CSE 373: Graph traversal

Michael Lee
Friday, Feb 16, 2018
Goal: How do we traverse graphs?

Today’s goal: how do we traverse graphs?

Idea 1: Just get a list of the vertices and loop over them.
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**Idea 1:** Just get a list of the vertices and loop over them

**Problem:** What if we want to traverse graphs following the edges?

For example, can we...

▶ Traverse a graph to find if there’s a connection from one node to another?
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Problem: What if we want to traverse graphs following the edges? For example, can we...

- Traverse a graph to find if there’s a connection from one node to another?
- Determine if we can start from our node and touch every other node?
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For example, can we...

▶ Traverse a graph to find if there’s a connection from one node to another?
▶ Determine if we can start from our node and touch every other node?
▶ Find the shortest path between two nodes?
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- Traverse a graph to find if there’s a connection from one node to another?
- Determine if we can start from our node and touch every other node?
- Find the shortest path between two nodes?
Goal: How do we traverse graphs?

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Problem: What if we want to traverse graphs following the edges?

For example, can we...

▶ Traverse a graph to find if there’s a connection from one node to another?
▶ Determine if we can start from our node and touch every other node?
▶ Find the shortest path between two nodes?

Solution: Use graph traversal algorithms like breadth-first search and depth-first search
Breadth-first search (BFS) example

search(v):
    visited = empty set

queue.enqueue(v)
visited.add(v)

while (queue is not empty):
    curr = queue.dequeue()

    for (w : v.neighbors()):
        if (w not in visited):
            queue.enqueue(w)
            visited.add(curr)

Current node:
Queue: a,
Visited: a,
Breadth-first search (BFS) example

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```

Current node: a

Queue:

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while (queue is not empty):
    curr = queue.dequeue()

    for (w : v.neighbors()):
        if (w not in visited):
            queue.enqueue(w)
            visited.add(curr)

Current node: a
Queue: b, d,
Visited: a, b, d,
Breadth-first search (BFS) example

\[
\text{search}(v): \\
\quad \text{visited} = \text{empty set} \\
\text{queue}.\text{enqueue}(v) \\
\text{visited}.\text{add}(v) \\
\text{while (queue is not empty)}: \\
\quad \text{curr} = \text{queue}.\text{dequeue}() \\
\quad \text{for (w : v.neighbors())}: \\
\quad \quad \text{if (w not in visited)}: \\
\quad \quad \quad \text{queue}.\text{enqueue}(w) \\
\quad \quad \quad \text{visited}.\text{add}(\text{curr})
\]

Current node: b
Queue: d,
Visited: a, b, d,
Breadth-first search (BFS) example

search(v):
    visited = empty set

queue.enqueue(v)
visited.add(v)

while (queue is not empty):
    curr = queue.dequeue()

    for (w : v.neighbors()):
        if (w not in visited):
            queue.enqueue(w)
            visited.add(curr)

Current node: b
Queue: d, c, e,
Visited: a, b, d, c, e,
Breadth-first search (BFS) example

search(v):
    visited = empty set

    queue.enqueue(v)
    visited.add(v)

    while (queue is not empty):
        curr = queue.dequeue()

        for (w : v.neighbors()):
            if (w not in visited):
                queue.enqueue(w)
                visited.add(curr)

Current node: d

Queue: c, e,

Visited: a, b, d, c, e,
Breadth-first search (BFS) example

search(v):
    visited = empty set
    queue.enqueue(v)
    visited.add(v)

while (queue is not empty):
    curr = queue.dequeue()
    for (w : v.neighbors()):
        if (w not in visited):
            queue.enqueue(w)
            visited.add(curr)

Current node: d
Queue: c, e, f, g,
Visited: a, b, d, c, e, f, g,
Breadth-first search (BFS) example

search(v):
    visited = empty set

queue.enqueue(v)
visited.add(v)

while (queue is not empty):
    curr = queue.dequeue()
    for (w : v.neighbors()):
        if (w not in visited):
            queue.enqueue(w)
    visited.add(curr)

