CSE 373: Hash functions and open addressing

Michael Lee
Wednesday, Jan 24, 2018
Warmup

Consider an IntegerDictionary using separate chaining with an internal capacity of 10. Assume our buckets are implemented using a linked list where we append new key-value pairs to the end.

Now, suppose we insert the following key-value pairs. What does the dictionary internally look like?

(1, a), (5, b), (11, a), (7, d), (12, e), (17, f), (1, g), (25, h)
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Written HW 1 due tonight at 11:30pm

PSA:

► For questions involving math, make sure it's easy for us to follow your work
  ► Don’t just spit out equations without context, add some text to (briefly) explain what you’re doing
  ► Neatly label or circle your final answer

► Make sure you’re submitting to the right place on Canvas
Project 2 released, due Wed Jan 24

- Partner selection due Thursday
  Can work with same partner or different one
Announcements

Project 2 released, due Wed Jan 24

- Partner selection due Thursday
  Can work with same partner or different one
- About project
  - Bulk of project is spent implementing a hash table, using separate chaining
  - Will need to add an iterator to ArrayDictionary and your hash table
  - Implementing iterator for hash table may be tricky, don’t leave it to the last moment
Core details
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Times:

- Midterm on Friday, Feb 2, in-class
- Will last 80 minutes (3:30 to 4:50)
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Review sessions

- Monday, Jan 29: Gowen 201, 4:30 to 6:30
- Tuesday, Jan 30: Gowen 201, 4:30 to 6:30
Midterm topics

Full list of topics available on course website now. Summary:

▶ Basic data structures (stacks, queues, list)
▶ Asymptotic analysis, modeling code
▶ Trees (BSTs and AVL trees)
▶ Hash tables
▶ Systems and B-Trees (on a high-level)

Topics NOT covered on the midterm

▶ Finding the closed form of summations or recurrences
▶ Sorting
▶ Heaps
▶ Anything about Java (generics, interfaces, junit, eclipse, etc)
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Practice

- Past CSE 373 midterms available on course website
- Past sections
- Questions on written homework 1 are representative of what will appear on midterm
Hash function

A hash function is a mapping from the key set $U$ to an integer.
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Or, in other words, a function that turns the input into an integer in some way.
How do we use a hash function?

1. We receive a key
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2. We run the hash function to get some integer
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1. We receive a key
2. We run the hash function to get some integer
3. We do the same thing we did for IntegerDictionary
Exercise: let’s convert a string into an integer.
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What we have:

```java
public class OurString {
    char[] chars;
    int size;

    // etc...
}
```
Analyzing hash function

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What we have:

```java
class OurString {
  char[] chars;
  int size;

  // etc...
}
```

Our goal:

```java
int hashCode(String str)
  // What goes here?
```
Analyzing hash functions

In math: \( h(s) = 1 \)

In pseudocode:

```java
int hashCode(str)
    return 1
```
Analyzing hash functions

In math: \( h(s) = 1 \)

In pseudocode:

```plaintext
int hashCode(str)
    return 1
```

Good idea? Bad idea?
Analyzing hash functions

In math: \( h(s) = 1 \)

In pseudocode:

```python
int hashCode(str)
    return 1
```

**Bad idea:** Every string has same hash code! Everything collides!

(But hey, at least it’s fast...)
Analyzing hash functions

In math: \[ h(s) = \sum_{i=0}^{s-1} s_i \]

In pseudocode:

```java
int hashCode(String str) {
    int out = 0;
    for (char c : str.chars) { // Use ASCII value of char
        out += c;
    }
    return out;
}
```

Better but not ideal: Still too many collisions! Ex: "baker" and "brake", and "break" all have same hash code!

