CSE 373: More on graphs; DFS and BFS

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Wednesday, Feb 14, 2018
Warmup:

Discuss with your neighbor:

- Remind your neighbor: what is a simple graph?
  - A simple graph has no...
    1. self-loops and 2. parallel edges

- Suppose we have a simple, directed graph with \( x \) nodes. What is the maximum number of edges it can have, in terms of \( x \)?
  - \( x(x-1) \)

- Now, suppose we have a different simple, undirected graph with \( y \) edges. What is the maximum number of vertices it can have, in terms of \( y \)?
  - \( \infty \)
Warmup:

Discuss with your neighbor:

- Remind your neighbor: what is a *simple graph*? A simple graph is a graph that has no self-loops and no parallel edges.
- Suppose we have a *simple, directed graph* with $x$ nodes. What is the maximum number of edges it can have, in terms of $x$? Each vertex can connect to $x - 1$ other vertices, so $x(x - 1)$.
- Now, suppose we have a different simple, undirected graph with $y$ edges. What is the maximum number of vertices it can have, in terms of $y$? Infinite: just keep adding nodes with no edges attached.
Warmup:

Some follow-up questions:

▶ Suppose we have a simple, undirected graph with \( x \) nodes. What is the maximum number of edges it can have? What if the graph is not simple?

If the graph is simple, the maximum number of edges is exactly half of what it would be if the graph were directed. So, \( \frac{x(x-1)}{2} \). If the graph is not simple, it's infinite: assuming \( x > 0 \), we can just keep adding more and more self-loops. Note that if \( x = 0 \), there can't be any edges at all.

▶ Now, suppose we have a different simple, undirected graph with \( y \) edges. What is the maximum number of vertices it can have? What if the graph is not simple?
Warmup:

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▶ Suppose we have a simple, undirected graph with \( x \) nodes. What is the maximum number of edges it can have? What if the graph is not simple?
If the graph is simple, the max number of edges is exactly half of what it would be if the graph were directed. So, \( \frac{x(x-1)}{2} \).
If the graph is not simple, it’s infinite: assuming \( x > 0 \), we can just keep adding more and more self-loops. Note that if \( x = 0 \), there can’t be any edges at all.

▶ Now, suppose we have a different simple, undirected graph with \( y \) edges. What is the maximum number of vertices it can have? What if the graph is not simple?
Either way, it’s still infinite, for the same reasons given previously.
Summary

What did we learn?

- In graphs with no restrictions, number of edges and number of vertices are independent.
What did we learn?

▶ In graphs with no restrictions, number of edges and number of vertices are independent.

▶ In simple graphs, if we know $|V|$ is some fixed value, we also know $|E| \in \mathcal{O}(|V|^2)$, for both directed and undirected graphs.
What did we learn?

- In graphs with no restrictions, number of edges and number of vertices are independent.
- In simple graphs, if we know $|V|$ is some fixed value, we also know $|E| \in \mathcal{O}(|V|^2)$, for both directed and undirected graphs.

**Dense graph**

If $|E| \in \Theta(|V|^2)$, we say the graph is **dense**.

To put it another way, dense graphs have “lots of edges”
Summary

What did we learn?

- In graphs with no restrictions, number of edges and number of vertices are independent.
- In simple graphs, if we know $|V|$ is some fixed value, we also know $|E| \in \Theta(|V|^2)$, for both directed and undirected graphs.

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<tr>
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How do we represent graphs in code?

So, how do we actually represent graphs in code?
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So, how do we actually represent graphs in code?

Two common approaches, with different tradeoffs:

- Adjacency matrix
- Adjacency list
Core idea:

▶ Assign each node a number from 0 to $|V| - 1$
▶ Create a $|V| \times |V|$ nested array of booleans or ints
▶ If $(x, y) \in E$, then $\text{nestedArray}[x][y] == \text{true}$
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Adjacency Matrix

What is the worst-case runtime to:

▸ Get out-edges: \(O(|V|)\)
▸ Get in-edges: \(O(|V|)\)
▸ Decide if an edge exists: \(O(1)\)
▸ Insert an edge: \(O(1)\)
▸ Delete an edge: \(O(1)\)

How much space do we use?

