries.
- **Heap**. Efficient priority queue for route finding.
- **K-d Tree**. Efficient 2-d nearest neighbors to find start and goal vertices.
- **A* Search**. Efficient route finding.
- **Rasterer**. Efficient map tile display.

Problem Decomposition in CS Theory

Decomposition. Taking a complex task and breaking it into smaller parts. This is the heart of computer science. Using appropriate abstractions makes problem solving vastly easier.

Perspective 2: Computational complexity theory.

Reduction. Using an algorithm for Problem **Q** to solve Problem **P**.

“If any subroutine for task Q can be used to solve P, we say P reduces to Q.”

Algorithms (Dasgupta, Papadimitriou, Vazirani)

Graph Problems and Their Solutions

Paths. Find a path from **s** to every reachable vertex.
  Depth-first search. O(**V** + **E**) runtime with adjacency list.

Unweighted Single-Source Shortest Paths.
  Find a shortest path from **s** to every reachable vertex.
  Breadth-first search. O(**V** + **E**) runtime with adjacency list.

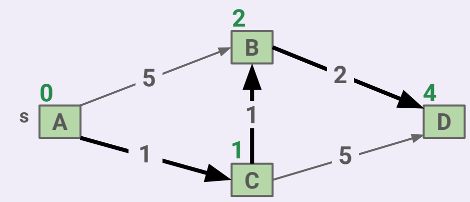
Weighted Single-Source Shortest Paths.
  Find a shortest path from **s** to every reachable vertex.
  Dijkstra's algorithm. O(**E** log **V** + **V** log **V**) runtime with adjacency list.

Weighted Single-Pair Shortest Paths.
  Find a shortest path from **s** to a single **goal** vertex.
  A* search. Dijkstra's algorithm with h(**v**, **goal**) as priority. Runtime depends on heuristic.

Q Algorithm for Finding a Shortest Paths Tree

Given a weighted, directed graph with **integer edge weights between 1 and 5**, find the single-source shortest paths tree from s to every other vertex in the graph.

Your algorithm should be faster than Dijkstra's algorithm.



PQ.add(**s**, 0)

For all other vertices **v**, PQ.add(**v**, infinity)

While PQ is not empty:

  p = PQ.removeSmallest()
  Relax all edges from **p**

Relaxing an edge (**v**, **w**) with **weight**:

If distTo[**w**] > distTo[**v**] + **weight**:

  distTo[**w**] = distTo[**v**] + **weight**
  edgeTo[**w**] = **v**
  PQ.changePriority(**w**, distTo[**w**])

Dijkstra's Runtime Analysis

ArrayHeapMinPQ implementation.

- **V** adds, each O(log **V**) time.
- **V** removals, each O(log **V**) time.
- **E** changePriority, each O(log **V**) time.

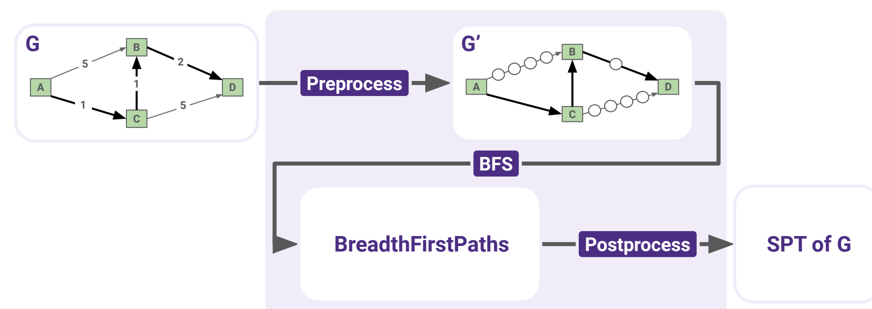
Overall: O(**V** log **V** + **V** log **V** + **E** log **V**).

Simple: O(**V** log **V** + **E** log **V**).

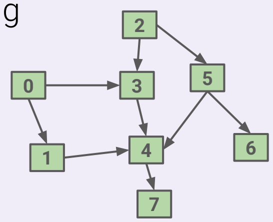
Assuming **E** > **V**, this is just O(**E** log **V**) for connected graphs.

Reductions

Given a graph **G**, we created a new graph **G'**, then fed it to a related (but different) algorithm, and finally interpreted the result.



Q Topological Ordering

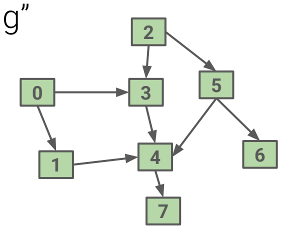


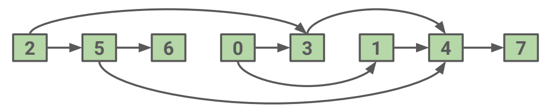
Suppose we have tasks 0 through 7, where an arrow from **v** to **w** indicates that **v** must happen before **w**.

Which graph algorithm could we use to find a valid ordering for these tasks?

Valid orderings include: [0, 2, 1, 3, 5, 4, 7, 6], [2, 0, 3, 5, 1, 4, 6, 7], ...

Topological "Sorting"





Reduction to Dijkstra's

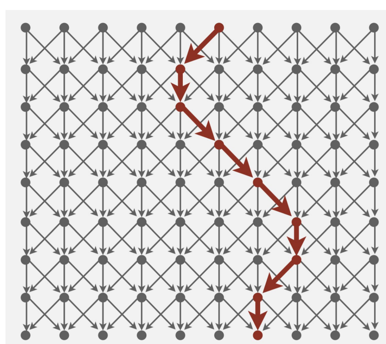
To find a **vertical seam**...

Vertex. Pixel in image.

Edge. Cost to go from a pixel to its 3 downward neighbors.

Weight. **Energy function** of 8 neighboring pixels.

Seam. Shortest path (sum of weights) from top to bottom.



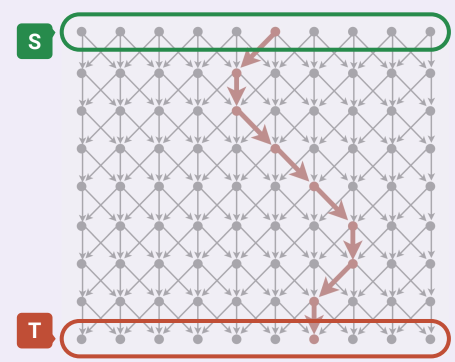
Shortest Paths (Robert Sedgewick, Kevin Wayne/Princeton)

Q Formal Problem Statement

Using AStarSolver, find the seam from any top vertex to any bottom vertex.

Given a digraph with positive edge weights, and two distinguished subsets of vertices **S** and **T**, find a shortest path from any vertex in **S** to any vertex in **T**.

Your algorithm should run in time proportional to **E** log **V** in the worst case.



Shortest Paths (Robert Sedgewick, Kevin Wayne/Princeton)