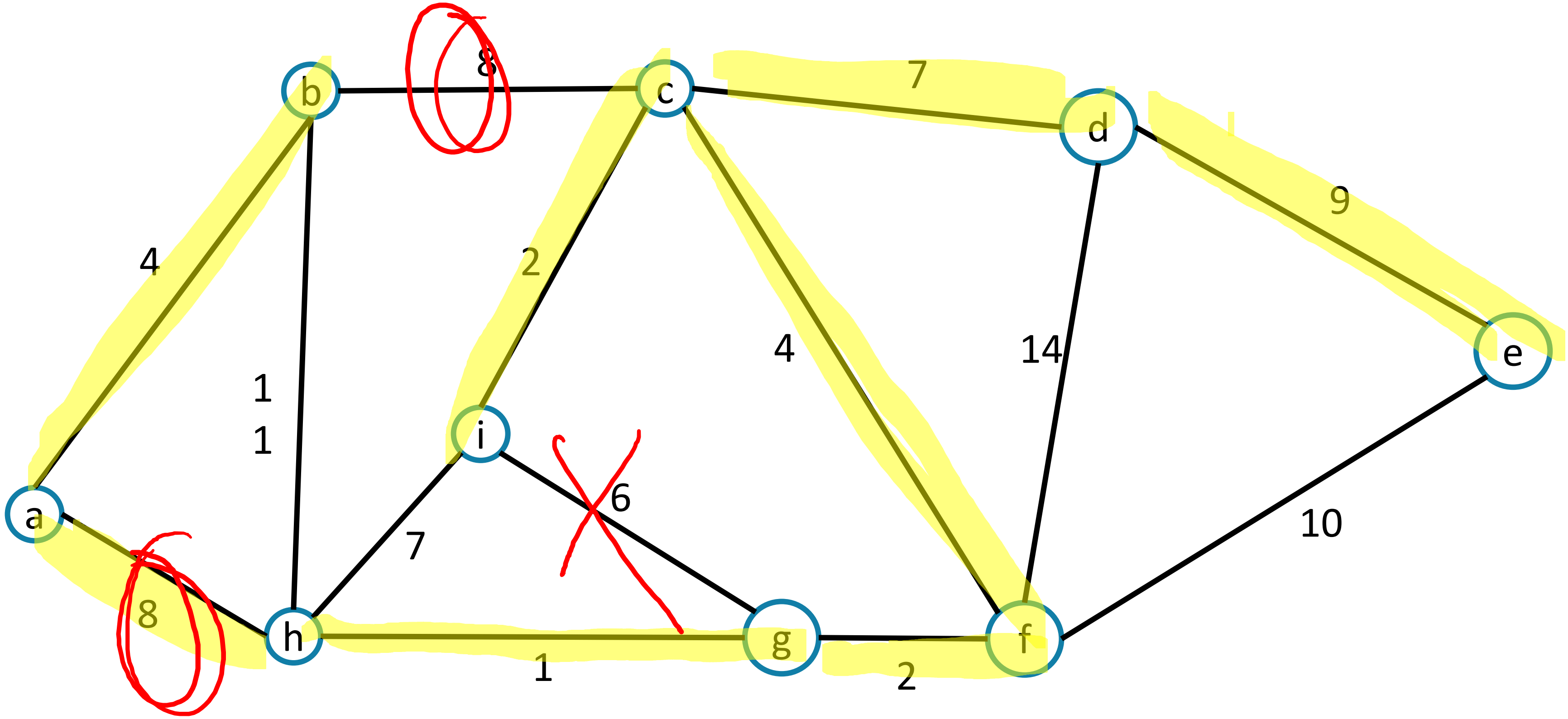



Disjoint Sets

Data Structures and Algorithms

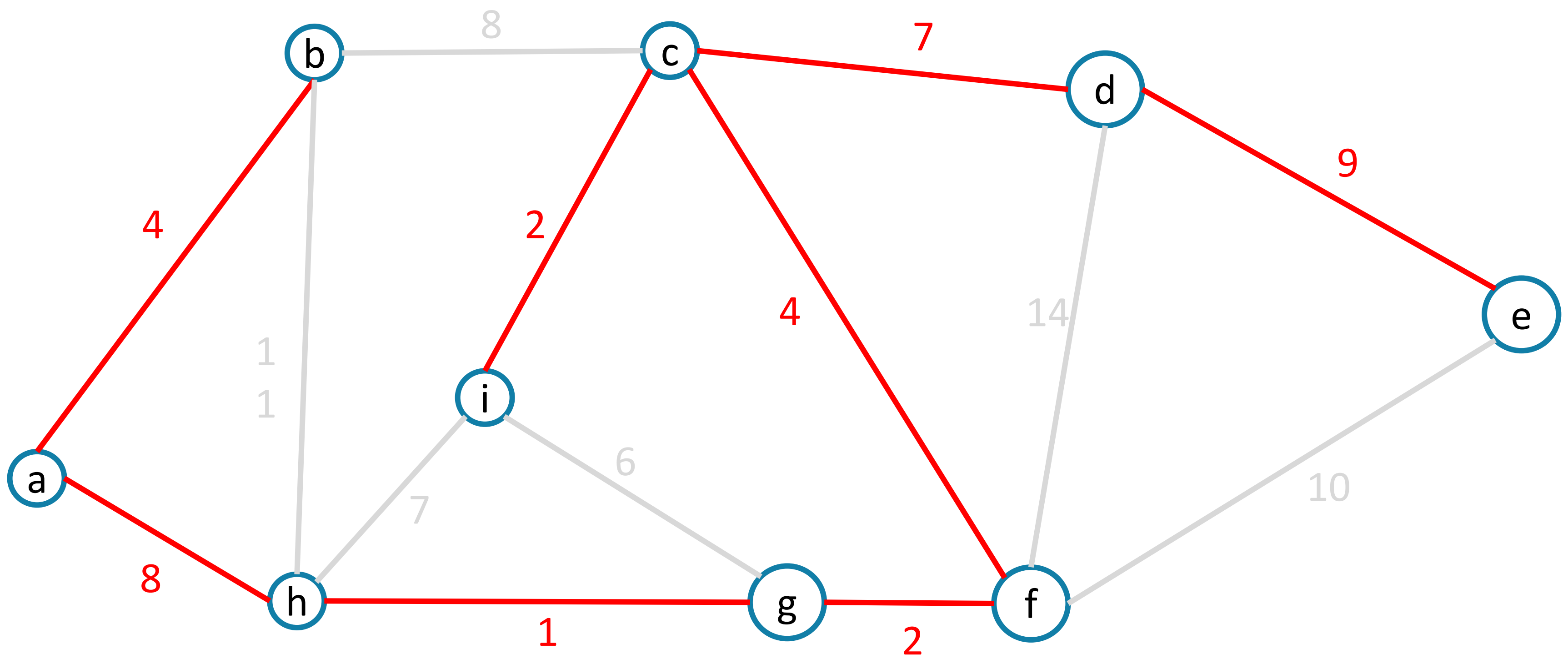
Warm Up

Finding a MST using Kruskal's algorithm



Warm Up

Finding a MST using Kruskal's algorithm



New ADT

Set ADT

state

Set of elements

- Elements must be unique!
- No required order

Count of Elements

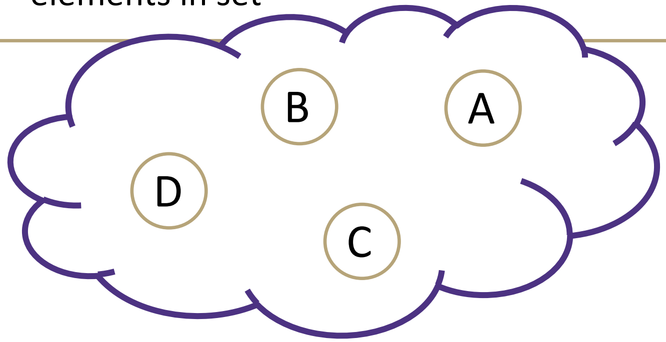
behavior

create(x) - creates a new set with a single member, x

add(x) - adds x into set if it is unique, otherwise add is ignored

remove(x) – removes x from set

size() – returns current number of elements in set



Disjoint-Set ADT

state

Set of Sets

- **Disjoint:** Elements must be unique across sets
- No required order
- Each set has representative

