CSE 373: Graph traversal

Michael Lee

Friday, Feb 16, 2018

Goal: How do we traverse graphs?

Today's goal: how do we traverse graphs?

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- Traverse a graph to find if there's a connection from one node to another?

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For example, can we...

- Traverse a graph to find if there's a connection from one node to another?
- Determine if we can start from our node and touch every other node?
- Find the shortest path between two nodes?

Solution: Use graph traversal algorithms like breadth-first search and depth-first search

Breadth-first search (BFS) example

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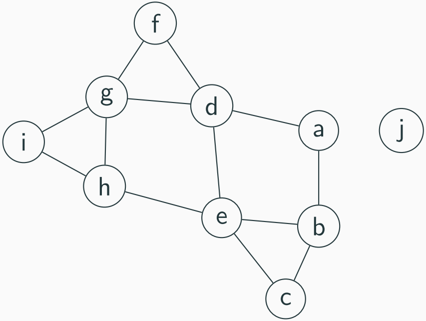
search(v):
    visited = empty set

    queue.enqueue(v)
    visited.add(v)

    while (queue is not empty):
        curr = queue.dequeue()

        for (w : v.neighbors()):
            if (w not in visited):
                queue.enqueue(w)
                visited.add(curr)
```



Current node:

Queue: a,

Visited: a,

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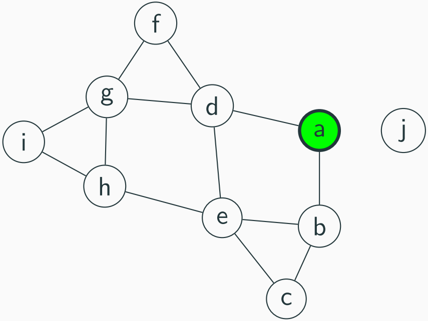

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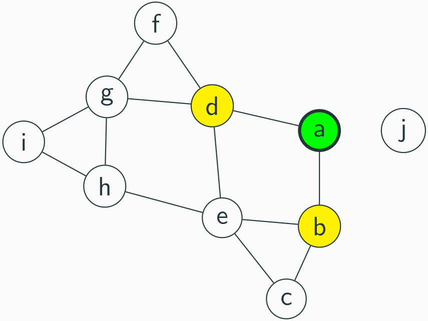

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Queue: b, d,

Visited: a, b, d,

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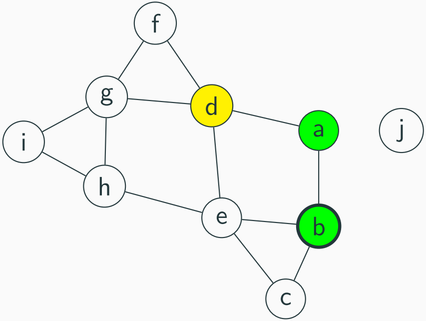

Current node: b

Queue: d,

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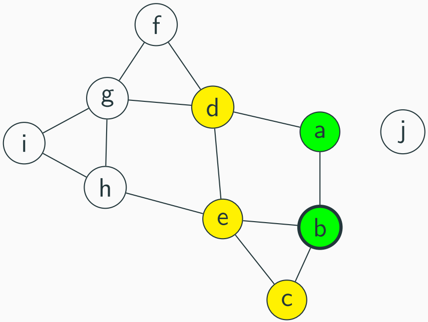

Current node: b

Queue: d, c, e,

Visited: a, b, d, c, e,

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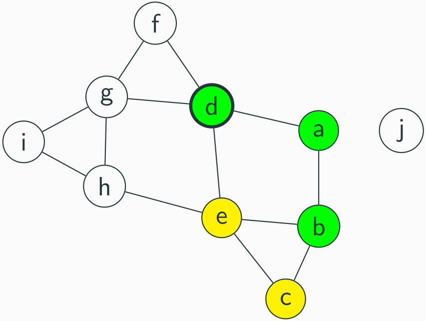

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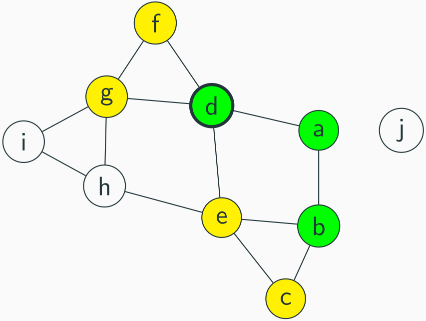

Current node: d

Queue: c, e, f, g,

Visited: a, b, d, c, e, f, g,

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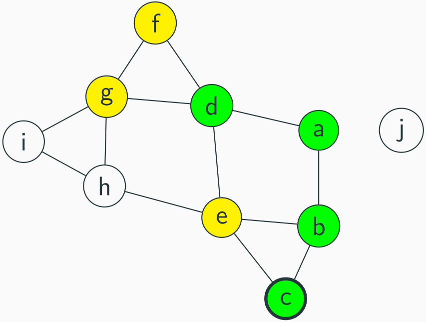

Current node: c

Queue: e, f, g,

Visited: a, b, d, c, e, f, g,

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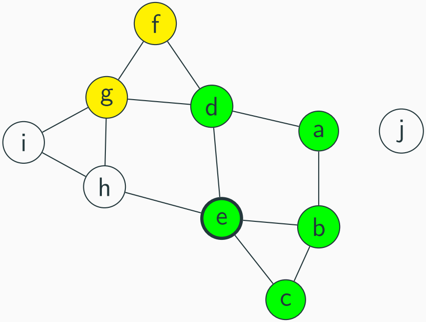

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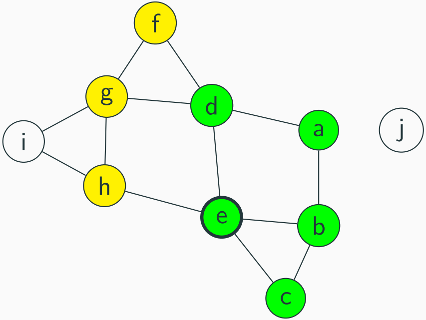

Current node: e

Queue: f, g, h,

Visited: a, b, d, c, e, f, g, h,

