Consider the following recursive function. You may assume that the input will be a multiple of 3.

```java
public int test(int n) {
    if (n <= 6) {
        return 2;
    } else {
        int curr = 0;
        for (int i = 0; i < n * n; i++) {
            curr += 1;
        }
        return curr + test(n - 3);
    }
}
```

(a) Write a recurrence modeling the worst-case runtime of test.

**Solution:**

\[
T(n) = \begin{cases} 
1 & \text{When } n \leq 6 \\
1 + \frac{n}{3} + \sum_{i=3}^{n/3} (3i)^2 & \text{Otherwise}
\end{cases}
\]

(b) Unfold the recurrence into a summation (for \( n \geq 6 \)).

**Solution:**

\[
1 + \sum_{i=3}^{n/3} (3i)^2
\]

Modeling this recurrence correctly is slightly challenging because we want to decrease \( n \) in increments of 3.

To do this, what we do is set the summation bounds to go up to \( n/3 \) instead of \( n \), and multiply \( i \) on the inside by 3, simulating changing \( i \) in those increments.

We then also set the lower summation bound to be 3 instead of 0 or 1. That way, our summation will only consider numbers in the range 9 to \( n \) — if we set \( i = 2 \) or lower, our summation would double-count \( n \leq 6 \), which should be handled by the base case.

Note: our model only works if \( n \) is a multiple of 3.
(c) Simplify the summation into a closed form (for \( n \geq 6 \)).

Solution:

\[
1 + \sum_{i=3}^{n/3} (3i)^2 = 1 + \sum_{i=0}^{n/3} (3i)^2 - \sum_{i=0}^{2} (3i)^2 \\
= 1 + 9 \sum_{i=0}^{n/3} i^2 - \sum_{i=0}^{2} (3i)^2 \\
= 1 + 9 \sum_{i=0}^{n/3} i^2 - (0 + 9 + 36) \\
= 9 \frac{n}{3} \left( \frac{n}{3} + 1 \right) \left( \frac{2n}{3} + 1 \right) - 44
\]

Adjusting summation bounds

Pulling out a constant

Evaluating the summation

Sum of squares

A “closed form”, within the context of this class, is just any expression that does not contain a summation or is recursive. This means we can stop here without needing to further simplify the expression.

That said, if you wanted to continue simplifying, we could:

\[
9 \frac{n}{3} \left( \frac{n}{3} + 1 \right) \left( \frac{2n}{3} + 1 \right) - 44 = \frac{9}{6} \left( \frac{n}{3} \left( \frac{n}{3} + 1 \right) \left( \frac{2n}{3} + 1 \right) \right) - 44 \\
= \frac{1}{2} \left( n \left( \frac{n}{3} + 1 \right) \left( \frac{2n}{3} + 1 \right) \right) - 44 \\
= \frac{1}{2} \left( n \left( \frac{2}{3} n^2 + n + 1 \right) \right) - 44 \\
= \frac{1}{9} n^3 + \frac{1}{2} n^2 + \frac{1}{2} n - 44
\]