Current node: c
Queue: e, f, g,
Visited: a, b, d, c, e, f, g,
Breadth-first search (BFS) example

search(v):
  visited = empty set

queue.enqueue(v)
visited.add(v)

while (queue is not empty):
  curr = queue.dequeue()
  for (w : v.neighbors()):
    if (w not in visited):
      queue.enqueue(w)
      visited.add(curr)

Current node: e
Queue: f, g,
Visited: a, b, d, c, e, f, g,
Breadth-first search (BFS) example

search(v):
    visited = empty set

queue.enqueue(v)
visited.add(v)

while (queue is not empty):
    curr = queue.dequeue()

    for (w : v.neighbors()):
        if (w not in visited):
            queue.enqueue(w)
            visited.add(curr)

Current node: e
Queue: f, g, h,
Visited: a, b, d, c, e, f, g, h,
Breadth-first search (BFS) example

```
search(v):
    visited = empty set

queue.enqueue(v)
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while (queue is not empty):
    curr = queue.dequeue()
    for (w : v.neighbors()):
        if (w not in visited):
            queue.enqueue(w)
            visited.add(curr)
```

Current node: f

Queue: g, h,

Visited: a, b, d, c, e, f, g, h,
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search(v):
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    curr = queue.dequeue()

    for (w : v.neighbors()):
        if (w not in visited):
            queue.enqueue(w)
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Current node: g
Queue: h,
Visited: a, b, d, c, e, f, g, h,
Breadth-first search (BFS) example

search(v):
    visited = empty set

queue.enqueue(v)
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Current node: g
Queue: h, i,
Visited: a, b, d, c, e, f, g, h, i,
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search(v):
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for (w : v.neighbors()):
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    queue.enqueue(w)
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Current node: h
Queue: i,
Visited: a, b, d, c, e, f, g, h, i,
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            queue.enqueue(w)
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Current node: i

Queue:

Visited: a, b, d, c, e, f, g, h, i,
Breadth-first search (BFS)

Breadth-first traversal, core idea:

1. Use something (e.g. a queue) to keep track of every vertex to visit

   - Runtime:
     - We visit each node once.
     - For each node, check each edge to see if we should add to queue
     - So we check each edge at most twice

   - So, \( O(|V| + 2|E|) \), which simplifies to \( O(|V| + |E|) \).
Breadth-first traversal, core idea:

1. Use something (e.g. a queue) to keep track of every vertex to visit
2. Add and remove nodes from queue until it’s empty
Breadth-first search (BFS)

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1. Use something (e.g. a queue) to keep track of every vertex to visit
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3. Use a set to store nodes we don’t want to recheck/revisit

Runtime:

- We visit each node once.
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So, $O(|V| + 2|E|)$, which simplifies to $O(|V| + |E|)$. 
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Breadth-first search (BFS)

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   ▶ We visit each node once.
Breadth-first search (BFS)

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\[ O(|V| + |E|) \]

which simplifies to \[ O(|V| + |E|) \].
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   - We visit each node once.
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So, $\mathcal{O}(|V| + 2|E|)$, which simplifies to $\mathcal{O}(|V| + |E|)$. 
Breadth-first search (BFS)

**Pseudocode:**

```
search(v):
    visited = empty set

    queue.enqueue(v)
    visited.add(v)

    while (queue is not empty):
        curr = queue.dequeue()

        for (w : v.neighbors()):
            if (w not in visited):
                queue.enqueue(w)
                visited.add(curr)
```
Note: We visited the nodes in “rings” – maintained a gradually growing “frontier” of nodes.
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An interesting property...

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What does this look like for trees?
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What does this look like for trees?