\(O(|V|^2)\)

Is this better for sparse or dense graphs?

Can we handle self-loops and parallel edges?

Self-loops yes, parallel edges, not easily
What is the worst-case runtime to:

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Adjacency list

Core idea:

- Assign each node a number from 0 to $|V| - 1$
- Create an array of size $|V|$
- Each element in the array stores its out edges in a list or set
- On a higher level: represent as `IDictionary<Vertex, Edges>`.
We can store edges using either sets or lists. Answer these questions for both.

What is the worst-case runtime to:

- Get out-edges: $O(1)$
- Get in-edges: $O(|V|+|E|)$
- Decide if an edge exists: $O(|E|)$
- Insert an edge: $O(1)$
- Delete an edge: $O(1)$

How much space do we use?

Is this better for sparse or dense graphs?

Can we handle self-loops and parallel edges?
Which do we pick?

So which do we pick?

Observations:
▶ Most graphs are sparse
▶ If we implement adjacency lists using sets, we can get comparable worst-case performance

So by default, pick adjacency lists.
So which do we pick?

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So by default, pick adjacency lists.
Walk

A **walk** is a list of vertices $v_0, v_1, v_2, \ldots, v_n$ where if $i$ is some int where $0 \leq i < v_n$, every pair $(v_i, v_{i+1}) \in E$ is true.

More intuitively, a walk is one continuous line following the edges.
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Path

A path is a walk that never visits the same vertex twice.
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**Path**

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# Walks and paths

## Walk

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## Path

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---

Walk

![Walk](image1.png)

Path

![Path](image2.png)
Connected graph

A graph is **connected** if every vertex is connected to every other vertex via some path.

E.g.: if we pick up the graph and shake it, nothing flies off.
Connected components

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Connected or not connected?

Connected or not connected?
Connected components

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E.g.: if we pick up the graph and shake it, nothing flies off.

![Connected and Not Connected Graphs](image)

**Connected component**
A connected component of a graph is any subgraph (part of a graph) where all vertices are connected to each other.

Note: A connected graph has only one connected component.
Trees vs graphs

Is this a graph or tree?

![Tree Diagram]

- a
  - b
    - c
    - d
  - e
    - f
    - g

Both!

Is this the same thing?

Yes! (If 'a' is the root...)

Tree

A tree is a connected and acyclic graph.

Rooted tree

A rooted tree is a tree where we call one special node the “root”.
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Detecting if a graph is connected

**Question:** How can we tell if a graph is connected or not?
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Idea: Let’s just find out! Pick a node and see if there’s a path to every other node!
Breadth-first search (BFS)

Breadth-first traversal, core idea:

1. Pick some node and “mark” it (or save it in a set, etc...)

Pseudocode, version 1:
s
search(v):
    visited = empty set
    queue.enqueue(v)
    while (queue is not empty):
        curr = queue.dequeue()
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4. Keep going until the data structure is empty.

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Current node:
Queue: a,
Breadth-first search (BFS) example

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Current node: a

Queue:
Breadth-first search (BFS) example

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search(v):
    queue.enqueue(v)
    
    while (queue is not empty):
        curr = queue.dequeue()
        for (w : v.neighbors()):
            queue.enqueue(w)
```

Current node: a
Queue: b, d,
Breadth-first search (BFS) example

```
search(v):
    queue.enqueue(v)

    while (queue is not empty):
        curr = queue.dequeue()
        for (w : v.neighbors()):
            queue.enqueue(w)
```

Current node: b
Queue: d,
Breadth-first search (BFS) example