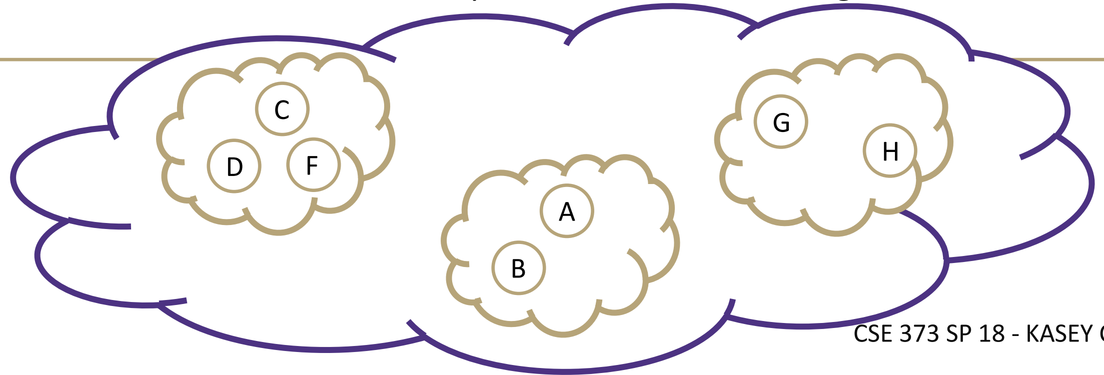
Count of Sets

behavior

makeSet(x) – creates a new set within the disjoint set where the only member is x. Picks representative for set

findSet(x) – looks up the set containing element x, returns representative of that set

union(x, y) – looks up set containing x and set containing y, combines two sets into one. Picks new representative for resulting set



Example

new()

makeSet(a)

makeSet(b)

makeSet(c)

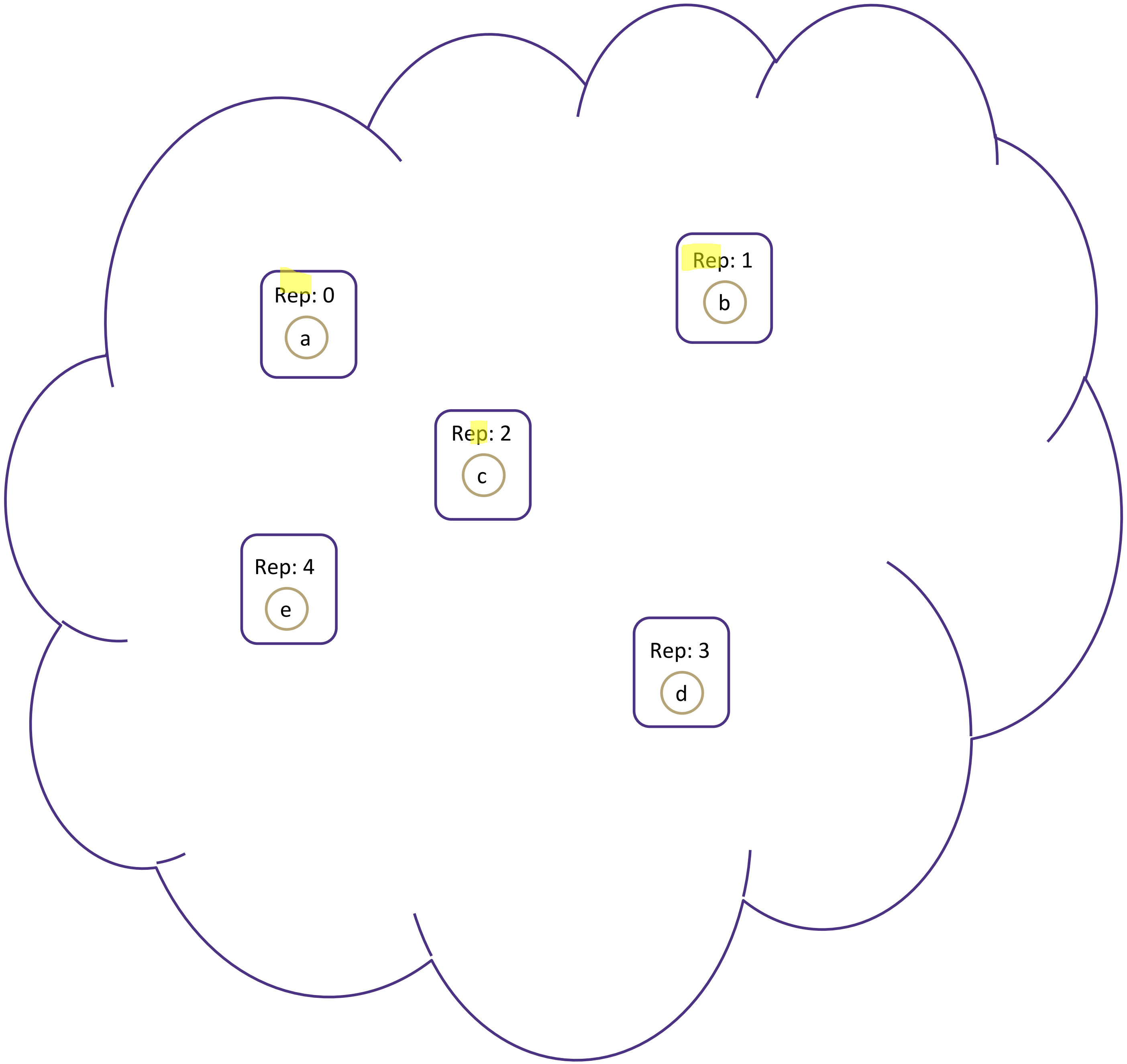
makeSet(d)

makeSet(e)

findSet(a) 0 3

findSet(d)

union(a, c)



Example

new()

makeSet(a)

makeSet(b)

makeSet(c)

makeSet(d)

