CSE 373: Asymptotic Analysis

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Warmup

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Remind your neighbor: what fields did our array list iterator need?

Comparing algorithms

Goal: *compare* algorithms

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- Memory used
- Number of network calls made
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Today: focus on comparing algorithms based on *how long it takes them to run in the worst case*.

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Algo 1	0.0018
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~~Which algorithm is better?~~

This is a trick question. Why isn't this table enough to let us decide which algorithm is better?

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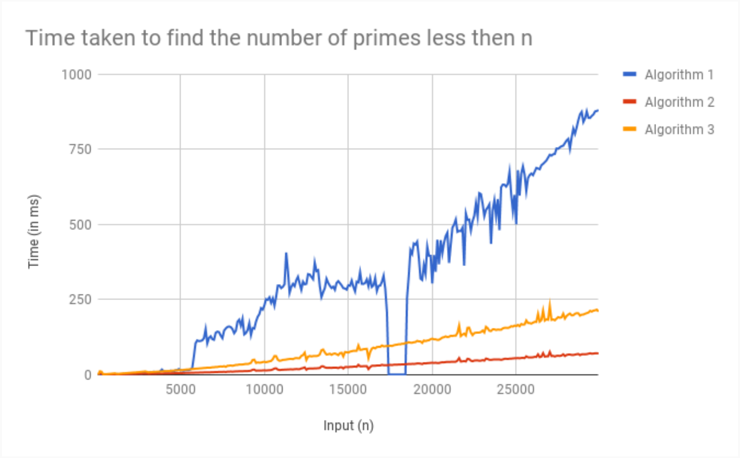

Which algorithm is better?

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- Rigorously discover overall trends without resorting to testing
  (what if we miss worst-case input? best-case input?)
- A way to analyze before coding!

Our process

Two step process:

1. **Model** what we care about as a mathematical function
2. **Analyze** that function using asymptotic analysis

Modeling: Assumptions

Assumption: basic operations take “constant” time

- Arithmetic (for fixed-width numbers)
- Variable assignment
- Accessing a field or array index
- Printing something out
- etc...

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Warning: These assumptions are over-simplifications.

But they're very useful approximations!

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- **Loops**
  Number of iterations $\times$ time for loop body

Modeling: exercise

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Algorithm 1: compare each pair of elements

```
public boolean hasDuplicate1(int[] array) {
    for (int i = 0; i < array.length; i++)
        for (int j = 0; j < array.length; j++)
            if (i != j && array[i] == array[j])
                return true;
    return false;
}
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Algorithm 2: compare each consecutive pairs of elements

```java
public boolean hasDuplicate2(int[] array) {
    for (int i = 0; i < array.length - 1; i++)
        if (array[i] == array[i + 1])
            return true;
    return false;
}
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Exercise: create a **mathematical function** modeling the amount of time taken in the worst case

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Two step process:

1. **Model** what we care about as a mathematical function
2. **Analyze** that function using asymptotic analysis
   Specifically: have a way to **compare** two functions

Next time

Next time: how do we compare functions?