```
search(v):
    queue.enqueue(v)

    while (queue is not empty):
        curr = queue.dequeue()
        for (w : v.neighbors()):
            queue.enqueue(w)
```

Current node: b
Queue: d, a, b, e,
Breadth-first search (BFS) example

search(v):
    queue.enqueue(v)

    while (queue is not empty):
        curr = queue.dequeue()
        for (w : v.neighbors()):
            queue.enqueue(w)

Current node: d
Queue: a, b, e,
Breadth-first search (BFS) example

search(v):
    queue.enqueue(v)

    while (queue is not empty):
        curr = queue.dequeue()
        for (w : v.neighbors()):
            queue.enqueue(w)

    Current node: d
    Queue: a, b, e, f, g,
Breadth-first search (BFS) example

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  queue.enqueue(v)

  while (queue is not empty):
    curr = queue.dequeue()
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Current node: a
Queue: b, e, f, g,
Breadth-first search (BFS) example

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search(v):
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```

Current node: a

Queue: e, f, g,

What went wrong?
A broken traversal

**Problem:** We’re re-visiting nodes we already visited!
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**A fix:** Keep track of nodes we’ve already visited in a set!
Breadth-first search (BFS) example

search(v):
    visited = empty set
    queue.enqueue(v)
    visited.add(curr)

    while (queue is not empty):
        curr = queue.dequeue()

        for (w : v.neighbors()):
            if (w not in visited):
                queue.enqueue(w)
                visited.add(curr)

Current node:

Queue: a,

Visited: a,
Breadth-first search (BFS) example

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Current node: a
Queue:
Visited: a,
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for (w : v.neighbors()):
  if (w not in visited):
    queue.enqueue(w)
    visited.add(curr)

Current node: a
Queue: b, d,
Visited: a, b, d,
Breadth-first search (BFS) example

search(v):
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  queue.enqueue(v)
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        queue.enqueue(w)
        visited.add(curr)

Current node: b
Queue: d,
Visited: a, b, d,
Breadth-first search (BFS) example

search(v):
    visited = empty set
    queue.enqueue(v)
    visited.add(curr)

    while (queue is not empty):
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            if (w not in visited):
                queue.enqueue(w)
                visited.add(curr)

Current node: b
Queue: d, c, e,
Visited: a, b, d, c, e,
Breadth-first search (BFS) example

search(v):
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    for (w : v.neighbors()):
        if (w not in visited):
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Current node: d
Queue: c, e,
Visited: a, b, d, c, e,
Breadth-first search (BFS) example

search(v):
  visited = empty set

queue.enqueue(v)
visited.add(curr)

while (queue is not empty):
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  for (w : v.neighbors()):
    if (w not in visited):
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      visited.add(curr)

Current node: d
Queue: c, e, f, g,
Visited: a, b, d, c, e, f, g,
Breadth-first search (BFS) example

search(v):
    visited = empty set

queue.enqueue(v)
visited.add(curr)

while (queue is not empty):
    curr = queue.dequeue()

    for (w : v.neighbors()):
        if (w not in visited):
            queue.enqueue(w)
            visited.add(curr)

Current node: c
Queue: e, f, g,
Visited: a, b, d, c, e, f, g,
Breadth-first search (BFS) example

search(v):
  visited = empty set

queue.enqueue(v)
visited.add(curr)

while (queue is not empty):
  curr = queue.dequeue()

  for (w : v.neighbors()):
    if (w not in visited):
      queue.enqueue(w)
      visited.add(curr)

Current node: e
Queue: f, g,
Visited: a, b, d, c, e, f, g,
Breadth-first search (BFS) example

```
search(v):
    visited = empty set

queue.enqueue(v)
visited.add(curr)

while (queue is not empty):
    curr = queue.dequeue()
    for (w : v.neighbors()):
        if (w not in visited):
            queue.enqueue(w)
            visited.add(curr)
```

Current node: e
Queue: f, g, h,
Visited: a, b, d, c, e, f, g, h,
Breadth-first search (BFS) example

search(v):
    visited = empty set

queue.enqueue(v)
visited.add(curr)

while (queue is not empty):
    curr = queue.dequeue()

    for (w : v.neighbors()):
        if (w not in visited):
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Current node: f
Queue: g, h,
Visited: a, b, d, c, e, f, g, h,
Breadth-first search (BFS) example

search(v):
  visited = empty set

queue.enqueue(v)
visited.add(curr)

while (queue is not empty):
curr = queue.dequeue()

for (w : v.neighbors()):
  if (w not in visited):
    queue.enqueue(w)
    visited.add(curr)

Current node: g
Queue: h,
Visited: a, b, d, c, e, f, g, h,
Breadth-first search (BFS) example

search(v):
  visited = empty set

queue.enqueue(v)
visited.add(curr)

while (queue is not empty):
curr = queue.dequeue()

for (w : v.neighbors()):
  if (w not in visited):
    queue.enqueue(w)
    visited.add(curr)

Current node: g
Queue: h, i,
Visited: a, b, d, c, e, f, g, h, i,
Breadth-first search (BFS) example

search(v):
    visited = empty set
    queue.enqueue(v)
    visited.add(curr)

    while (queue is not empty):
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        for (w : v.neighbors()):
            if (w not in visited):
                queue.enqueue(w)
                visited.add(curr)

Current node: h
Queue: i,
Visited: a, b, d, c, e, f, g, h, i,
Breadth-first search (BFS) example