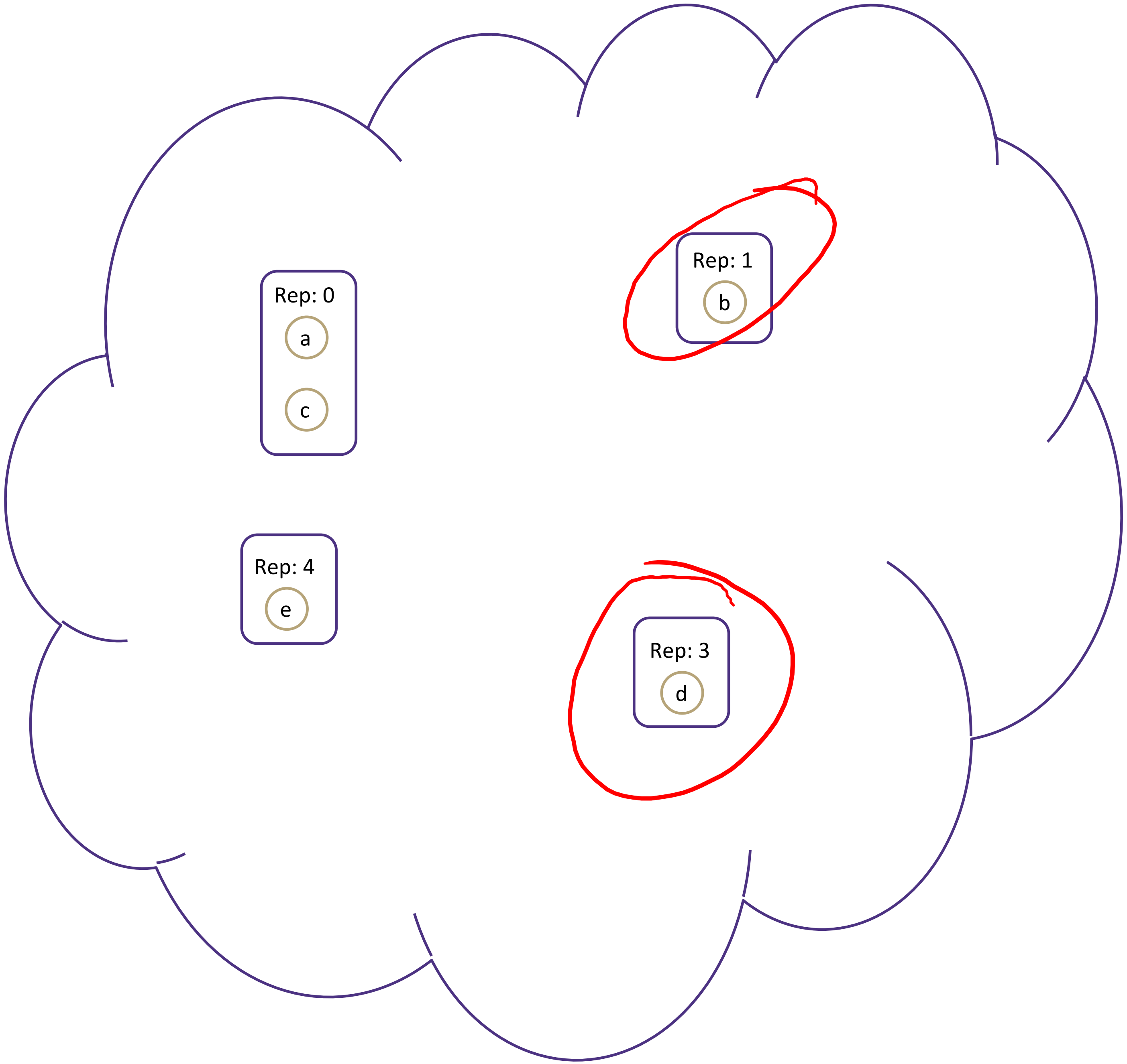
makeSet(e)

findSet(a)

findSet(d)

union(a, c)

union(b, d)



Example

new()

makeSet(a)

makeSet(b)

makeSet(c)

makeSet(d)

makeSet(e)

findSet(a)

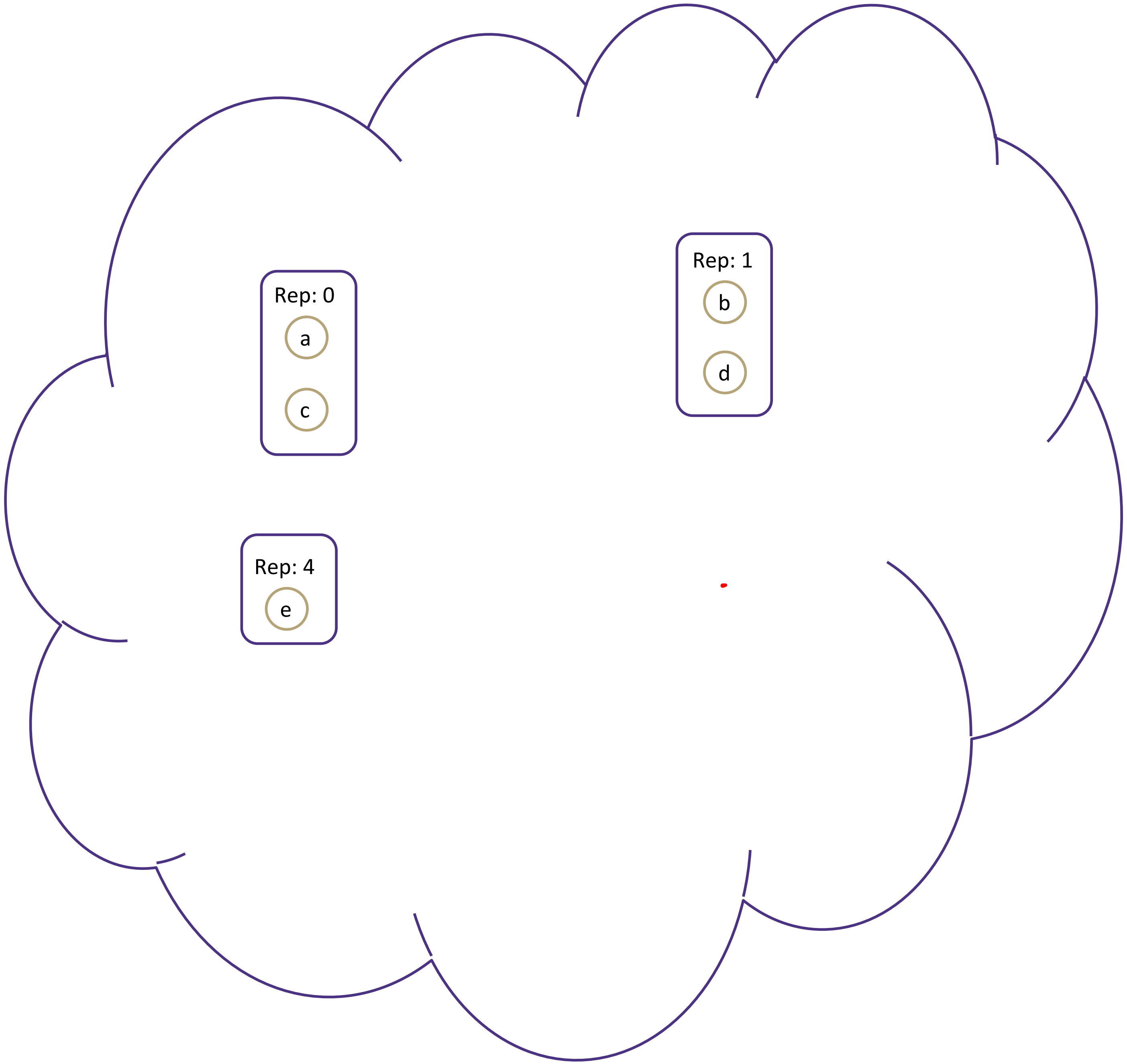
findSet(d)

union(a, c)

union(b, d)

findSet(a) == findSet(c)

findSet(a) == findSet(d)



