Overall Asymptotic Runtime Bound for dup1

\[ R_{\text{best}}(N) = 2 \]
\[ R_{\text{worst}}(N) = \frac{N^2 + 3N + 2}{2} \]

Give an overall asymptotic runtime bound for \( R \) as a combination of \( \Theta \), \( O \), and/or \( \Omega \) notation. Take into account both the best and the worst case runtimes (\( R_{\text{best}} \) and \( R_{\text{worst}} \)).

Q1: Give an overall asymptotic runtime bound for \( R \) as a combination of \( \Theta \), \( O \), and/or \( \Omega \) notation. Take into account both the best and the worst case runtimes (\( R_{\text{best}} \) and \( R_{\text{worst}} \)).

Print Party: Attempt 1

```java
void printParty(int N) {
    for (int i = 1; i <= N; i *= 2) {
        for (int j = 0; j < i; j += 1) {
            System.out.println("hello");
        }
    }
}
```

Q1: Find a simple \( f(N) \) such that the runtime \( R(N) \in \Theta(f(N)) \).

A. \( 1 \)
B. \( \log N \)
C. \( N \)
D. \( N \log N \)
E. \( N^2 \)
F. Other

Q: How do we know that there's only one case to consider?
Let the cost model \( C(N) \) be the number of calls to println for a given \( N \). This is our representative operation for figuring out the runtime.

? : For each \( N \), predict \( C(N) \).

Real world programs are often messy and difficult to model.

? : What’s different between these two summations?

? : How did we apply these strategies to analyze printParty?
Informal Recursion Analysis

Find a simple $f(N)$ such that the runtime $R(N) \in \Theta(f(N))$.

Inspect the example and give the order of growth of the runtime as a function of $N$.

A. $1$
B. $\log N$
C. $N$
D. $N^2$
E. $2^N$

Q: What does each node represent in the tree on the right?

Q1: Find a simple $f(N)$ such that the runtime $R(N) \in \Theta(f(N))$.

Recursion and Exact Counts

Find a simple $f(N)$ such that the runtime $R(N) \in \Theta(f(N))$.

Approach 2: Count number of calls to $f_3$, given by $C(N)$.

Give a simple, exact expression for $C(N)$.

$C(4) = 1 + 2 + 4 + 8$

$C(N) = 1 + 2 + 4 + 8 + \ldots + ?$

Q: What is the exact value of the last term in the sum for $C(N)$?

Q1: Give a simple, exact expression for $C(N)$. 

Q1: Find a simple $f(N)$ such that the runtime $R(N) \in \Theta(f(N))$. 

Q: What does each node represent in the tree on the right?
The Merge Operation

Given two sorted arrays, the merge operation combines them into a single sorted array by successively copying the smallest item from the two arrays into a target array.

2 3 6 10 11 4 5 7 8

?: What is a cost model that we can use to evaluate the runtime of the merge operation?

Merge Sort

Merge sort algorithm merges every layer.
1. If array is of size 1, return.
2. Merge sort the left half.
3. Merge sort the right half.
4. Merge the two sorted halves.

For $N = 64$, the total runtime is $\sim 384$ AU.

- Top layer: $\sim 64$ AU
- Second layer: $2 \cdot (\sim 32$ AU) = $\sim 64$ AU
- Third layer: $4 \cdot (\sim 16$ AU) = $\sim 64$ AU
- $i^{th}$ layer: $2^{i-1} \cdot (\sim 64$ AU / $2^{i-1}) = \sim 64$ AU

?: How does the call tree for merge sort differ from the example we saw in f3?

?: How do these differences affect our runtime analysis?