The algorithm traverses the width, or “breadth” of the tree.
Depth-first search (DFS)

**Question:** Why a queue? Can we use other data structures?

---

The BFS algorithm:

```python
search(v):
    visited = empty set
    queue.enqueue(v)
    visited.add(v)
    while (queue is not empty):
        curr = queue.dequeue()
        for (w : v.neighbors()):
            if (w not in visited):
                queue.enqueue(w)
                visited.add(curr)
```

The DFS algorithm:

```python
search(v):
    visited = empty set
    stack.push(v)
    visited.add(v)
    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)
        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
                visited.add(v)
```
Question: Why a queue? Can we use other data structures?
Answer: Yes! Any kind of list-like thing that supports appends and removes works! For example, what if we try using a stack?
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**Answer:** Yes! Any kind of list-like thing that supports appends and removes works! For example, what if we try using a stack?

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**Depth-first search (DFS)**

**Question:** Why a queue? Can we use other data structures?

**Answer:** Yes! Any kind of list-like thing that supports appends and removes works! For example, what if we try using a stack?

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        curr = stack.pop()
        visited.add(curr)
        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
                visited.add(v)
```
Depth-first search (DFS) example

```python
search(v):
    visited = empty set
    stack.push(v)

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        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
```

Current node: a,
Stack: a,
Visited: a,
Depth-first search (DFS) example

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            if (w not in visited):
                stack.push(w)

Current node: a
Stack:
Visited: a,
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        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)

Current node: a
Stack: b, d,
Visited: a, b, d,
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)

Current node: d
Stack: b,
Visited: a, b, d,
Depth-first search (DFS) example

search(v):
    visited = empty set
    stack.push(v)

while (stack is not empty):
    curr = stack.pop()
    visited.add(curr)

    for (w : v.neighbors()):
        if (w not in visited):
            stack.push(w)

Current node: d
Stack: b, e, f, g
Visited: a, b, d, e, f, g,
Depth-first search (DFS) example

```python
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
```

Current node: g
Stack: b, e, f,
Visited: a, b, d, e, f, g,
Depth-first search (DFS) example

search(v):
    
    visited = empty set
    stack.push(v)

    while (stack is not empty):
    
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)

Current node: g
Stack: b, e, f, h, i,
Visited: a, b, d, e, f, g, h, i,
Depth-first search (DFS) example

search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)

Current node: i
Stack: b, e, f, h,
Visited: a, b, d, e, f, g, h, i,
Depth-first search (DFS) example

search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)

Current node: h
Stack: b, e, f,
Visited: a, b, d, e, f, g, h, i,
Depth-first search (DFS) example

search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)

Current node: f
Stack: b, e,
Visited: a, b, d, e, f, g, h, i, e,
Depth-first search (DFS) example

```python
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
```

Current node: e

Stack: b, e,

Visited: a, b, d, e, f, g, h, i, e,
Depth-first search (DFS) example

search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)

Current node: e
Stack: b, e, c,
Visited: a, b, d, e, f, g, h, i, e, c,
Depth-first search (DFS) example

search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)

Current node: c

Stack: b,

Visited: a, b, d, e, f, g, h, i, e, c,
search(v):
    visited = empty set
    stack.push(v)

while (stack is not empty):
    curr = stack.pop()
    visited.add(curr)

    for (w : v.neighbors()):
        if (w not in visited):
            stack.push(w)

Current node: b

Stack:

Visited: a, b, d, e, f, g, h, i, e, c,
Depth-first search (DFS) example

search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)

Current node:
Stack:
Visited: a, b, d, e, f, g, h, i, e, c,
Depth-first search (DFS)

Depth-first traversal, core idea:

1. Instead of using a queue, use a stack. Otherwise, keep everything the same.
Depth-first search (DFS)

Depth-first traversal, core idea:

1. Instead of using a queue, use a stack. Otherwise, keep everything the same.
2. Runtime: also $O(|V| + |E|)$ for same reasons as BFS
Depth-first search (DFS)

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1. Instead of using a queue, use a stack. Otherwise, keep everything the same.
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Pseudocode:

```python
search(v):
    visited = empty set
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        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
                visited.add(curr)
```
An interesting property...

**Note:** Rather the growing the node in “rings”, we randomly wandered through the graph until we got stuck, then “backtracked”.

![Graph Diagram](image-url)
An interesting property...

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![Graph Diagram]

- [a]
- [b]
- [c]
- [d]
- [e]
- [f]
- [g]
- [h]
- [i]
- [j]
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![Graph diagram](image)
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![Diagram of a graph with nodes and edges]
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```
+---+  +---+
    |  |   |
  +---+  +---+