Implementation

Disjoint-Set ADT

state

Set of Sets

- **Disjoint:** Elements must be unique across sets
- No required order
- Each set has representative

Count of Sets

behavior

makeSet(x) – creates a new set within the disjoint set where the only member is x. Picks representative for set

findSet(x) – looks up the set containing element x, returns representative of that set

union(x, y) – looks up set containing x and set containing y, combines two sets into one. Picks new representative for resulting set

1

TreeDisjointSet<E>

state

`Collection<TreeSet> forest`

`Dictionary<NodeValues, NodeLocations> nodeInventory`

behavior

`makeSet(x)`-create a new tree of size 1 and add to our forest

`findSet(x)`-locates node with x and moves up tree to find root

`union(x, y)`-append tree with y as a child of tree with x

2

~~TreeSet<E>~~

state

`SetNode overallRoot`

behavior

`TreeSet(x)`

`add(x)`

`remove(x, y)`

`getRep()`-returns data of overallRoot

SetNode<E>

state

`E data`

`Collection<SetNode> children`

behavior

`SetNode(x)`

`addChild(x)`

`removeChild(x, y)`

Implement makeSet(x)

makeSet(0)

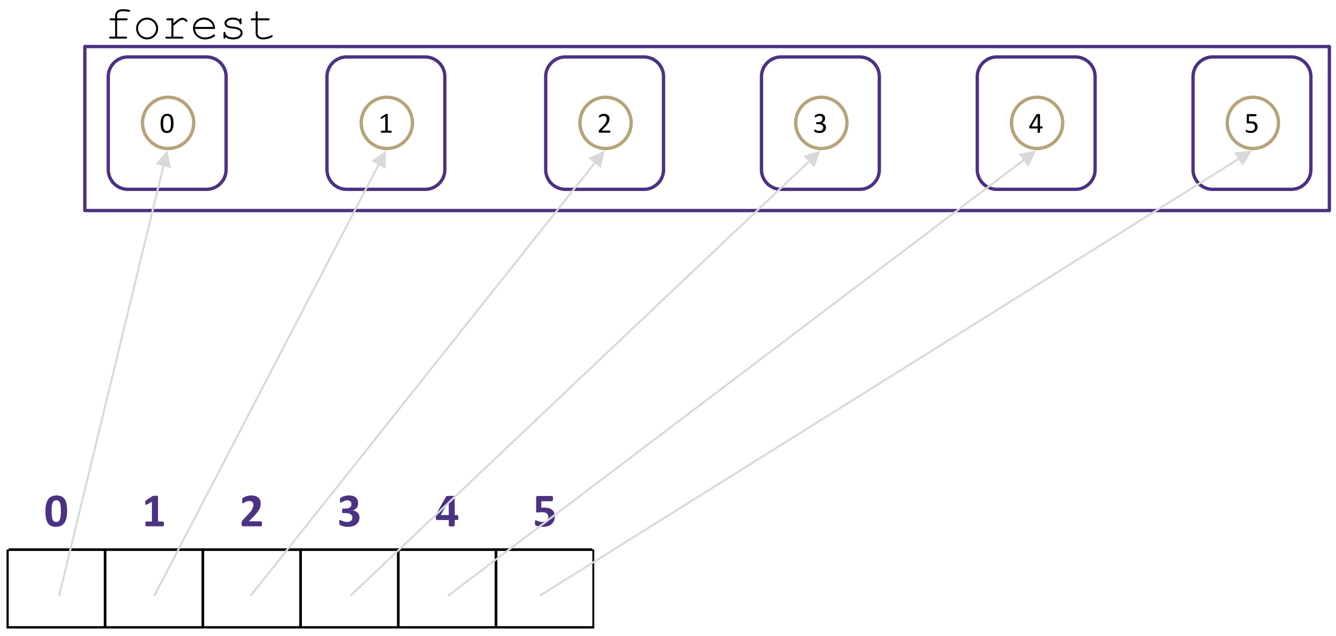
makeSet(1)

makeSet(2)

makeSet(3)

makeSet(4)

makeSet(5)



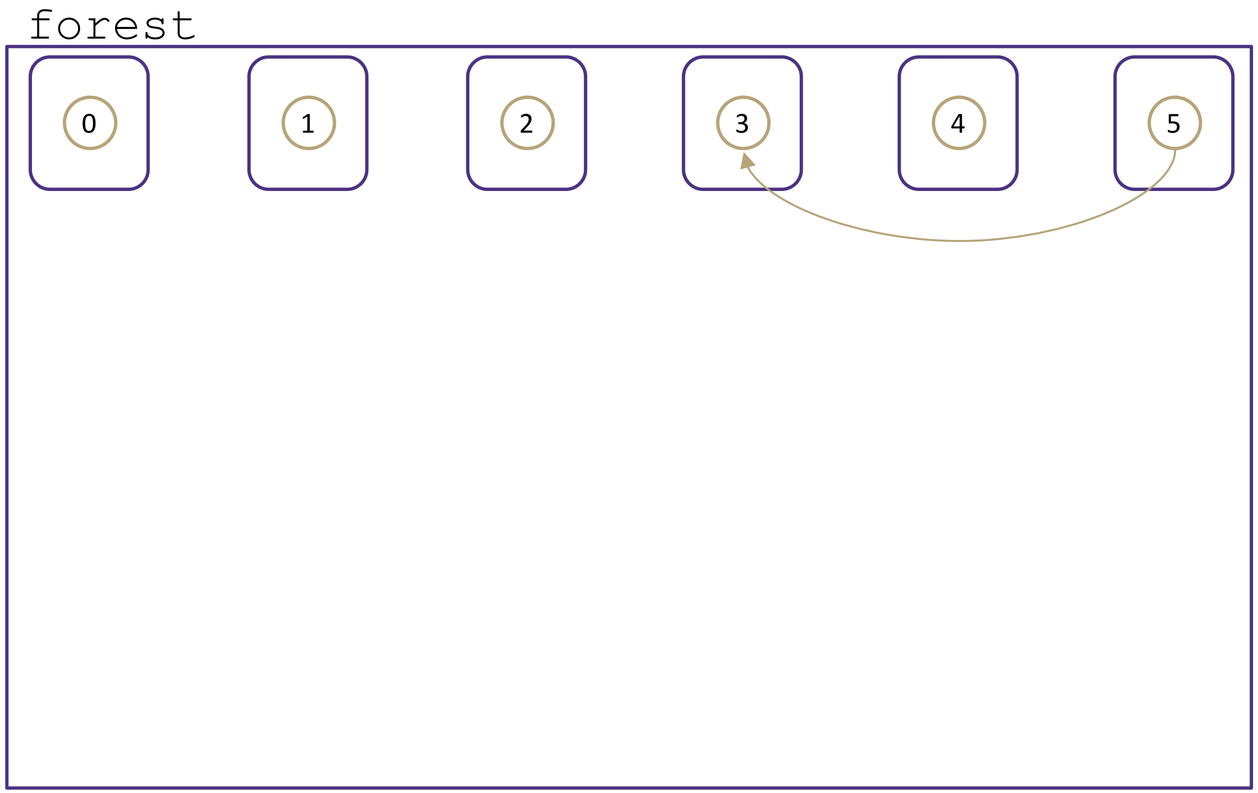
TreeDisjointSet<E>
state
`Collection<TreeSet> forest`
`Dictionary<NodeValues, NodeLocations> nodeInventory`
behavior
`makeSet(x)`-create a new tree of size 1 and add to our forest
`findSet(x)`-locates node with x and moves up tree to find root
`union(x, y)`-append tree with y as a child of tree with x

Worst case runtime?

