CSE 373: Data Structures and Algorithms

Graph Traversals

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Graph Vocabulary

Review

Graph Direction

- Undirected graph – edges have no direction and are two-way

  V = { A, B, C }

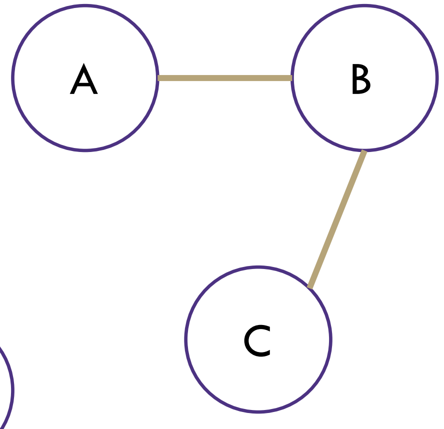
  E = { (A, B), (B, C) } *inferred (B, A) and (C,B)*

- Directed graphs – edges have direction and are thus one-way

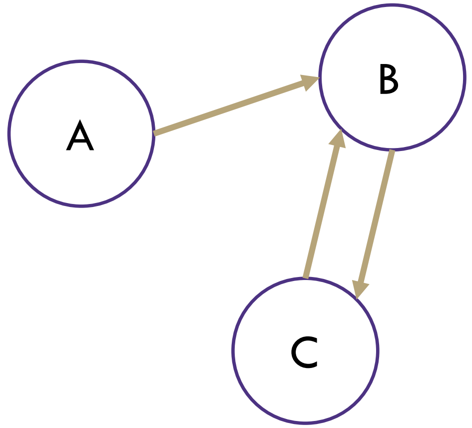
  V = { A, B, C }

  E = { (A, B), (B, C), (C, B) }

Undirected Graph:



Directed Graph:



Degree of a Vertex

- Degree – the number of edges containing that vertex

  A : 1, B : 2, C : 1

- In-degree – the number of directed edges that point to a vertex

  A : 0, B : 2, C : 1

- Out-degree – the number of directed edges that start at a vertex

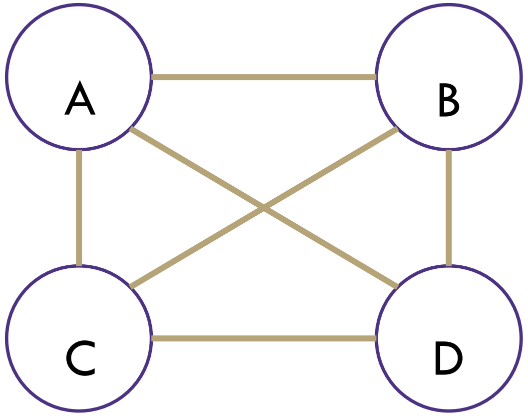
  A : 1, B : 1, C : 1

Graph Vocabulary

Review

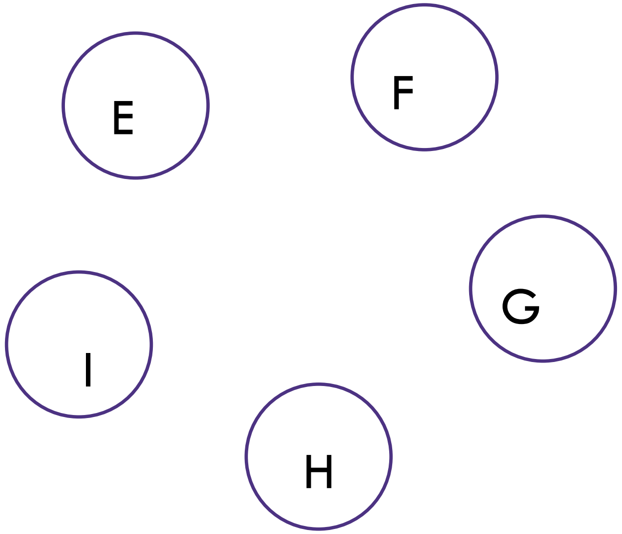
Dense Graph – a graph with a lot of edges