Runtime: still pretty decent, relatively speaking

Insight: can we use character positions somehow?
Analyzing hash functions

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In math: \( h(s) = 2^{s_0} \cdot 3^{s_1} \cdot 5^{s_2} \cdot 7^{s_3} \cdot 11^{s_4} \ldots \)

In pseudocode:

```java
int hashCode(String str) {
    int out = 1;
    for (char c : str.chars) {
        int nextPrime = get next prime number
        out *= Math.pow(nextPrime, c);
    }
    return out;
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**Not ideal:** Hideously expensive, creates gigantic integers

(But hey, at least every string maps to a unique int!)
Analyzing hash functions

In math:
\[ h(s) = \sum_{i=0}^{s-1} 31^i \cdot s_i \]

In code:
```java
int hashCode(String str)
{
    int accum = 1;
    int out = 0;
    for (char c : str.toCharArray())
    {
        out += accum * c;
        accum *= 31;
    }
    return out;
}
```

Good idea:
- Uses both character values and positions.
- Strikes good balance between efficiency and reducing collisions.

(Why use 31? People tried a bunch of different strategies, and this one seemed to work well "in practice").
Analyzing hash functions

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So, what does a good hash function look like?

**Using hash functions inside dictionaries: useful properties**

A hash function that is intended to be used for a dictionary should ideally have the following properties:

- **Low collision rate:**
  The hash of two different inputs should usually be different. We want to *minimize collisions* as much as possible.

```java
// In Java, there are 2^32 32-bit ints. So, the probability that the hash function returns any individual int should be 1/2^32.
```
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- **Uniform distribution of outputs:**
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- **Uniform distribution of outputs:**
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- **Low computational cost:**
  We will be computing the hash function a lot, so we need it to be very easy to compute.
Who implements the hash function? The client, or the dictionary?
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**Client responsibilities**

- Responsible for implementing a “good” hash function.
- The hash function avoids “wasting” information in the key or the output bits while still being “fast”.

**Dictionary/implementor responsibilities**

- Responsible for calling the hash function
- Responsible for managing the internal array
- Responsible for keeping track of collisions
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So, how does this work in Java?
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Every object has a default `equals` and `hashCode` implementation. Override these two methods.
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Every object has a default equals and hashCode implementation. Override these two methods.

**Important invariants**

When implementing hashCode, you MUST respect these invariants!

- IF you implement `hashCode(...)`, THEN you MUST also implement `equals(...)`
- IF `a.equals(b)`, THEN you MUST make sure that `a.hashCode() == b.hashCode()`
Handling multiple fields

What if an object has multiple fields?

General considerations:
▶ Trade-off: hashing time vs collision avoidance
▶ Are some fields redundant? Do you need to hash all of them?

Tips for creating hashes
▶ Use all 32 bits (including negative numbers!)
▶ Use different overlapping bits for different parts of the hash
▶ If keys are known ahead of time, choose a perfect hash
▶ Use expertise of others: consult books, have your IDE auto-generate a hash function...
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Handling collisions

Insight:

The majority of our time is spent handling collisions

Our strategy so far:

▶ Design a good hash function to minimize chance of collision
▶ If we do have a collision, store both in a "bucket"

Are there other strategies for storing collisions?

Yes: something called open addressing
Handling collisions

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Open addressing

*Open addressing* is a kind of collision resolution strategy that resolves collisions by choosing a different location when the natural choice is full.
Open addressing: linear probing

Exercise: assume internal capacity of 10, insert the following keys:

1, 5, 11, 7, 12, 17, 6, 25

set(12, ...)
Open addressing: linear probing

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1, 5, 11, 7, 12, 17, 6, 25

```
<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>11</td>
<td>12</td>
<td></td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>17</td>
<td>25</td>
</tr>
</tbody>
</table>
```
Strategy: Linear probing

If we collide, checking each next element until we find an open slot.

So, \( h'(k, i) = (h(k) + i) \mod T \), where \( T \) is the table size

\[
i = 0
\]
\[
\text{while} \ (\text{index in use})
\]
\[
\text{try} \ (\text{hash(key)} + i) \% \text{array.length}
\]
\[
i += 1
\]