O(1)

Implement union(x, y)

`union(3, 5)`



0	1	2	3	4	5
->	->	->	->	->	->

TreeDisjointSet<E>

state

Collection<TreeSet> forest

Dictionary<NodeValues, NodeLocations> nodeInventory

behavior

makeSet(x)-create a new tree of size 1 and add to our forest

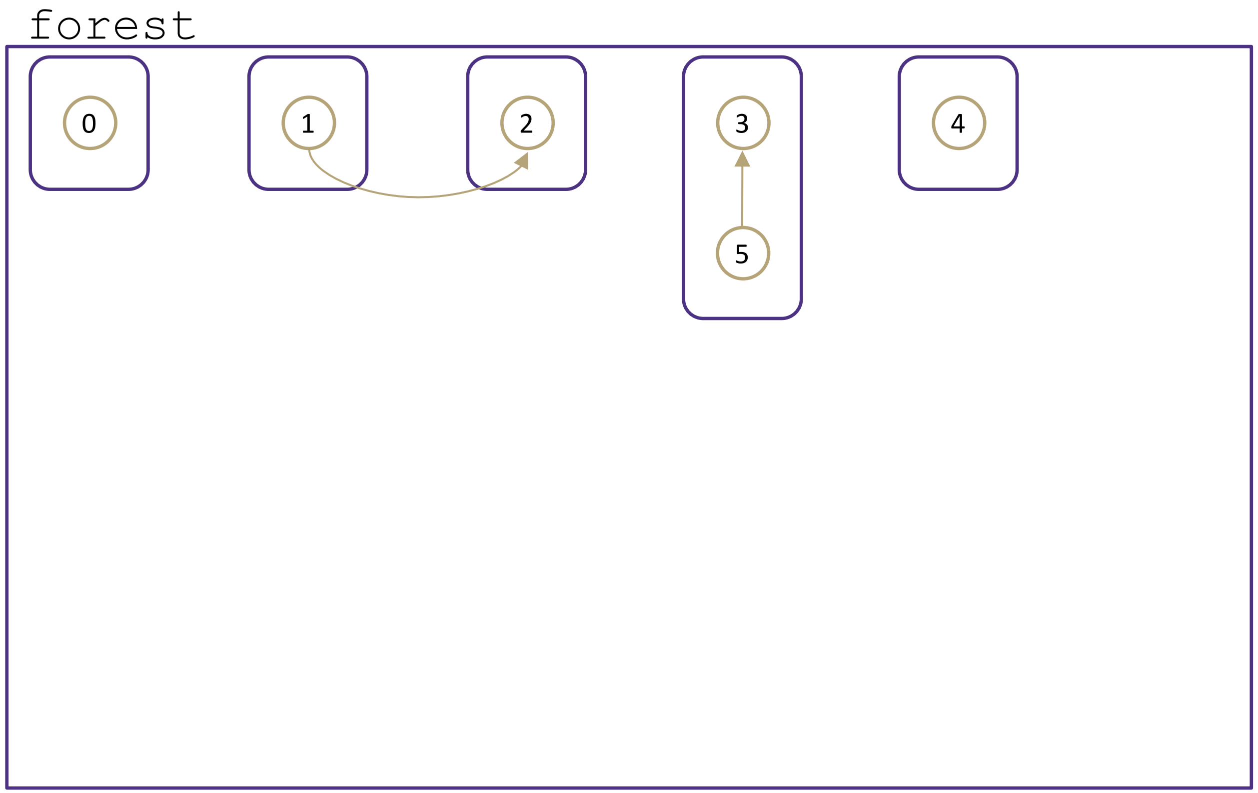
findSet(x)-locates node with x and moves up tree to find root

union(x, y)-append tree with y as a child of tree with x

Implement union(x, y)

```
union(3, 5)

union(2, 1)
```



0	1	2	3	4	5
->	->	->	->	->	->

TreeDisjointSet<E>

state

```
Collection<TreeSet> forest
Dictionary<NodeValues,
NodeLocations> nodeInventory
```

behavior

`makeSet(x)`-create a new tree of size 1 and add to our forest

`findSet(x)`-locates node with x and moves up tree to find root

`union(x, y)`-append tree with y as a child of tree with x

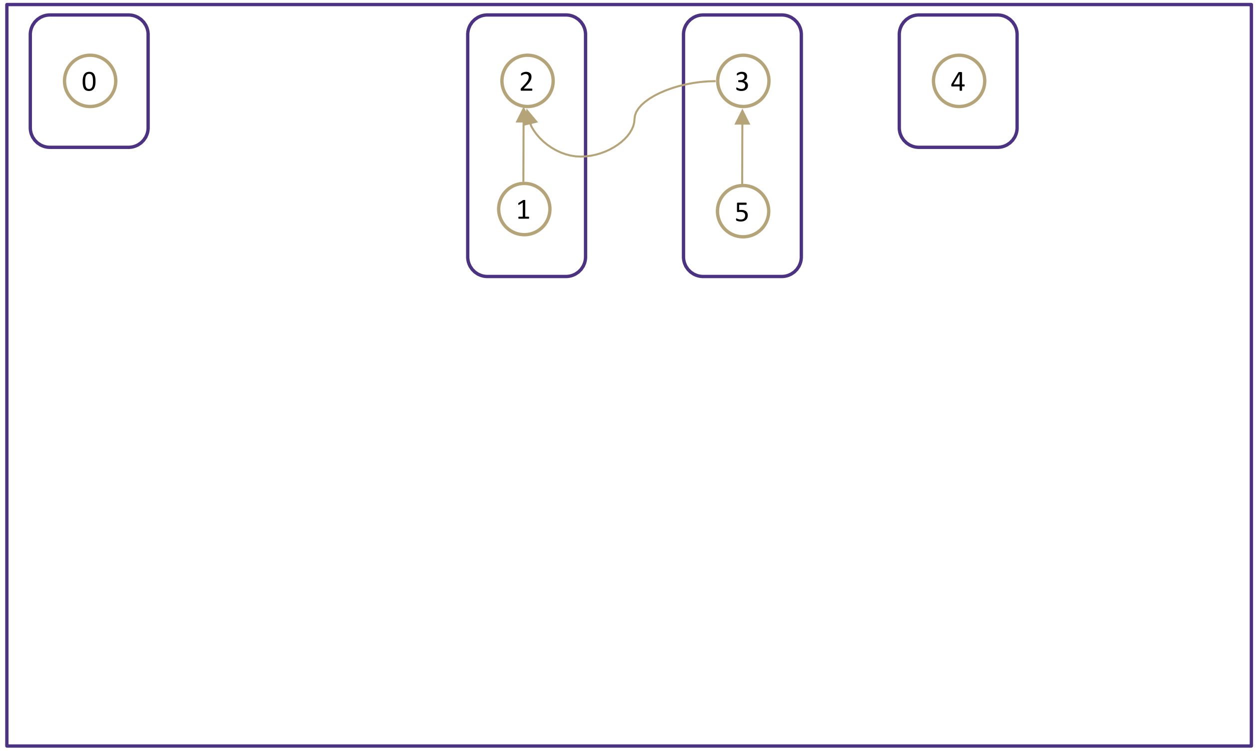
Implement union(x, y)

```
union(3, 5)

union(2, 1)

union(2, 5)
```