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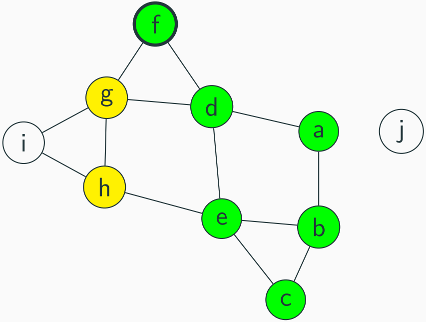

Current node: f

Queue: g, h,

Visited: a, b, d, c, e, f, g, h,

Breadth-first search (BFS) example

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search(v):
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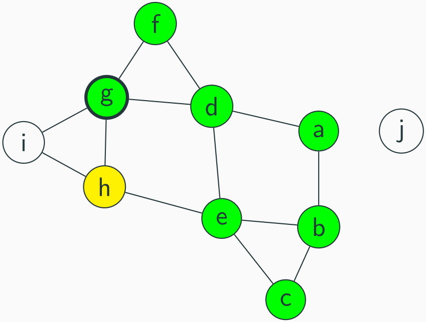

Current node: g

Queue: h,

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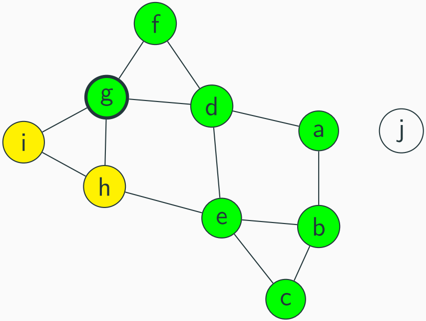

Current node: g

Queue: h, i,

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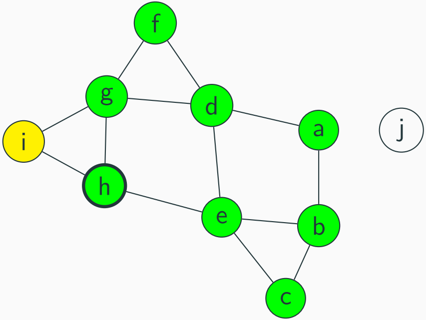

Current node: h

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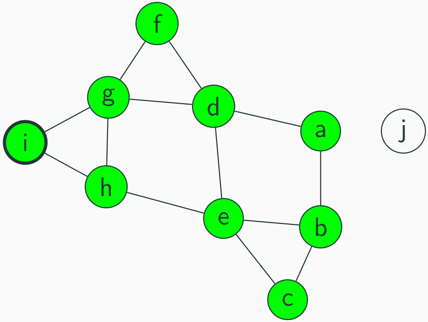

Current node: i

Queue:

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Breadth-first traversal, core idea:

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    So, $\mathcal{O}(|V| + 2|E|)$, which simplifies to $\mathcal{O}(|V| + |E|)$.

Breadth-first search (BFS)

Pseudocode:

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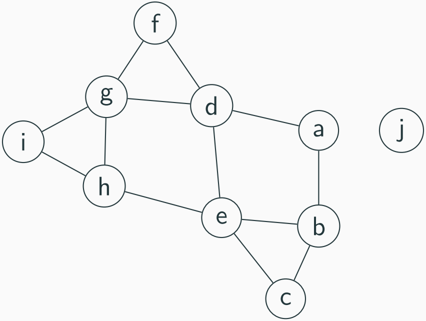
An interesting property...

Note: We visited the nodes in “rings” – maintained a gradually growing “frontier” of nodes.



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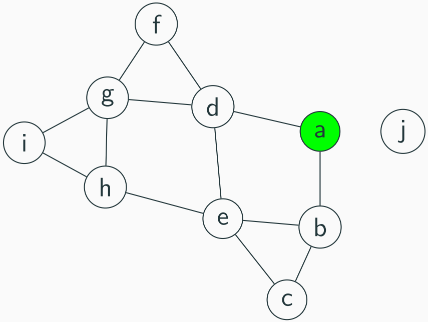

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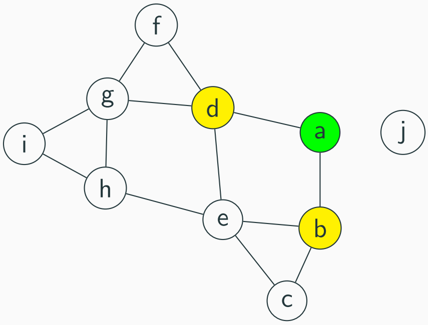

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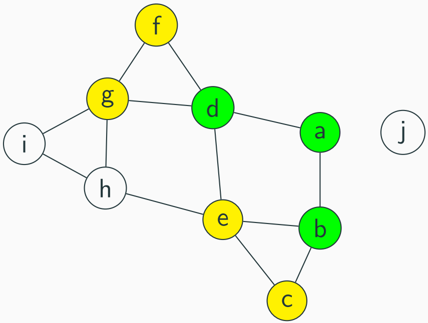

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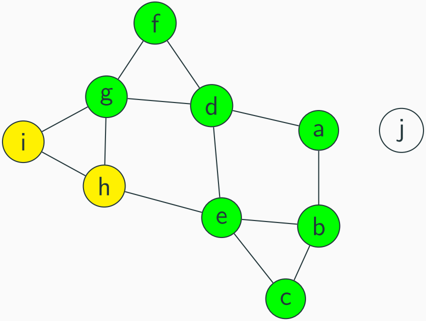

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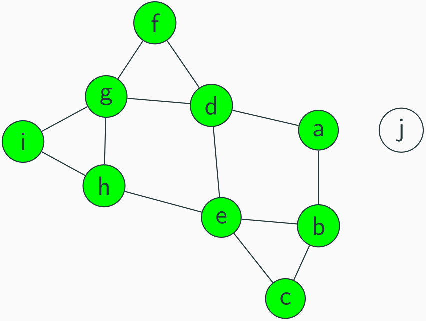

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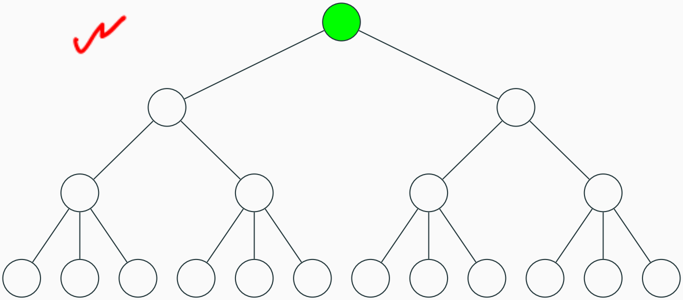
What does this look like for trees?



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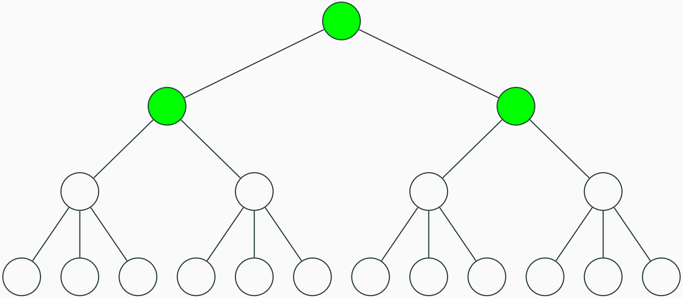

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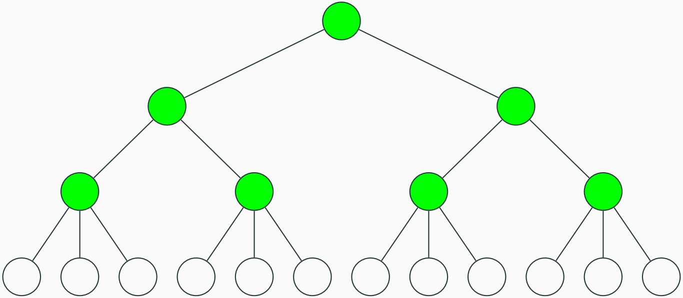

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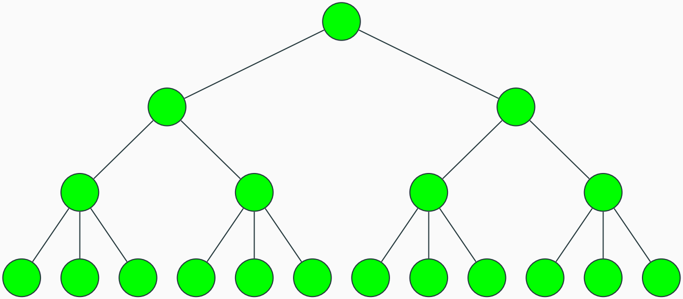

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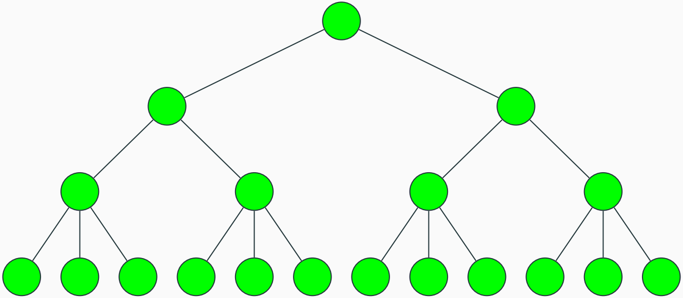

The algorithm traverses the width, or “breadth” of the tree

Depth-first search (DFS)

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```

The DFS algorithm:

```
search(v):
    visited = empty set
    stack.push(v)
    visited.add(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
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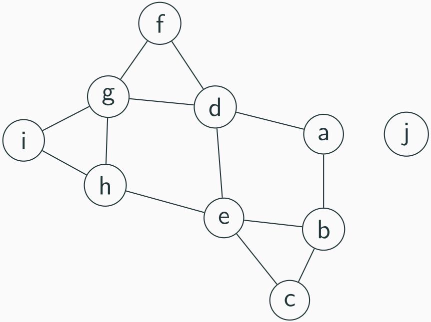

Current node:

Stack: a,

Visited: a,

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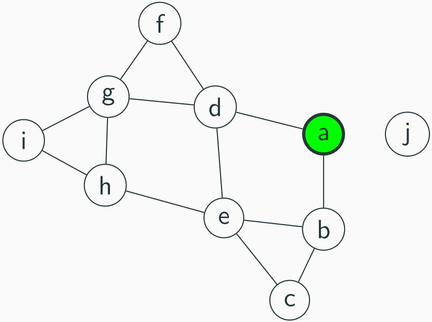

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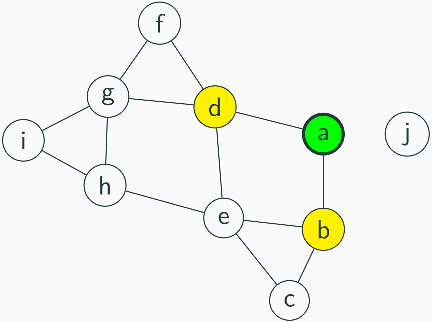

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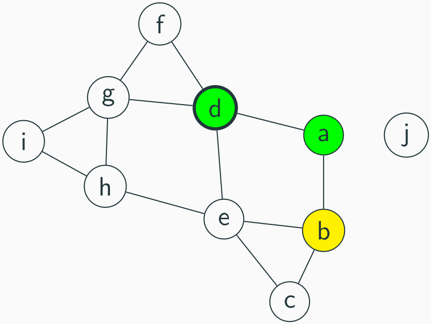


Current node: d

Stack: b,

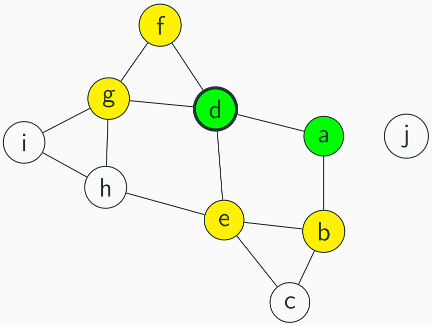
Visited: a, b, d,

Depth-first search (DFS) example

```
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
```



