

For answers that involve filling-in a , fill-in the shape completely: .

1. Suppose we create a new sorting algorithm called `PARTITIONHYBRIDSORT(INPUT, LO, HI, SUBSORT)`, where `INPUT` is an array of integers, `LO` is the lowest index in the array to be sorted, `HI` is the highest index in the array to be sorted, and `SUBSORT` is the sorting algorithm to use in step 3:
  1. If  $HI - LO \leq 1$ , return.
  2. Partition around `INPUT[LO]`, i.e. the leftmost item of the current subproblem.
  3. Use `SUBSORT` to sort the left and right subproblems.

Give the **worst-case runtime** for `PARTITIONHYBRIDSORT` using different `SUBSORT` algorithms in terms of  $N$ , the size of the `INPUT`.

- (a) `PARTITIONHYBRIDSORT(INPUT, 0, N, INSERTIONSORT)` Worst-case:  $\Theta(N^2)$
  - (b) `PARTITIONHYBRIDSORT(INPUT, 0, N, MERGESORT)` Worst-case:  $\Theta(N \log N)$
  - (c) `PARTITIONHYBRIDSORT(INPUT, 0, N, LSDRADIXSORT)` Worst-case:  $\Theta(N)$
  - (d) `PARTITIONHYBRIDSORT(INPUT, 0, N, PARTITIONHYBRIDSORT)` Worst-case:  $\Theta(N^2)$
2. Suppose we replace the counting sort used in LSD radix sort with merge sort, resulting in a new radix-based sorting algorithm called `LSD RADIX MERGE SORT`. The merge sort used as a subsort by `LSD RADIX MERGE SORT` is exactly like regular merge sort, except that its merge operation compares only one digit of an input to decide which is larger. For example, the merge operation would ordinarily consider 361 to be less than 410, but if we're sorting on the final digit, it will consider 361 to be larger than 430 since  $1 > 0$ .

Just like regular LSD radix sort, `LSD RADIX MERGE SORT` would sort by the last digit, then second to last digit, and so forth. We can define `LSD RADIX QUICKSORT` in a similar way. Assume that `LSD RADIX QUICKSORT` always picks the leftmost pivot and uses in-place Hoare partitioning.

Give the **worst-case runtime** of both sorting algorithms in terms of  $N$  and  $W$ , where  $W$  is the number of digits in each key. For simplicity, assume all keys have the same number of digits. Don't worry about the alphabet size  $R$ . Also state whether or not they always return a correct sort and explain.

- (a) `LSDRADIXMERGESORT` Worst-case:  $\Theta(WN \log N)$

Correct?  Yes  No

Explanation: Merge sort is stable.

- (b) `LSDRADIXQUICKSORT` Worst-case:  $\Theta(WN^2)$

Correct?  Yes  No

Explanation: Quicksort is unstable.