forest



0	1	2	3	4	5
->	->	->	->	->	->

TreeDisjointSet<E>

state

Collection<TreeSet> forest

Dictionary<NodeValues, NodeLocations> nodeInventory

behavior

makeSet(x)-create a new tree of size 1 and add to our forest

findSet(x)-locates node with x and moves up tree to find root

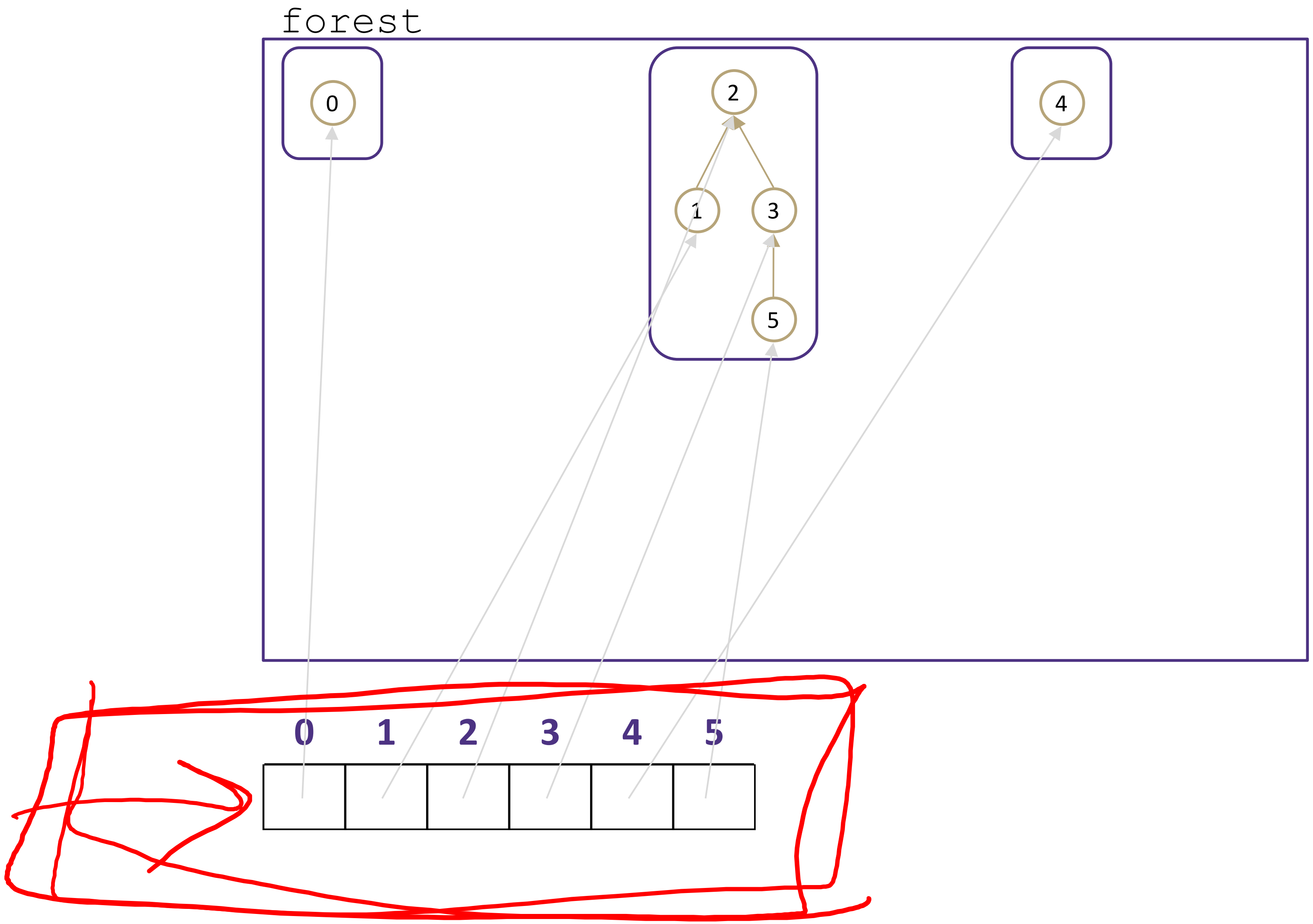
union(x, y)-append tree with y as a child of tree with x

Implement union(x, y)

```
union(3, 5)

union(2, 1)

union(2, 5)
```



TreeDisjointSet<E>

state

```
Collection<TreeSet> forest
Dictionary<NodeValues,
NodeLocations> nodeInventory
```

