CSE 373: Hash functions and open addressing

Michael Lee

Wednesday, Jan 24, 2018

Warmup

Consider an IntegerDictionary using separate chaining with an internal capacity of 10. Assume our buckets are implemented using a linked list where we append new key-value pairs to the end.

Now, suppose we insert the following key-value pairs. What does the dictionary internally look like?

(1, a), (5, b), (11, a), (7, d), (12, e), (17, f), (1, g), (25, h)

0	1	2	3	4	5	6	7	8	9

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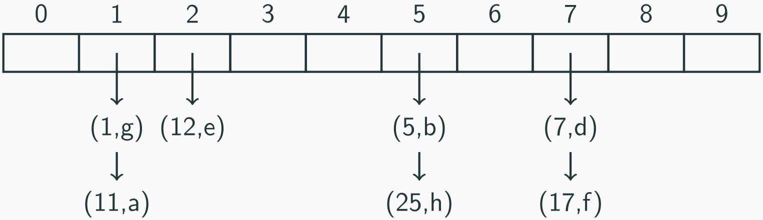

Announcements

Written HW 1 due tonight at 11:30pm

PSA:

- For questions involving math, make sure it's easy for us to follow your work
  - Don't just spit out equations without context, add some text to (briefly) explain what you're doing
  - Neatly label or circle your final answer
- Make sure you're submitting to the right place on Canvas

Announcements

Project 2 released, due Wed Jan 24

- Partner selection due Thursday

  Can work with same partner or different one

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  Can work with same partner or different one
- About project
  - Bulk of project is spent implementing a hash table, using separate chaining
  - Will need to add an iterator to ArrayDictionary and your hash table
  - Implementing iterator for hash table may be tricky, don't leave it to the last moment

Midterm

Core details

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Times:

- Midterm on Friday, Feb 2, in-class
- Will last 80 minutes (3:30 to 4:50)

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Review sessions

- Monday, Jan 29: Gowen 201, 4:30 to 6:30
- Tuesday, Jan 30: Gowen 201, 4:30 to 6:30

Midterm

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Full list of topics available on course website now. Summary:

- Basic data structures (stacks, queues, list)
- Asymptotic analysis, modeling code
- Trees (BSTs and AVL trees)
- Hash tables
- Systems and B-Trees (on a high-level)

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Topics NOT covered on the midterm

- Finding the closed form of summations or recurrences
- Sorting
- Heaps
- Anything about Java (generics, interfaces, junit, eclipse, etc)

Midterm

Practice

- Past CSE 373 midterms available on course website
- Past sections
- Questions on written homework 1 are representative of what will appear on midterm

Hash functions

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A hash function is a mapping from the key set U to an integer.

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Or, in other words, a function that turns the input into an integer in some way.

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1. We receive a key

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1. We receive a key
2. We run the hash function to get some integer
3. We do the same thing we did for IntegerDictionary

Analyzing hash function

Exercise: let's convert a string into an integer.

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What we have:

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    char[] chars;
    int size;

    // etc...
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Our goal:

```
int hashCode(str)
    // What goes here?
```

Analyzing hash functions

In math: $h(s) = 1$

In pseudocode:

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int hashCode(str)
    return 1
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Good idea? Bad idea?

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In pseudocode:

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Bad idea: Every string has same hash code! Everything collides!

(But hey, at least it's fast...)

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In math: $h(s) = \sum_{i=0}^{|s|-1} s_i$

In pseudocode:

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int hashCode(str)
    int out = 0
    for (char c : str.chars) {
        // Use ASCII value of char
        out += c
    return out
```

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Better but not ideal: Still too many collisions! Ex: "baker" and "brake", and "break" all have same hash code!

Runtime: still pretty decent, relatively speaking

Insight: can we use character positions somehow?

Analyzing hash functions

In math: $h(s) = 2^{s_0} \cdot 3^{s_1} \cdot 5^{s_2} \cdot 7^{s_3} \cdot 11^{s_4} \cdots$

In pseudocode:

```
int hashCode(str)
    int out = 1
    for (char c : str.chars)
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        out *= Math.pow(nextPrime, c)
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Not ideal: Hideously expensive, creates gigantic integers

(But hey, at least every string maps to a unique int!)

Analyzing hash functions

In math: $h(s) = \sum_{i=0}^{|s|-1} 31^i \cdot s_i$

In code:

```
int hashCode(str)
    int accum = 1
    int out = 0
    for (char c : s.chars)
        out += accum * c
        accum *= 31
    return out
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Good idea: Uses both character values and positions.

Strikes good balance between efficiency and reducing collisions.

(Why use 31? People tried a bunch of different strategies, and this one seemed to work well "in practice")

Hash functions

So, what does a good hash function look like?

Using hash functions inside dictionaries: useful properties

A hash function that is intended to be used for a dictionary should ideally have the following properties:

- **Low collision rate:**
  The hash of two different inputs should usually be different.
  We want to *minimize collisions* as much as possible.

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  In Java, there are 2^{32} 32-bit ints. So, the probability that the hash function returns any individual int should be $\frac{1}{2^{32}}$.

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- **Uniform distribution of outputs:**
  In Java, there are 2^{32} 32-bit ints. So, the probability that the hash function returns any individual int should be $\frac{1}{2^{32}}$.
- **Low computational cost:**
  We will be computing the hash function a lot, so we need it to be very easy to compute.

Client vs implementor

Who implements the hash function? The client, or the dictionary?

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Client responsibilities

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Dictionary/implementor responsibilities

- Responsible for calling the hash function
- Responsible for managing the internal array
- Responsible for keeping track of collisions

A Java interlude...

So, how does this work in Java?

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Every object has a default `equals` and `hashCode` implementation.

Override these two methods.

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Override these two methods.

Important invariants

When implementing hashCode, you MUST respect these invariants!

- IF you implement `hashCode(...)`,
  THEN you MUST also implement `equals(...)`
- IF `a.equals(b)`,
  THEN you MUST make sure that `a.hashCode() == b.hashCode()`

Handling multiple fields

What if an object has multiple fields?

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- Are some fields redundant? Do you need to hash all of them?

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Tips for creating hashes

- Use all 32 bits (including negative numbers!)
- Use different overlapping bits for different parts of the hash
- If keys are known ahead of time, choose a perfect hash
- Use expertise of others: consult books, have your IDE auto-generate a hash function...

Handling collisions

Insight:

The majority of our time is spent handling collisions

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Our strategy so far:

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- If we do have a collision, store both in a “bucket”

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Are there other strategies for storing collisions?

Yes: something called **open addressing**

Open addressing

Open addressing

Open addressing is a kind of collision resolution strategy that resolves collisions by chosing a different location when the natural choice is full.

Open addressing: linear probing

Exercise: assume internal capacity of 10, insert the following keys:

1, 5, 11, 7, 12, 17, 6, 25

0	1	2	3	4	5	6	7	8	9

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	1	11	12		5	6	7	17	25

Open addressing: linear probing

Strategy: Linear probing

If we collide, checking each next element until we find an open slot.

So, $h'(k, i) = (h(k) + i) \bmod T$, where T is the table size

```
i = 0
while (index in use)
    try (hash(key) + i) % array.length
    i += 1
```