$E \in \Theta(V^2)$



Sparse Graph – a graph with “few” edges

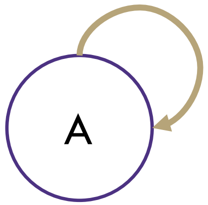
$E \in \Theta(V)$



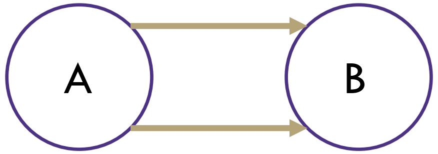
Graph Vocabulary

Review

Self loop – an edge that starts and ends at the same vertex



Parallel edges – two edges with the same start and end vertices

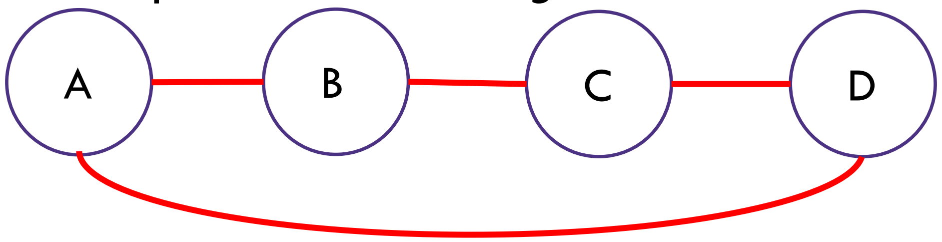


Simple graph – a graph with no self-loops and no parallel edges

Graph Vocabulary

Review

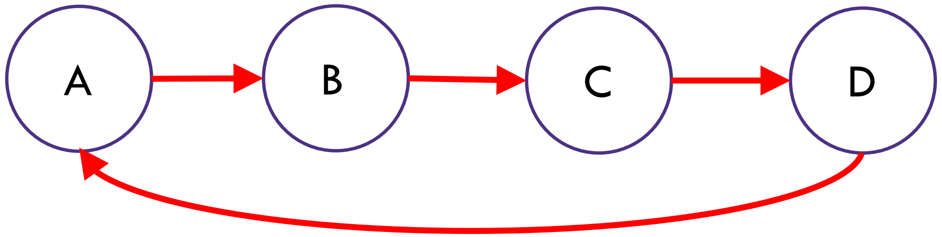
Walk – A sequence of **adjacent** vertices. Each connected to next by an edge.



A,B,C,D is a walk.

So is A,B,A

(Directed) Walk–must follow the direction of the edges



A,B,C,D,A is a directed walk.

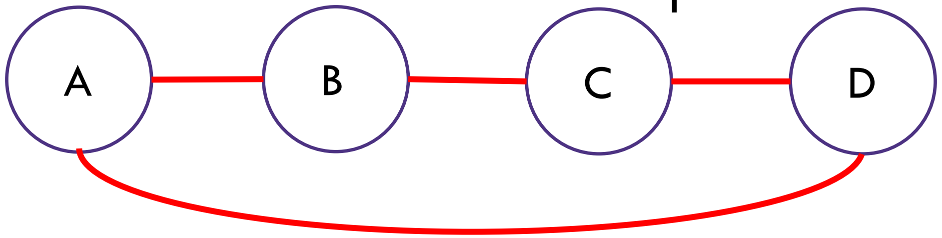
A,B,A is not.

Length – The number of edges in a walk

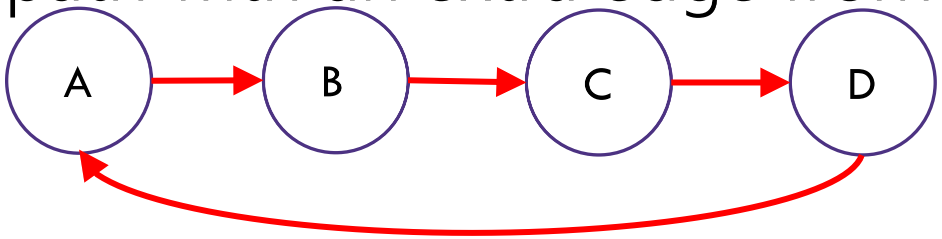
- (A,B,C,D) has length 3.

Graph Vocabulary

Path – A walk that doesn't repeat a vertex. A,B,C,D is a path. A,B,A is not.



Cycle – path with an extra edge from last vertex back to first.



Be careful looking at other sources.

Some people call our "walks" "paths" and our "paths" "simple paths"

Use the definitions on these slides.