behavior

`makeSet(x)`-create a new tree of size 1 and add to our forest

`findSet(x)`-locates node with x and moves up tree to find root

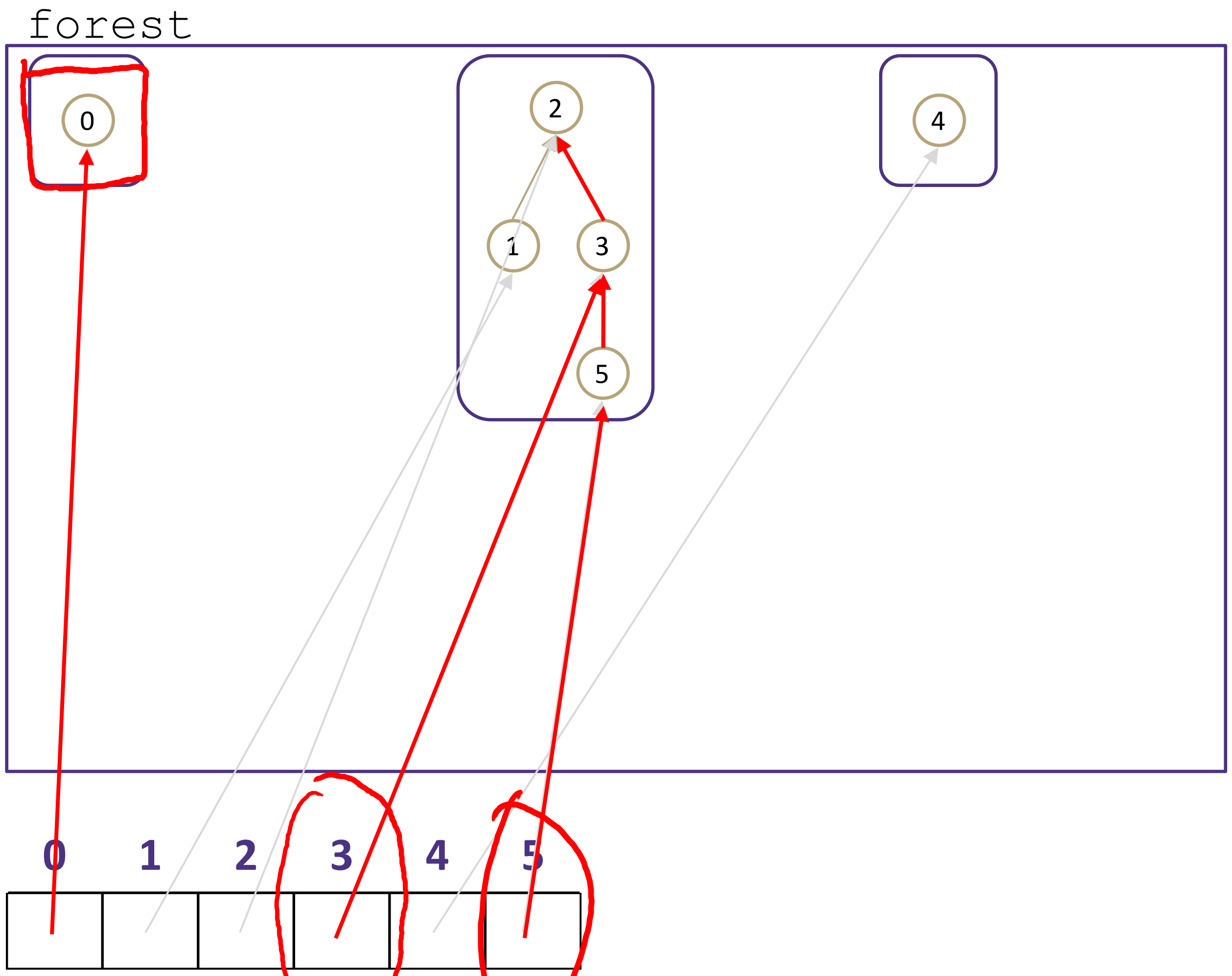
`union(x, y)`-append tree with y as a child of tree with x

Implement findSet(x)

findSet(0) 0

findSet(3) 2

findSet(5) 2



2n + Cu

TreeDisjointSet<E>

state

```
Collection<TreeSet> forest
Dictionary<NodeValues,
NodeLocations> nodeInventory
```

behavior

```
makeSet(x)-create a new tree
of size 1 and add to our
forest
findSet(x)-locates node with x
and moves up tree to find root
union(x, y)-append tree with y
as a child of tree with x
```

union(5, 4)
findSet(5)
findSet(4)
union

Worst case runtime?

O(n)

addn

Worst case runtime of union?

O(n)

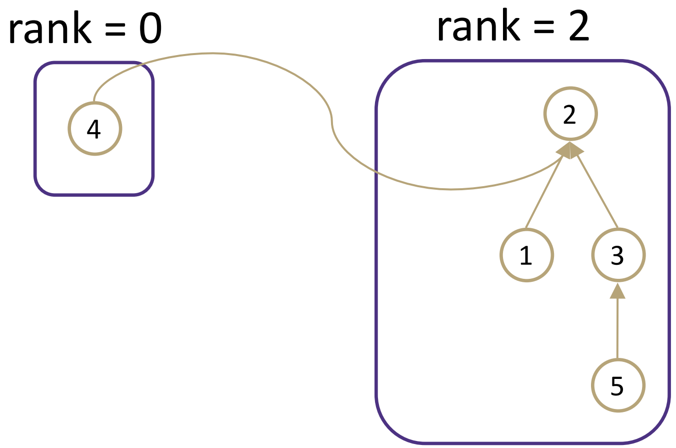
Improving union

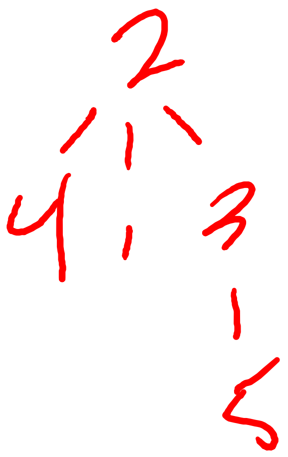
→ DEGENERATE!

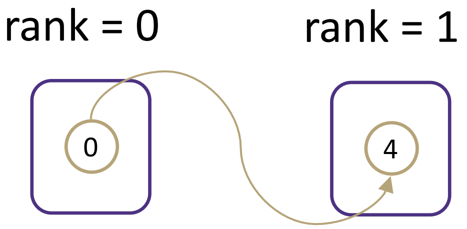
Problem: Trees can be unbalanced

Solution: Union-by-rank!

- let rank(x) be a number representing the upper bound of the height of x so rank(x) >= height(x)
- Keep track of rank of all trees
- When unioning make the tree with larger rank the root
- If it’s a tie, pick one randomly and increase rank by one

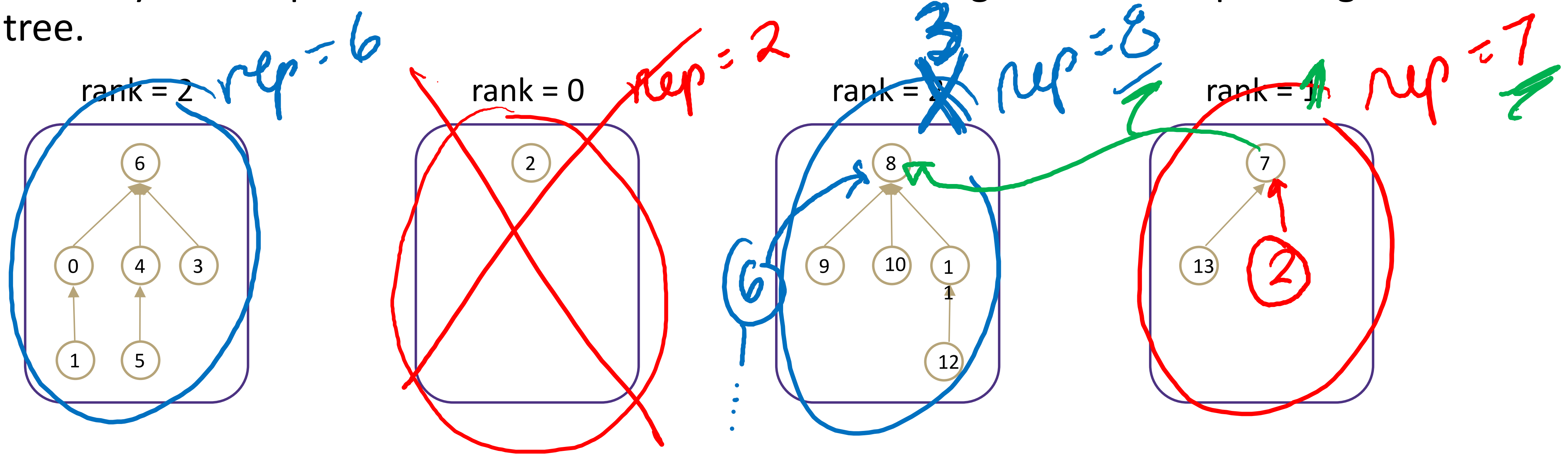






Practice

Given the following disjoint-set what would be the result of the following calls on union if we add the “union-by-rank” optimization. Draw the forest at each stage with corresponding ranks for each tree.