Current node: d

Stack: b, e, f, g,

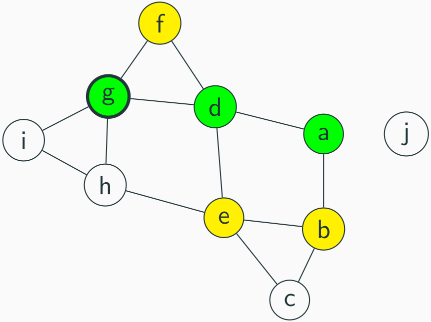
Visited: a, b, d, e, f, g,

Depth-first search (DFS) example

```
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
```



Current node: g

Stack: b, e, f,

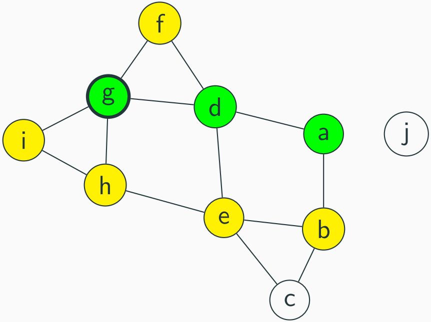
Visited: a, b, d, e, f, g,

Depth-first search (DFS) example

```
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
```



Current node: g

Stack: b, e, f, h, i,

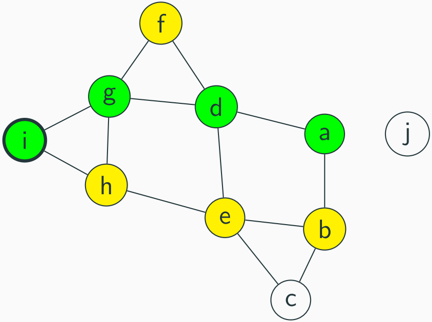
Visited: a, b, d, e, f, g, h, i,

Depth-first search (DFS) example

```
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
```



Current node: i

Stack: b, e, f, h,

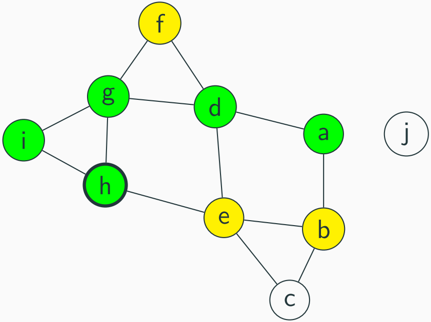
Visited: a, b, d, e, f, g, h, i,

Depth-first search (DFS) example

```
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
```



Current node: h

Stack: b, e, f,

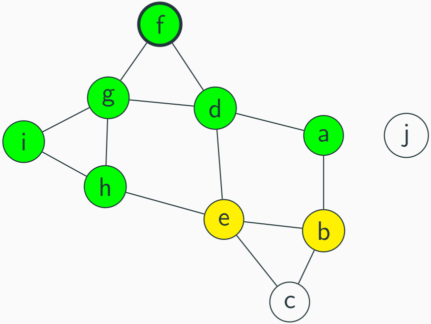
Visited: a, b, d, e, f, g, h, i,

Depth-first search (DFS) example

```
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
```



Current node: f

Stack: b, e,

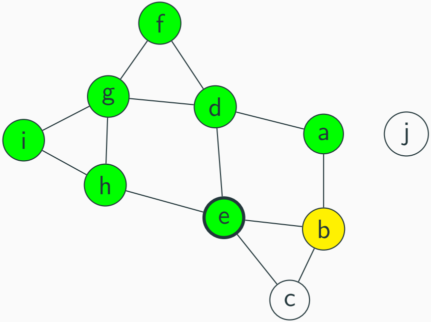
Visited: a, b, d, e, f, g, h, i, e,

Depth-first search (DFS) example

```
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
```



Current node: e

Stack: b, e,

Visited: a, b, d, e, f, g, h, i, e,

Depth-first search (DFS) example

```
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
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        visited.add(curr)

        for (w : v.neighbors()):
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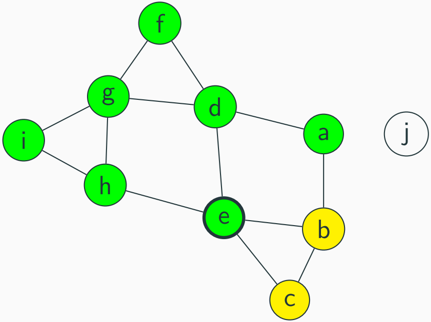
                stack.push(w)
```



Current node: e

Stack: b, e, c,

Visited: a, b, d, e, f, g, h, i, e, c,

Depth-first search (DFS) example

```
search(v):
    visited = empty set
    stack.push(v)

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        for (w : v.neighbors()):
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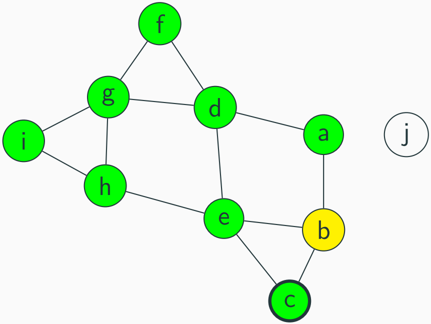

Current node: c

Stack: b,

Visited: a, b, d, e, f, g, h, i, e, c,

Depth-first search (DFS) example

```
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
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                stack.push(w)
```
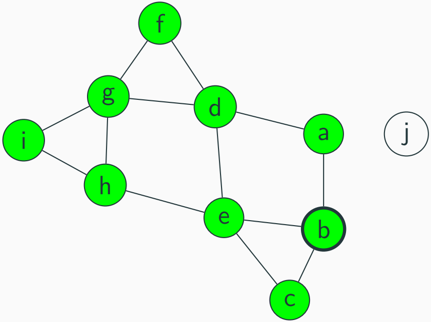



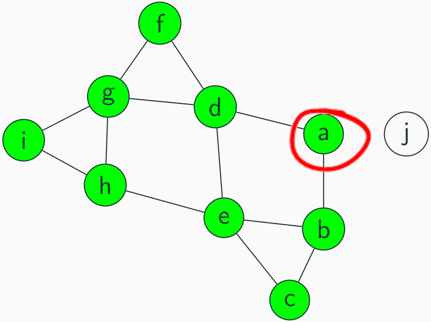
Current node: b

Stack:

Visited: a, b, d, e, f, g, h, i, e, c,

Depth-first search (DFS) example