Paths and Reachability

Common questions:

- Is there a path between two vertices? (Can I drive from Seattle to LA?)
- What is the length of the shortest path between two vertices? (How long will it take?)
- List vertices that can reach the maximum number of nodes with a path of length 2.
- Can every vertex reach every other on a short path?
  - Length of the longest shortest path is the “diameter” of a graph

Implementing a Graph

Two main ways to implement a graph:

1. Adjacency Matrix
2. Adjacency List

Adjacency Matrix

Assign each vertex a number from 0 to V – 1

Create a V x V array of Booleans (or Int, as 0 and 1)

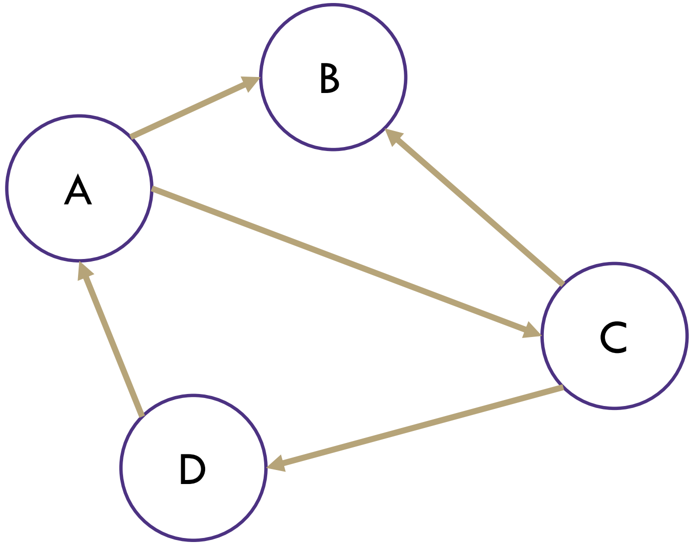
If (x,y) ∈ E then arr[x][y] = true

	A	B	C	D
A		T	T	
B				
C		T		T
D	T			

Time complexity (in terms of V and E)

- Get in-edges:
- Get out–edges:
- Decide if an edge (u, w) exists:
- Insert an edge:
- Delete an edge:

Space complexity:



Adjacency Matrix

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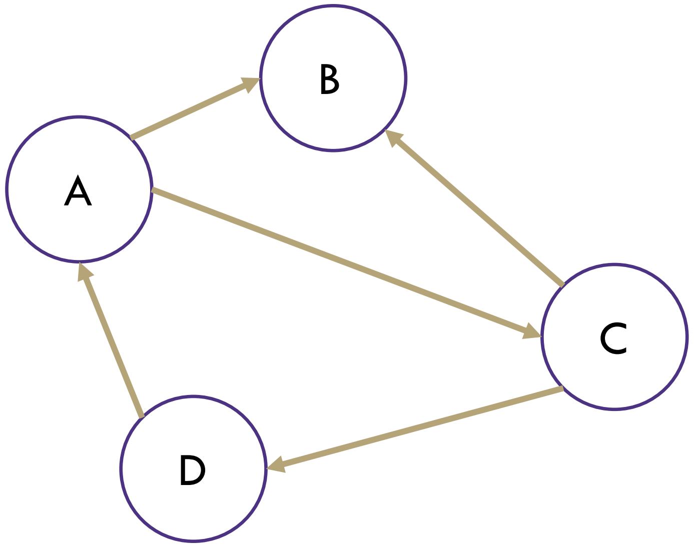
If (x,y) ∈ E then arr[x][y] = true

	A	B	C	D
A		T	T	
B				
C		T		T
D	T			

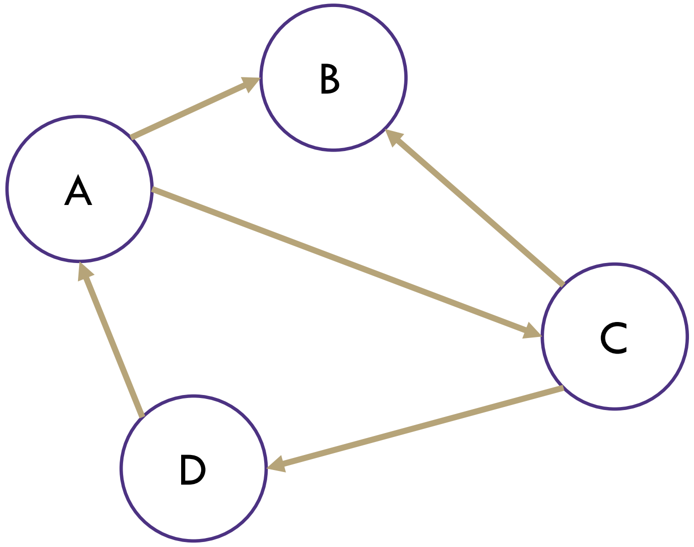
Time complexity (in terms of V and E)