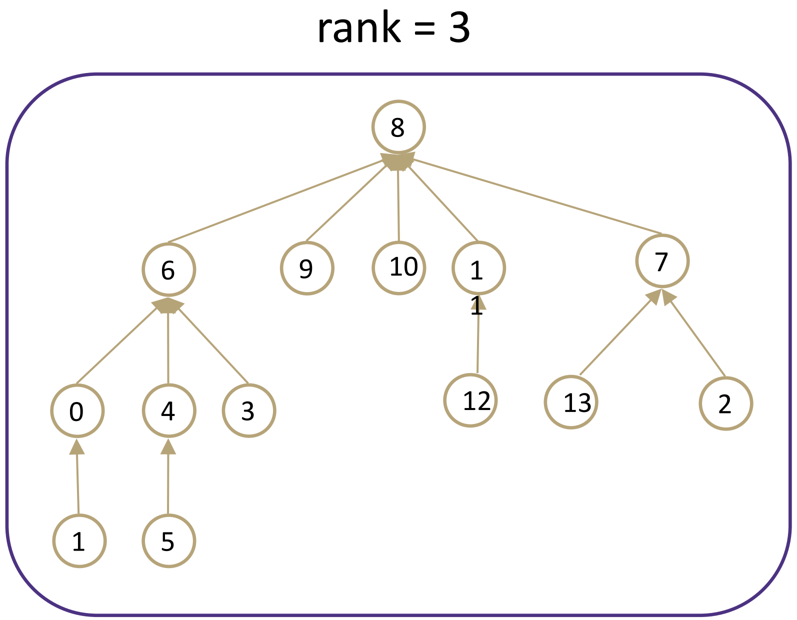
1 `union(2, 13)`

2 `union(4, 12)`

3 `union(2, 8)`

Practice

Given the following disjoint-set what would be the result of the following calls on union if we add the “union-by-rank” optimization. Draw the forest at each stage with corresponding ranks for each tree.




```
union(2, 13)
union(12, 4)
union(2, 8)
```

Does this improve the worst case runtimes?

findSet is more likely to be O(log(n)) than O(n)

Improving findSet()

Problem: Every time we call findSet() you must traverse all the levels of the tree to find representative

Solution: Path Compression

- Collapse tree into fewer levels by updating parent pointer of each node you visit
- Whenever you call findSet() update each node you touch's parent pointer to point directly to overallRoot

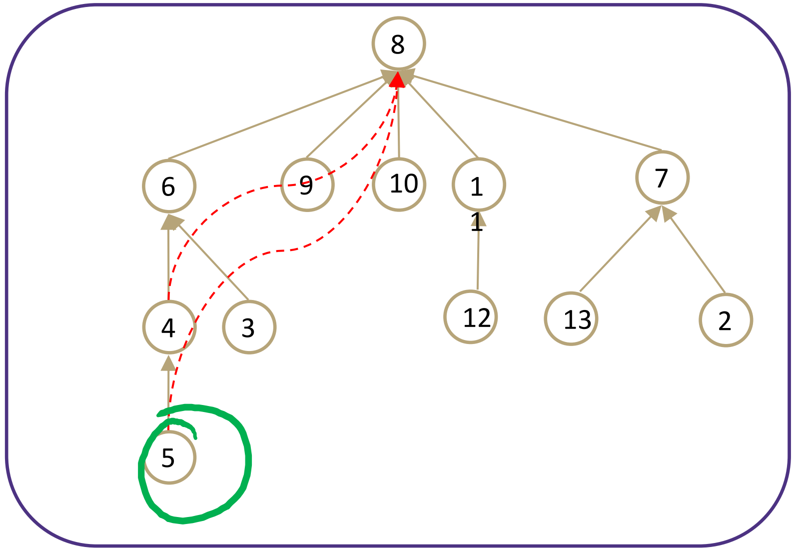
```
findSet(5)

findSet(4)
```

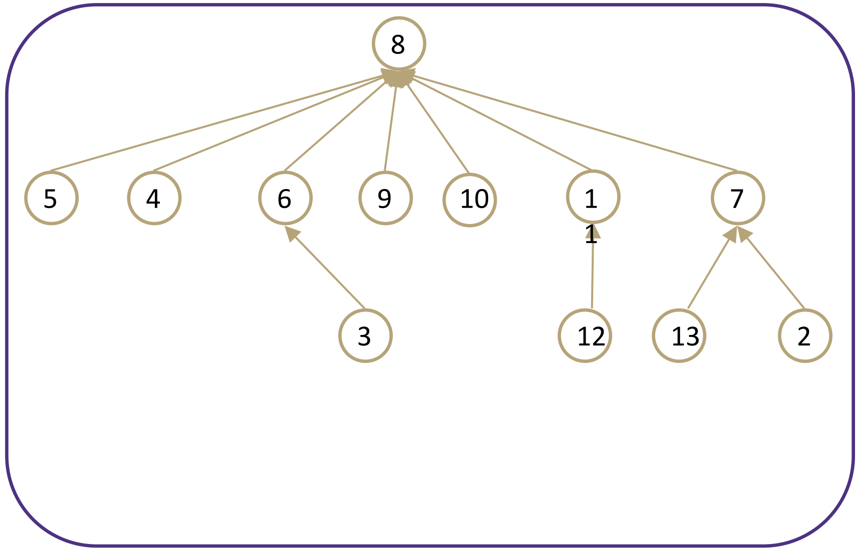
Does this improve the worst case runtimes?

findSet is more likely to be O(1) than O(log(n))

rank = 3



rank = 2

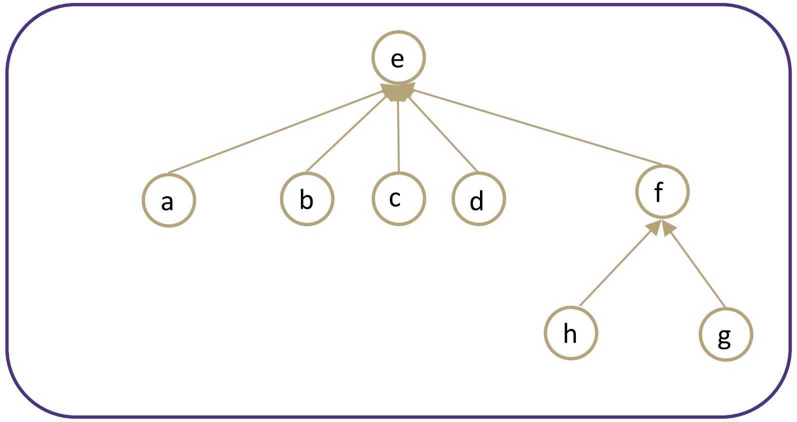


Example

Using the union-by-rank and path-compression optimized implementations of disjoint-sets draw the resulting forest caused by these calls:

```
1. makeSet(a)
2. makeSet(b)
3. makeSet(c)
4. makeSet(d)
5. makeSet(e)
6. makeSet(f)
7. makeSet(h)
8. union(c, e)
9. union(d, e)
10.union(a, c)
11.union(g, h)
12.union(b, f)
13.union(g, f)
14.union(b, c)
```

rank = 2



Array Representation

Like heaps, pretend the tree exists, but use an Array for actual implementation