```
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
```



Current node:

Stack:

Visited: a, b, d, e, f, g, h, i, e, c,

Depth-first traversal, core idea:

1. Instead of using a queue, use a stack. Otherwise, keep everything the same.

Depth-first search (DFS)

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Depth-first traversal, core idea:

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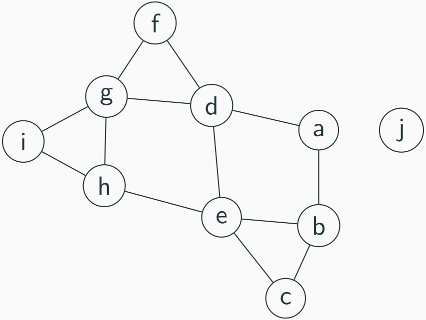
Pseudocode:

```
search(v):
    visited = empty set

    stack.push(v)
    visited.add(v)

    while (stack is not empty):
        curr = stack.pop()

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
                visited.add(curr)
```

An interesting property...

Note: Rather the growing the node in “rings”, we randomly wandered through the graph until we got stuck, then “backtracked”.



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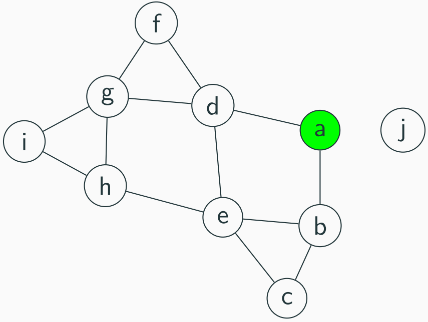

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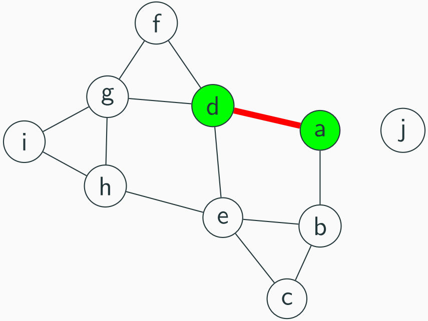

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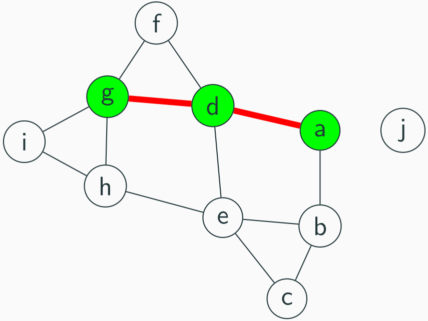

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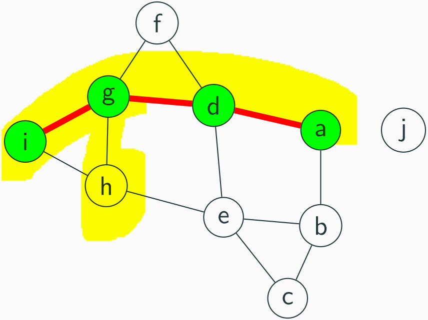

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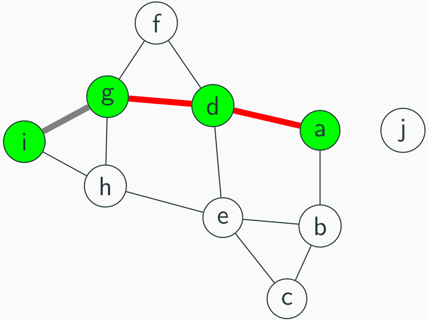

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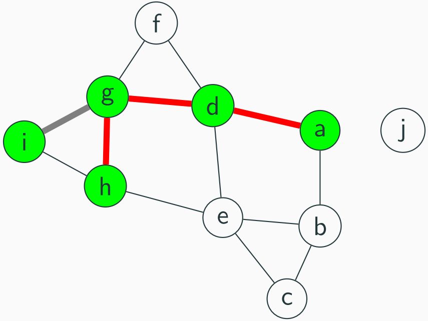

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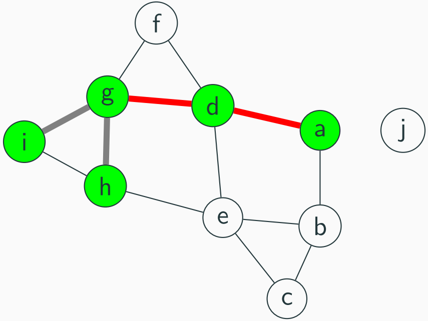

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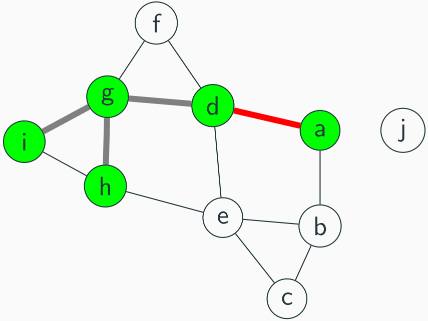

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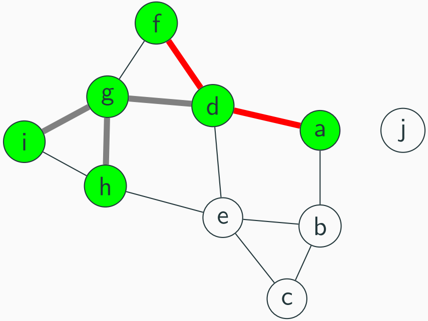

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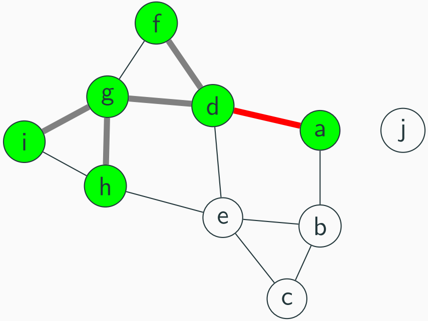

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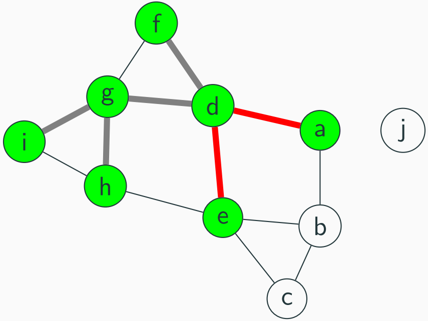

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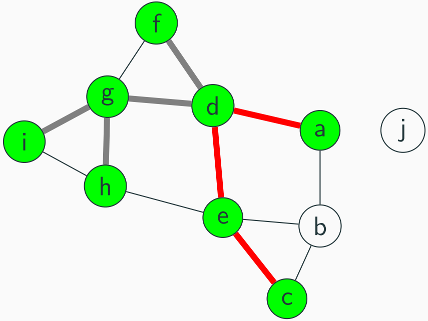

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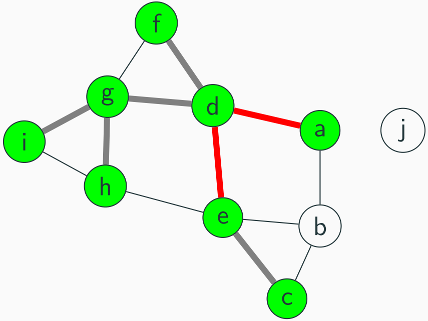

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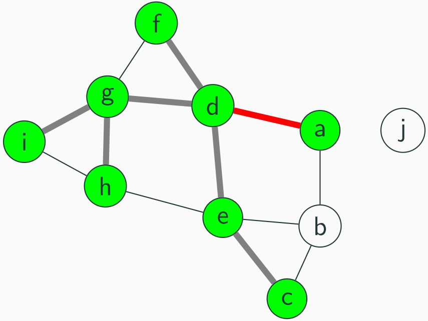