- Get in-edges: $O(|V|)$
- Get out–edges: $O(|V|)$
- Decide if an edge (u, w) exists: $O(1)$
- Insert an edge: $O(1)$
- Delete an edge: $O(1)$

Space complexity: $O(|V|^2)$



Adjacency List

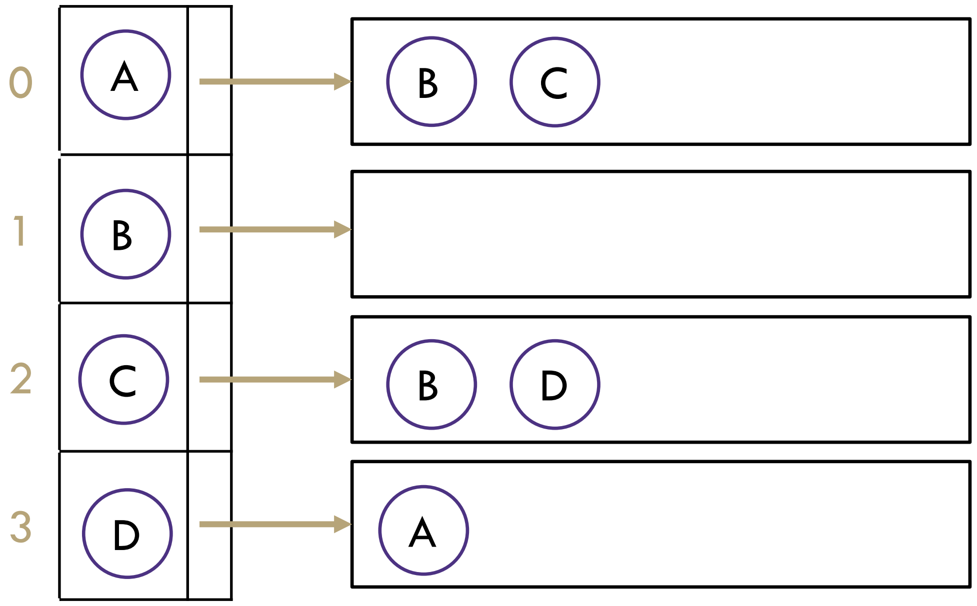


Create a Dictionary of size V from type V to Collection of E

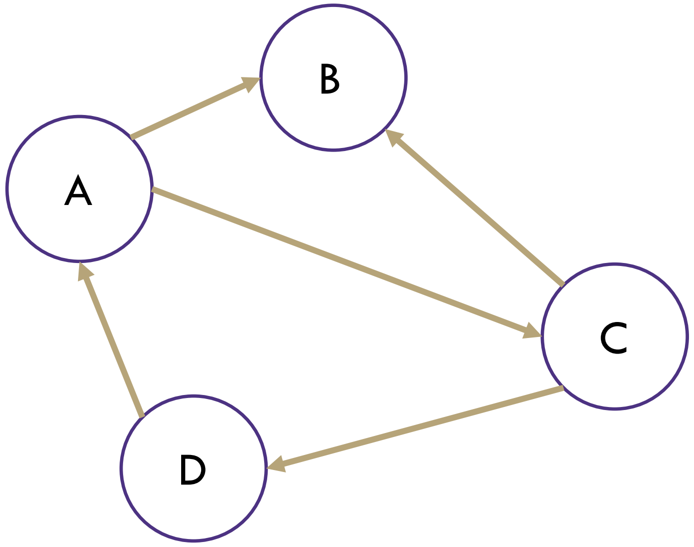
If (x,y) ∈ E then add y to the set associated with the key x

Time complexity
- Get in-edges:
- Get out–edges:
- Decide if an edge (u, w) exists:
- Insert an edge:
- Delete an edge:

Space complexity:



Adjacency List



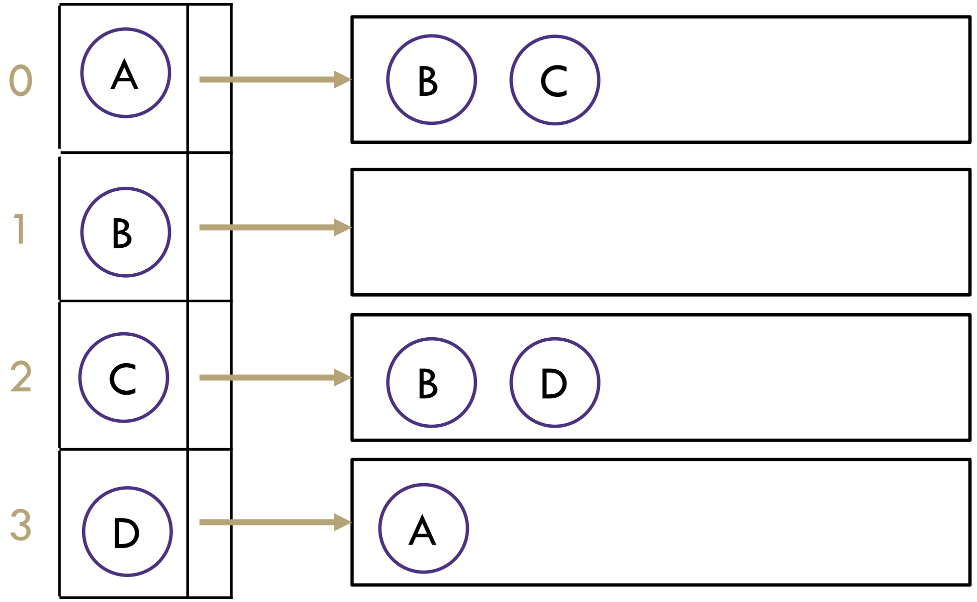
Create a Dictionary of size V from type V to Collection of E

If (x,y) ∈ E then add y to the set associated with the key x

Time complexity

- Get in-edges: $O(|V| + |E|)$
- Get out–edges: $O(1)$
- Decide if an edge (u, w) exists: $O(1)$
- Insert an edge: $O(1)$
- Delete an edge: $O(1)$