```python
search(v):
    visited = empty set
    queue.enqueue(v)
    visited.add(curr)
    while (queue is not empty):
        curr = queue.dequeue()
        for (w : v.neighbors()):
            if (w not in visited):
                queue.enqueue(w)
                visited.add(curr)

Current node: i
Queue:
Visited: a, b, d, c, e, f, g, h, i,
Note: We visited the nodes in “rings” – maintained a gradually growing “frontier” of nodes.
An interesting property...

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BFS analysis

```python
search(v):
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```

Questions:

- What is the worst-case runtime? (Let $|V|$ be the number of vertices, let $|E|$ be the number of edges)

- What is the worst-case amount of memory used?
search(v):
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queue.enqueue(v)
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while (queue is not empty):
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Questions:

- What is the worst-case runtime? (Let $|V|$ be the number of vertices, let $|E|$ be the number of edges)
  
  We visit each vertex once, and each edge once, so $O(|V| + |E|)$.

- What is the worst-case amount of memory used?
  
  Whatever the largest "horizon size" is. In the worst case, the horizon will contain $|V| - 1$ nodes, so $O(|V|)$. Note: $O(|V| + |E|)$ is also called "graph linear".
Describe how you would use or modify BFS to solve the following:

- Determine if some graph is also a tree.
- Print out all the elements in a tree level by level.
- Find the shortest path from one node to another.
Depth-first search (DFS)

**Question:** Why a queue? Can we use other data structures?

The BFS algorithm:

```
search(v):
    visited = empty set
    queue.enqueue(v)
    visited.add(v)
    while (queue is not empty):
        curr = queue.dequeue()
        for (w : v.neighbors()):
            if (w not in visited):
                queue.enqueue(w)
                visited.add(curr)
```

The DFS algorithm:

```
search(v):
    visited = empty set
    stack.push(v)
    visited.add(v)
    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)
        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
                visited.add(v)
```
Depth-first search (DFS)

**Question:** Why a queue? Can we use other data structures?

**Answer:** Yes! Any kind of list-like thing that supports appends and removes works! For example, what if we try using a stack?
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    queue.enqueue(v)
    visited.add(v)

    while (queue is not empty):
        curr = queue.dequeue()
        visited.add(curr)
        for (w : v.neighbors()):
            if (w not in visited):
                queue.enqueue(w)
                visited.add(curr)
```

The DFS algorithm:

```python
search(v):
    visited = empty set
    stack.push(v)
    visited.add(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)
        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
                visited.add(curr)
```
**Question:** Why a queue? Can we use other data structures?

**Answer:** Yes! Any kind of list-like thing that supports appends and removes works! For example, what if we try using a stack?

The BFS algorithm:

```python
search(v):
    visited = empty set
    queue.enqueue(v)
    visited.add(v)

    while (queue is not empty):
        curr = queue.dequeue()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                queue.enqueue(w)
                visited.add(curr)
```

The DFS algorithm:

```python
search(v):
    visited = empty set
    stack.push(v)
    visited.add(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
                visited.add(v)
```
**Depth-first search (DFS) example**

```
search(v):
    visited = empty set
    stack.push(v)

    while (stack is not empty):
        curr = stack.pop()
        visited.add(curr)

        for (w : v.neighbors()):
            if (w not in visited):
                stack.push(w)
```

**Current node:**

**Stack:**

**Visited:**