0. Another half point. (0.5 points). Write your name, login, and ID on the front page. Write your exam room. Write the IDs of your neighbors. Write the given statement and sign. Write your login in the corner of every page. Enjoy your free half point.

1. Basic Operations (6 Points).

a. To the right of the BST below, draw a BST that results if we delete 20 from the BST. You should use the deletion procedure discussed in class (i.e. no more than 4 references should change).

![BST Diagram]

b. To the right of the minHeap below, draw the minHeap that results if we delete the smallest item from the minHeap.

![MinHeap Diagram]

c. To the right of the External Chaining Hash Set below, draw the External Chaining Hash Set that results if we insert 5. As part of this insertion, you should also resize from 4 buckets to 8 (in other words, the implementer of this data structure seems to be resizing when the load factor reaches 1.5). Assume that we’re using the default hashCode for integers, which simply returns the integer itself.

![Hash Set Diagram]
5. Binary heaps. (6 points)

(a) Consider the following binary tree representation of a max-heap.

```
          Y
         /\  
        X  H
       /\  /\  
      G  T C  A
     /\  /\  /\  /\ 
    F  B Q  R U L
```

Give the array representation of the heap.

```
0  1  2  3  4  5  6  7  8  9  10  11  12
-  -  -  -  -  -  -  -  -  -  -  -  -
```

(b) Insert the key P into the binary heap above, circling any entries that changed.

```
0  1  2  3  4  5  6  7  8  9  10  11  12
-  -  -  -  -  -  -  -  -  -  -  -  -
```

(c) A delete-the-max operation in the binary heap at left results in the binary heap at right.

```
          Z
         /\  
        Y  R
       /\  /\  
      L  U N  K
     /\  /\  /\  /\ 
    A  K H G ?  -
```

Which of the keys below could be the one labeled with a question mark? Circle all possibilities.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
3. (11 points) Consider a hashtable with separate chaining with $N$ buckets and $k$ items currently in the table.

(a) $k/N$ is the definition of a term used when discussing hashing. What is this term?

(b) Is it necessary that $k < N$?

(c) What is the average number of items in a bucket?

(d) In the worst-case, how many items could be in a single bucket?

(e) If $k > N$, is it possible that any buckets are empty?

(f) If we resize the table to a table of size $2N$, what is the asymptotic running time in terms of $k$ and $N$ to put all the items in the new table?
(b) For each function on the left, give the best matching order of growth of the \textit{running time} on the right.

\begin{align*}
\text{B.} & \quad \text{public static int } f1\text{(int } N) \{ \\
& \quad \quad \text{int } x = 0; \\
& \quad \quad \text{for (int } i = 0; i < N; i++) \\
& \quad \quad \quad x++; \\
& \quad \quad \text{return } x; \\
& \} \\
& \quad \text{A. } \log N \\
\end{align*}

\begin{align*}
\text{D.} & \quad \text{public static int } f2\text{(int } N) \{ \\
& \quad \quad \text{int } x = 0; \\
& \quad \quad \text{for (int } i = 0; i < N; i++) \\
& \quad \quad \quad \text{for (int } j = 0; j < i; j++) \\
& \quad \quad \quad \quad x++; \\
& \quad \quad \text{return } x; \\
& \} \\
& \quad \text{D. } N^2 \\
\end{align*}

\begin{align*}
\text{E.} & \quad \text{public static int } f3\text{(int } N) \{ \\
& \quad \quad \text{if (} N == 0 \text{) return } 1; \\
& \quad \quad \text{int } x = 0; \\
& \quad \quad \text{for (int } i = 0; i < N; i++) \\
& \quad \quad \quad x += f3\text{(} N-1 \text{);} \\
& \quad \quad \text{return } x; \\
& \} \\
& \quad \text{E. } 2^N \\
\end{align*}

\begin{align*}
\text{F.} & \quad \text{public static int } f4\text{(int } N) \{ \\
& \quad \quad \text{if (} N == 0 \text{) return } 0; \\
& \quad \quad \text{return } f4\text{(} N/2 \text{) + } f1\text{(} N \text{) + } f4\text{(} N/2 \text{);} \\
& \} \\
& \quad \text{F. } N! \\
\end{align*}

\begin{align*}
\text{C.} & \quad \text{public static int } f5\text{(int } N) \{ \\
& \quad \quad \text{int } x = 0; \\
& \quad \quad \text{for (int } i = N; i > 0; i = i/2) \\
& \quad \quad \quad x += f1\text{(} i \text{);} \\
& \quad \quad \text{return } x; \\
& \} \\
& \quad \text{C. } N \log N \\
\end{align*}

\begin{align*}
\text{G.} & \quad \text{public static int } f6\text{(int } N) \{ \\
& \quad \quad \text{if (} N == 0 \text{) return } 1; \\
& \quad \quad \text{return } f6\text{(} N-1 \text{) + } f6\text{(} N-1 \text{);} \\
& \} \\
& \quad \text{G. } N^2 \\
\end{align*}

\begin{align*}
\text{H.} & \quad \text{public static int } f7\text{(int } N) \{ \\
& \quad \quad \text{if (} N == 1 \text{) return } 0; \\
& \quad \quad \text{return } 1 + f7\text{(} N/2 \text{);} \\
& \} \\
& \quad \text{H. } N^2 \\
\end{align*}
1. Create the multiway trie for the following keys (drawing the nodes as shown), inserted in this order (once you get the hang of it, you don’t have to draw the whole thing):
   apple, ape, applet, cape, tall, tap, tape

2. Does the structure of the trie depend on the order in which you inserted the words?
   A. Yes  B. No

3. Assume you insert 100 keys of length 5 digits into a multi-way trie. How tall is your trie in the worst case?

4. If N is the number of keys you insert in your trie, and D is the length of the longest key, what is the maximum height of your trie in terms of N and/or D in the worse case?

5. Assume an alphabet of 6 characters, as above. From this alphabet, you can create $6^5 = 7776$ different keys (strings) of length 5. If you insert all 7776 of these strings into a MWT and a BST, which will be shorter? How tall is each? Is this true in general (consider different size alphabets, different length strings)? You can assume the BST is perfectly balanced.

6. What is the main drawback of a MWT compared to a BST, especially when the MWT isn’t full?
7. List all of the words you can find in this TST. Nodes that end words have extra-thick circles.

8. Create the TST for the following strings (same set as on the last page)
apple, ape, applet, cape, tall, tap, tape

9. If you add the word “ace” to the TST you just created, how many nodes do you have to add?

10. Does the structure of a TST depend on the order in which the keys were inserted?