Space complexity: $O(|V| + |E|)$

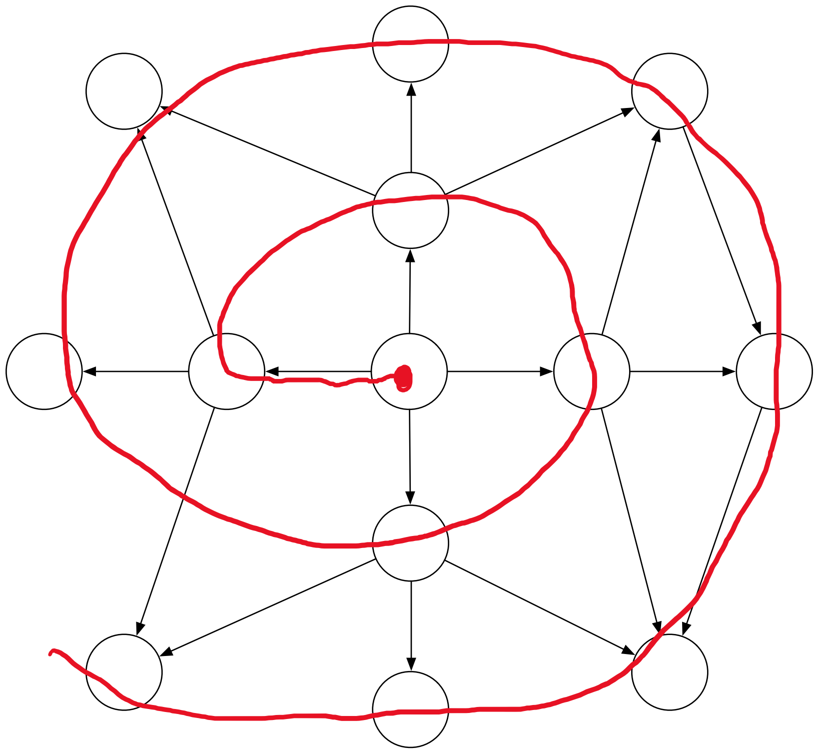


Traversing a graph

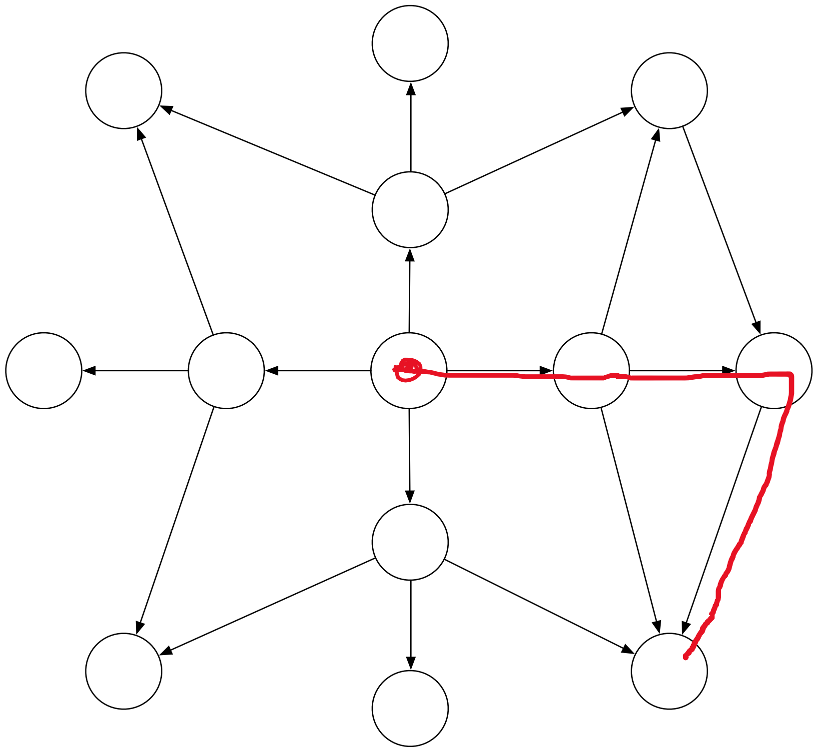
Four collections: ‘unvisited’, ‘visited’, ‘to be visited’, and ‘current’.

1. The vertex you are currently processing is your ‘current’ vertex.
2. Pick any vertex to start
3. Put all neighbors of the *current* vertex in a “to be visited” collection
4. Mark the current vertex “visited”
5. Move onto next vertex in “to be visited” collection
6. Put all unvisited neighbors in “to be visited”
7. Move onto next vertex in “to be visited” collection
8. Repeat…