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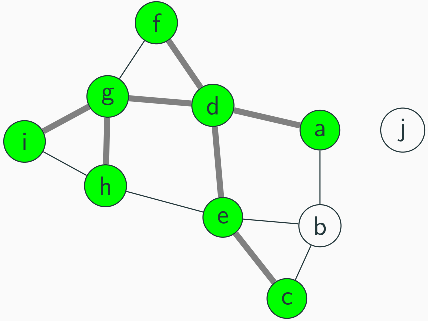

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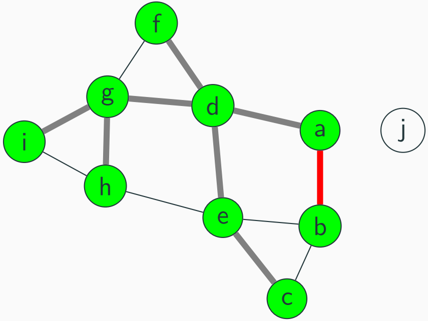

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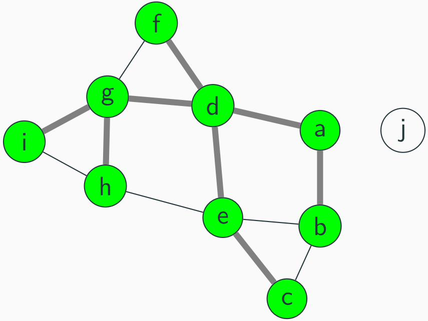

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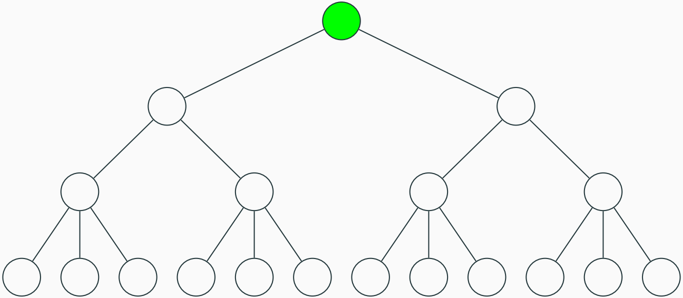
What does this look like for trees?



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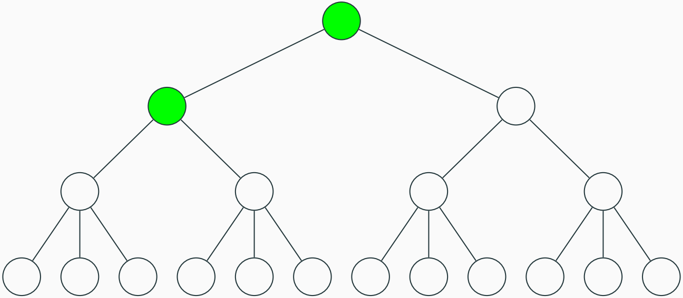

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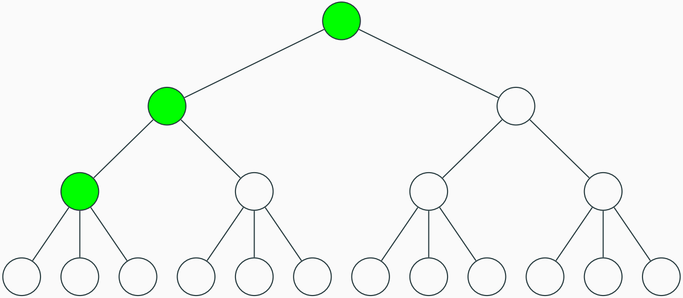

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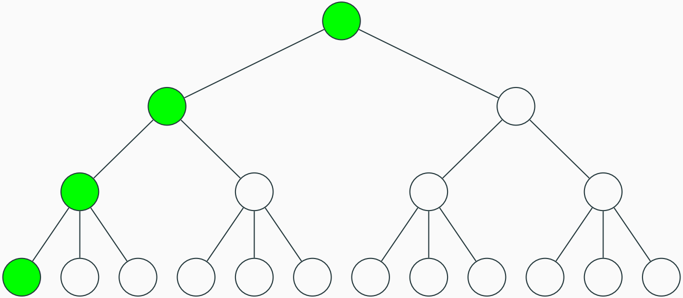

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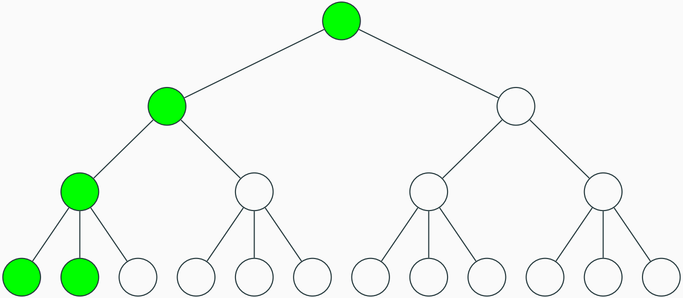

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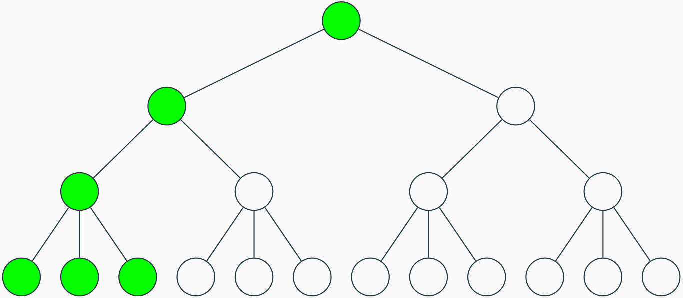

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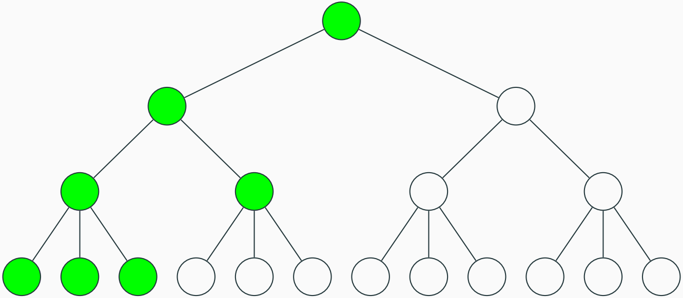

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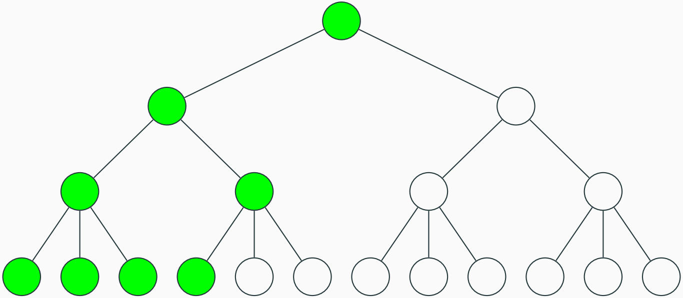

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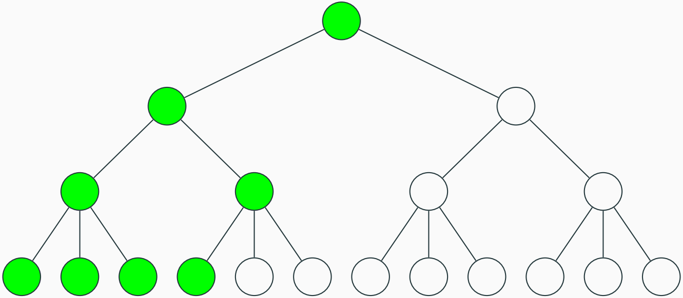

The algorithm traverses to the bottom first: it prioritizes the "depth" of the tree

Note: rest of algorithm omitted

Compare and contrast

Question: When do we use BFS vs DFS?

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Related question: How much memory does BFS and DFS use in the worst case?

- BFS:
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orst possible
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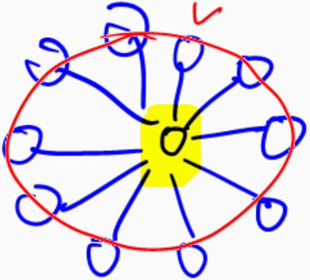

$|V|$

Compare and contrast

Question: When do we use BFS vs DFS?

Related question: How much memory does BFS and DFS use in the worst case?

- BFS: $\mathcal{O}(|V|)$ – what if every node is connected to the start?
- DFS: $\mathcal{O}(|V|)$

So, in the worst case, BFS and DFS both have the same worst-case runtime and memory usage.

They only differ in what order they visit the nodes.

Compare and contrast

How much memory does BFS and DFS use in the **average** case?

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For graphs:

- Use BFS if graph is "narrow", or if solution is "near" start
- Use DFS if graph is "wide"

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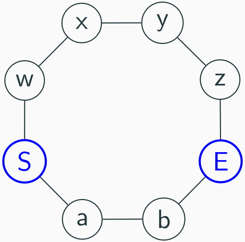
For graphs:

- Use BFS if graph is "narrow", or if solution is "near" start
- Use DFS if graph is "wide"

In practice, graphs are often large/very wide, so DFS is often a good default choice. (It's also possible to implement DFS recursively!)

Design challenge

Question: How would you modify BFS to find the shortest path between every node?



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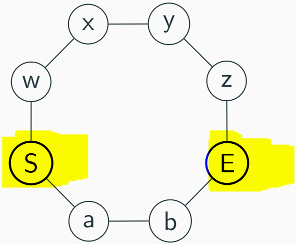

Observation: Since BFS moves out in rings, we will reach the end node via the path of length 3 first.

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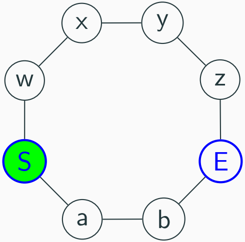

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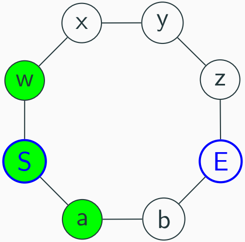

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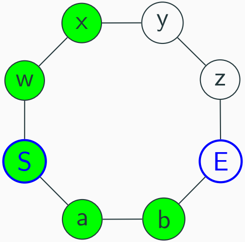

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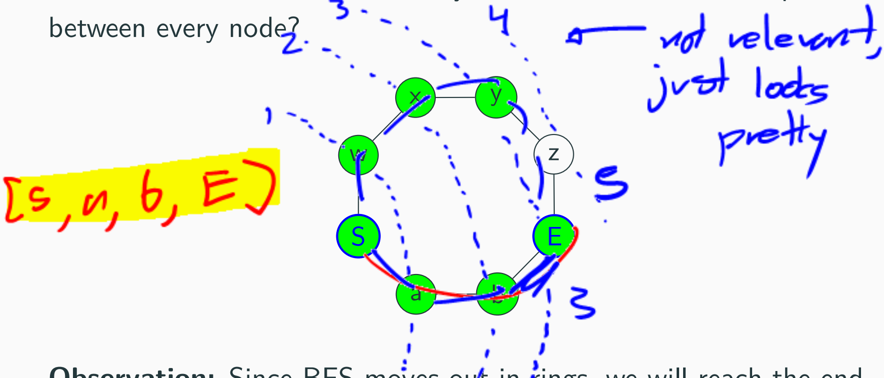

Observation: Since BFS moves out in rings, we will reach the end node via the path of length 3 first.

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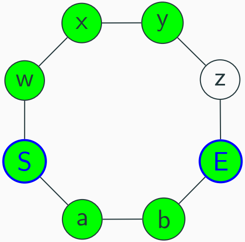

Observation: Since BFS moves out in rings, we will reach the end node via the path of length 3 first.

Idea: when we enqueue, store where we came from in some way. (e.g. mark node, use a dictionary...)

After BFS is done, *backtrack*.

Design challenge: pathfinding

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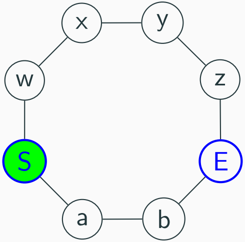

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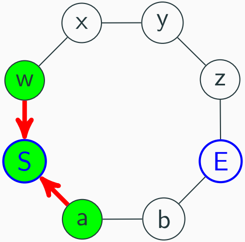

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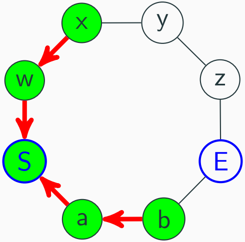

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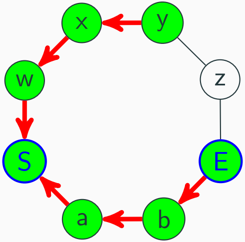

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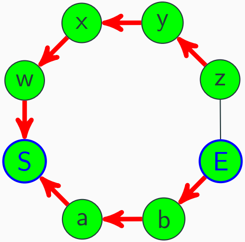

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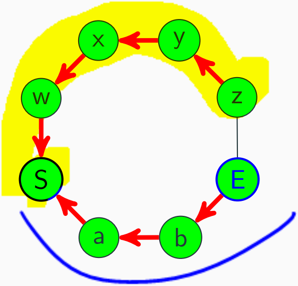