```

```
  +---+  +---+  +---+  +---+  +---+
    |  |   |   |   |   |   |
  +---+  +---+  +---+  +---+  +---+

```

```
  f---g---h---i---d---a---j
```

```
  e---b---c
```
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![Graph Diagram]

- Nodes: a, b, c, d, e, f, g, h, i, j
- Edges: Various connections between nodes
- Key: Node j is highlighted or marked in some way
**Note:** Rather the growing the node in “rings”, we randomly wandered through the graph until we got stuck, then “backtracked”.
An interesting property...

What does this look like for trees?
An interesting property...

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The algorithm traverses to the bottom first: it prioritizes the “depth” of the tree.
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Note: rest of algorithm omitted
Question: When do we use BFS vs DFS?

- **BFS:** $O(|V|)$ – what if every node is connected to the start?
- **DFS:** $O(|V|)$ – what if the nodes are arranged like a linked list?

So, in the worst case, BFS and DFS both have the same worst-case runtime and memory usage. They only differ in what order they visit the nodes.
Question: When do we use BFS vs DFS?

Related question: How much memory does BFS and DFS use in the worst case?

- **BFS:**
- **DFS:**

![Diagram of BFS and DFS]
Compare and contrast

**Question:** When do we use BFS vs DFS?

Related question: How much memory does BFS and DFS use in the worst case?

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- DFS: $O(|V|)$

So, in the worst case, BFS and DFS both have the same worst-case runtime and memory usage.

They only differ in what order they visit the nodes.
Compare and contrast

How much memory does BFS and DFS use in the *average* case?

- **BFS:** \(O(\text{"width" of tree}) = O(\text{num leaves})\)
- **DFS:** \(O(\text{height})\)

For graphs:
- Use BFS if graph is "narrow", or if solution is "near" start
- Use DFS if graph is "wide"

In practice, graphs are often large/very wide, so DFS is often a good default choice. (It's also possible to implement DFS recursively!)
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Related question: how much memory do they use when we want to traverse a tree?
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**Question:** How would you modify BFS to find the shortest path between every node?

Diagram:
```
  S   E
  |   |
 a — b — w — x — y — z
```

Observation: Since BFS moves out in rings, we will reach the end node via the path of length 3 first.

Idea: When we enqueue, store where we came from in some way. (e.g. mark node, use a dictionary...) After BFS is done, backtrack.
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Design challenge: pathfinding

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Now, start from any node, follow arrows, then reverse to get path.
Question: What if the edges have weights?

Weighted graph

A weighted graph is a kind of graph where each edge has a numerical "weight" associated with it. This number can represent anything, but is often (but not always!) used to indicate the "cost" of traveling down that edge.
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**Design challenge: pathfinding**

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![Weighted graph](image)

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We need a better algorithm.
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Today: **Dijkstra’s algorithm**
Dijkstra’s algorithm

Core idea:

1. Assign each node an initial cost of \( \infty \)
Dijkstra’s algorithm

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2. Set our starting node’s cost to 0

Metaphor:

Treat edges as canals and edge weights as distance. Imagine opening a dam at the starting node. How long does it take for the water to reach each vertex?

Caveat:

Dijkstra’s algorithm only guaranteed to work for graphs with no negative edge weights.

Pronunciation:

DYKE-struh (“dijk” rhymes with “bike”)
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Suppose we start at vertex “a”:

![Graph Diagram](image-url)
Dijkstra’s algorithm

Some implementation details...

▶ How do we keep track of the node costs?

- Could use a dictionary
- Could manually mark each node
- How do we find the node with the smallest cost?
  - Could maintain a sorted list
  - Could use a heap!
  - If we’re using a heap, how do we update node costs?
    - Could add a `changeKeyPriority(...)` method to heap
    - Alternatively, add the node and the cost to the heap again (and ignore duplicates)
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