Traversing a graph

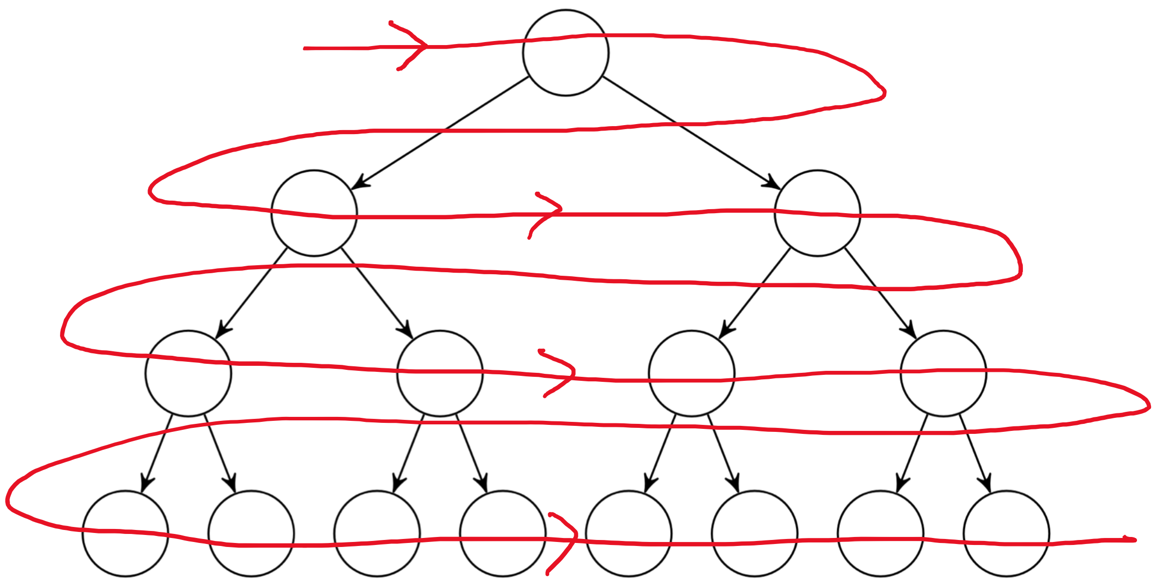


Breadth first search

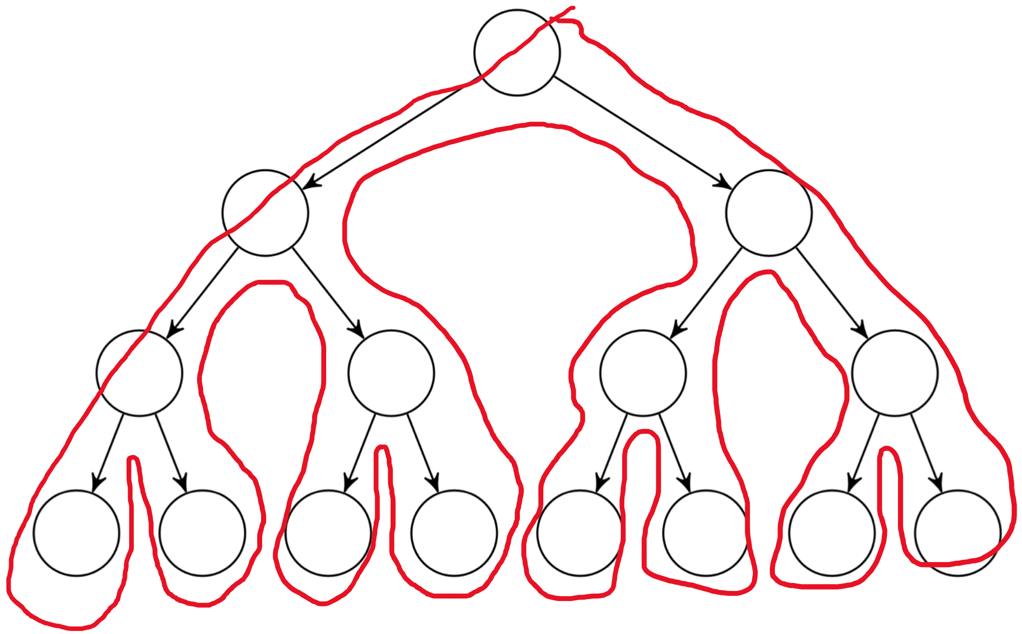


Depth first search

BFS and DFS on Trees



Breadth first search
(Level order traversal)



Depth first search
(pre-order, in-order, post-order traversals)

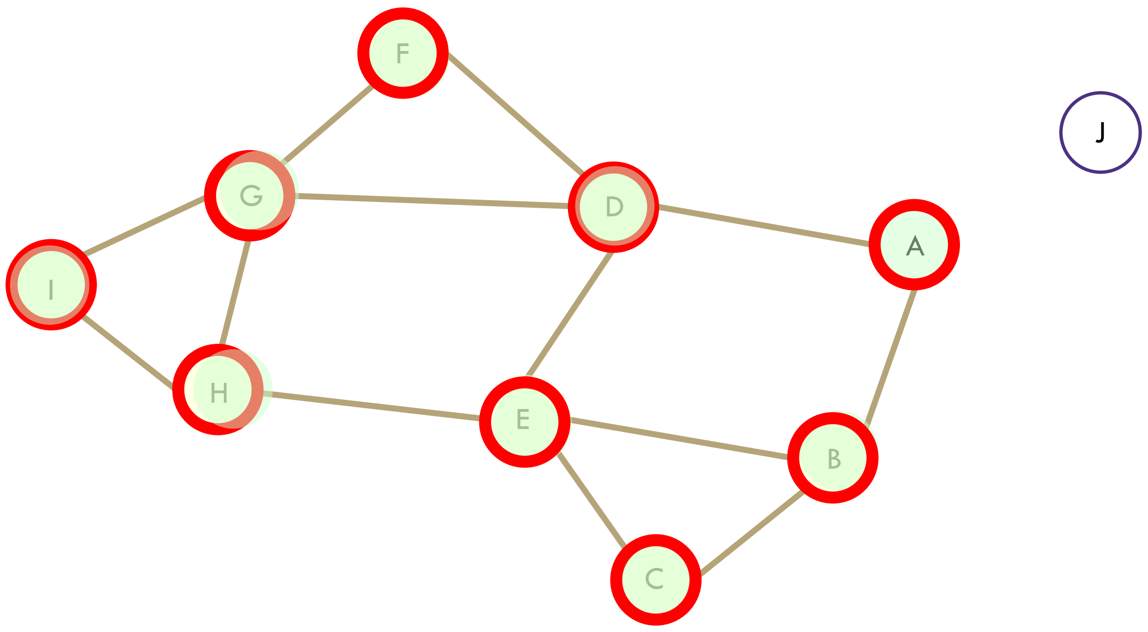
Breadth First Search

```
search(graph)
   toVisit.enqueue(first vertex)
   mark first vertex as visited
   while(toVisit is not empty)
      current = toVisit.dequeue()
      for (V : current.neighbors())
         if (v is not visited)
            toVisit.enqueue(v)
            mark v as visited
      finished.add(current)
```