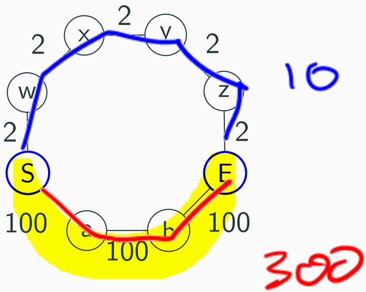


Now, start from any node, follow arrows, then reverse to get path.

Design challenge: pathfinding

Question: What if the edges have weights?



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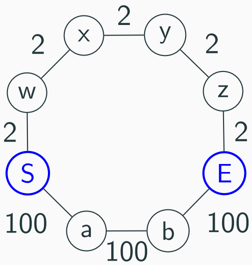

Weighted graph

A **weighted graph** is a kind of graph where each edge has a numerical “weight” associated with it.

Design challenge: pathfinding

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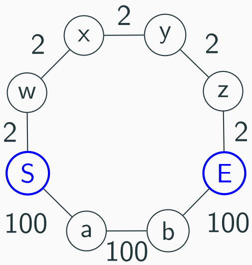

Weighted graph

A **weighted graph** is a kind of graph where each edge has a numerical "weight" associated with it.

This number can represent anything, but is often (but not always!) used to indicate the "cost" of traveling down that edge.

Pathfinding and DFS

We can use BFS to correctly find the shortest path between two nodes in an unweighted graph...

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...but it fails if the graph is weighted!

We need a better algorithm.

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Today: **Dijkstra's algorithm**

Dijkstra's algorithm

Core idea:

1. Assign each node an initial cost of ∞

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Metaphor: Treat edges as canals and edge weights as distance. Imagine opening a dam at the starting node. How long does it take for the water to reach each vertex?

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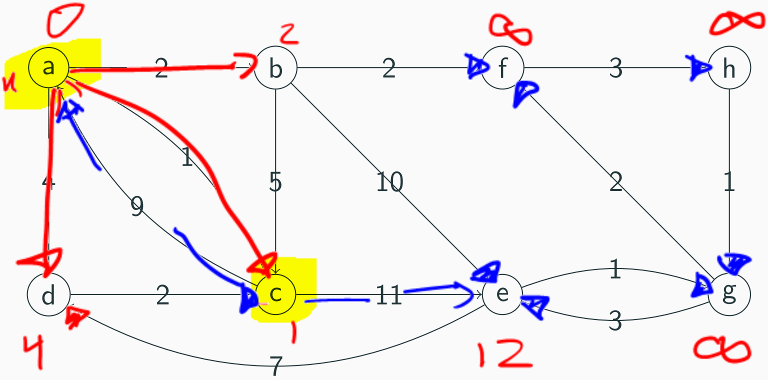
Pronunciation: DYKE-struh ("dijk" rhymes with "bike")

Dijkstra's algorithm

Suppose we start at vertex "a":



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Some implementation details...

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  - Could maintain a sorted list
  - Could use a heap!

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  - Could manually mark each node
- How do we find the node with the smallest cost?
  - Could maintain a sorted list
  - Could use a heap!
- If we're using a heap, how do we update node costs?
  - Could add a `changeKeyPriority(...)` method to heap
  - Alternatively, add the node and the cost to the heap again (and ignore duplicates)