CSE 373: P vs NP; reductions

Michael Lee

Wednesday, Mar 7, 2018

Warmup

Remind your neighbor:

- What is a decision-problem?
- What is P, EXP, and NP?

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- What is P, EXP, and NP?

  1. P is the set of all decision problems that can be *solved* in worst-case *polynomial* time
  2. EXP is the set of all decision problems that can be *solved* in worst-case *exponential* time
  3. NP is the set of all decision problems where we can *verify* all "yes" answers in worst-case *polynomial* time

Final

Final logistics:

- Thursday, March 15
- 2:30 to 4:20
- Gowen 301

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Review sessions:

- Monday, Mar 12: EEB 125, 4:30 to 6:30
- Tuesday, Mar 13: EEB 105, 4:30 to 6:30

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The final will be cumulative, but skewed towards new material.

Post-midterm topics:

1. Heaps

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    - Representation
    - Traversal
    - Dijkstra's
    - Topological sort
    - MSTs (Prim, Kruskal, disjoint sets)

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5. P and NP

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3. Caching and locality

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General study tips for mechanical problems:

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5. Think about what would happen if you were to tweak some aspect of a data structure or algorithm

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General tips when asked to write pseudocode:

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2. Try writing an algorithm that works on a specific example first, then figure out how to generalize.

Final

Syllabus change:

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Previously:

- Midterm was 20% of grade
- Final was 20% of grade

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Now:

- Your lowest-scoring exam will be 15% of grade
- Your highest-scoring exam will be 25% of grade

Recap

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- Introduced the complexity classes P and EXP
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- But many of those problems are also in NP!
- Question: if there are problems where we can *verify* answers efficiently, does that mean we can also *find* answers efficiently?

Is CIRCUIT-SAT in NP?

Question: is `CIRCUIT-SAT` in NP?

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Step 1: Assume you have a magical solver, and it said "yes" for some boolean expression B.

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Step 1: Assume you have a magical solver, and it said "yes" for some boolean expression B.

Step 2: Three questions to answer.

1. How do we modify the solver so it returns a convincing certificate for B?
2. How do we check the certificate, whatever it is?
3. Does our verifier actually run in polynomial time?

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`{a=true, b=false, c=true, d=false, ...}`

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2b: How do we check the certificate, whatever it is?

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boolean verifyCiruitSat(BooleanAst B, Dictionary<String, Boolean> certificate) {
    return evaluateExpr(B, certificate);
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private boolean evaluateExpr(B, certificate) {
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Yes: we visit each node and edge in the tree a constant number of times.

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- 3-COLOR easier or harder then CIRCUIT-SAT?

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a >= b

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Core ideas: If solving A lets us also solve B, then...

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- A was "harder then" (or the same as) B
- The B was really a special case of A all along!
- We've *reduced* the number of distinct problems in the world by one.

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Answer:

1. Start by adding a new vertex to the graph
2. Connect this vertex to all other nodes
3. Give this vertex some color. This forces all other vertices to have a only one of two colors!
4. Run the solver for 3-COLOR, return the result

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- To show two functions $f(n)$ and $g(n)$ are asymptotically the same, we can show that $f(n)$ both dominates and is dominated by $g(n)$
- To show two decision problems A and B are the same, we can show that A reduces to B and B reduces A!

LONG-PATH and HAM-PATH

LONG-PATH

Given a graph G and some integer k, does G contain some path that uses k edges?

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Step 1:

Reduce HAM-PATH to LONG-PATH

Note: paths can't revisit the same node

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boolean hamPathSolver(G) {
    return longPathSolver(G, |V| - 1)
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```

Step 2:

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```
boolean longPathSolver(G, k) {
    for (G2=(v1, v2, ..., vk) : G):
        if (hamPathSolver(G2)):
            return true;
    return false;
}
```

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Question: Are there other problems that are secretly the same problem in disguise?

Yes! It turns out that...

- `CIRCUIT-SAT`
- `3-COLOR`
- `HAM-PATH`
- `LONG-PATH`

...are all the same problem.

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NP-COMPLETE

A decision problem is NP-COMPLETE if it is both in NP and in NP-HARD.

NP-HARD and NP-COMPLETE

Punchline: If we have a way of solving any NP-HARD problem, we have a way of solving *every* problem we've looked at so far.

NP-HARD and NP-COMPLETE

How do these relate?

NP-HARD and NP-COMPLETE

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How do all relate to P?

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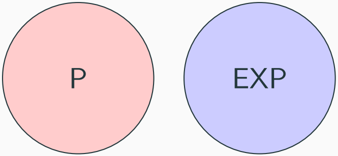
Answer 1: The sets are disjoint

E.g. if a problem is in P, it's not in EXP.



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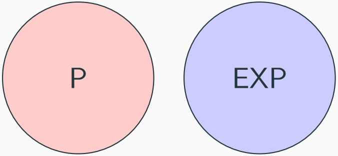
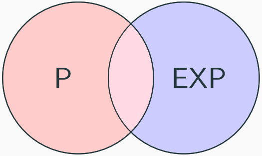


Answer 2: The sets overlap
E.g. some, but not all problems in P are in EXP



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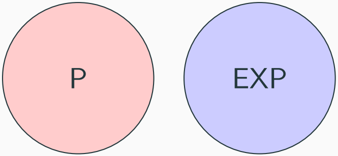

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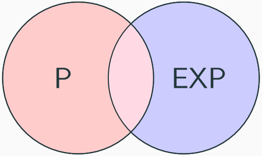

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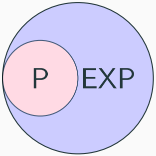

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It turns out, yes, P is indeed a subset of EXP:

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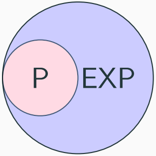

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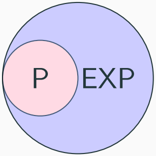

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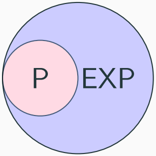

Reason: EXP is the set of decision problems where there exists an algorithm that solves the problem in *worst-case exponential time*.

So, if we can find a polynomial-time algorithm to a problem, we can definitely find an exponential one!

Is P a subset of NP?

New question: is a P a subset of **NP**?

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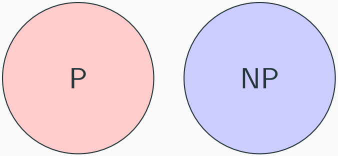
Answer 1: The sets are disjoint

E.g. if a problem is in P, it's not in NP.



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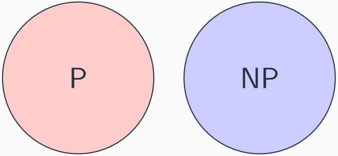
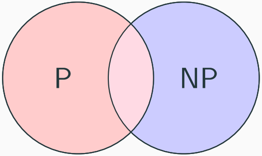


Answer 2: The sets overlap
E.g. some, but not all problems in P are in NP



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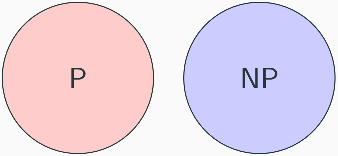

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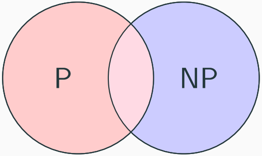

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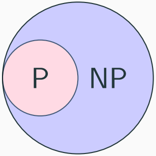
All problems in P are also in NP



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It turns out, yes.

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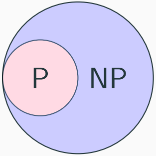

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3. Does our verifier actually run in polynomial time?
   Yep. If X was originally in P, then we know by definition `solverX` runs in polynomial time.

Is P a subset of NP?

Punchline: For any problem in P, we can build a verifier by just re-using the solver!

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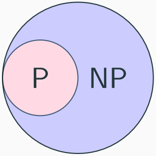

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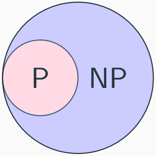
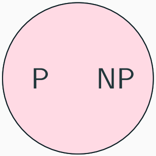


Answer 2: Yes

Not only is a P a subset of NP, they're exactly the same



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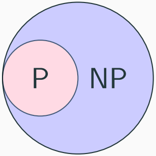

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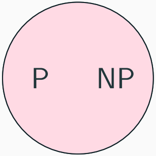

Answer: We don't know.

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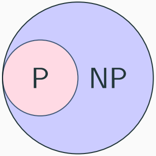

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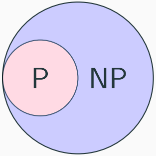

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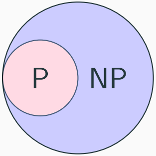

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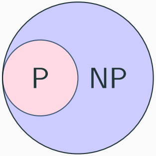

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- Something something quantum computing? (Lots of caveats, not practical right now, doesn't solve everything, even if they work.)

What if P = NP?

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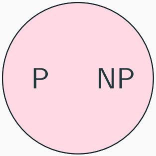
What if this is reality?



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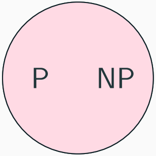

AND what if we have an efficient way of solving any NP-COMPLETE problem?

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