Current node: I

Queue: B D E C F G H I

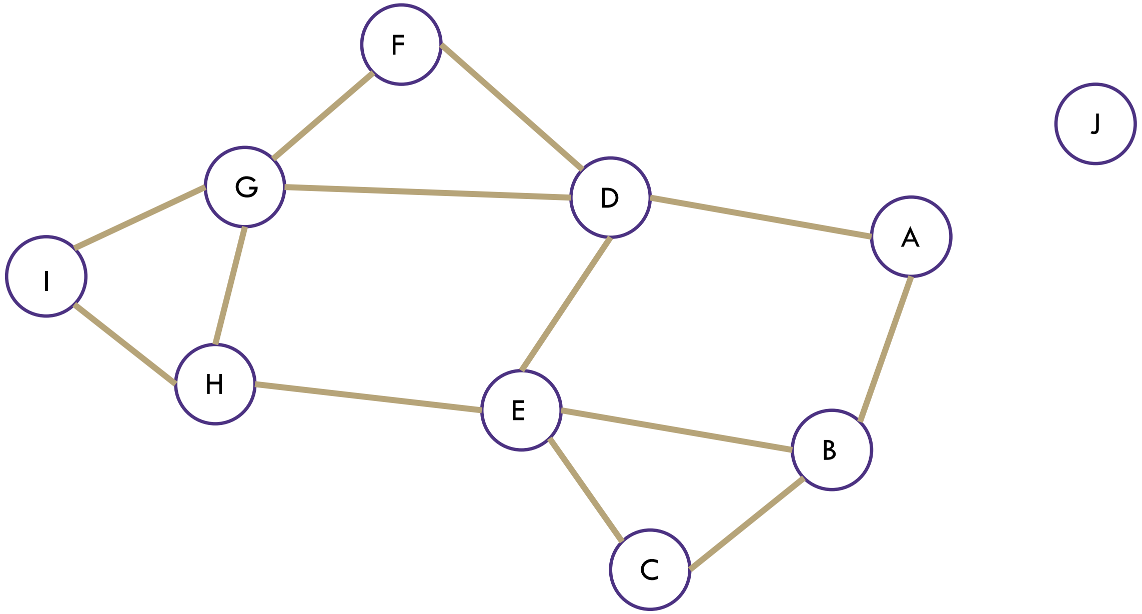
Finished: A B D E C F G H I



Breadth First Search Analysis

```
search(graph)
   toVisit.enqueue(first vertex)
   mark first vertex as visited
   while(toVisit is not empty)
      current = toVisit.dequeue()
      for (V : current.neighbors())
         if (v is not visited)
            toVisit.enqueue(v)
            mark v as visited
      finished.add(current)
```

Visited: A B D E C F G H I



How many times do you visit each node?

1 time each

How many times do you traverse each edge?

Max 2 times each

- Putting them into toVisit
- Checking if they're visited

Runtime? $O(V + 2E) = O(V + E)$ "graph linear"

Depth First Search (DFS)

BFS uses a queue to order which vertex we move to next

Gives us a growing “frontier” movement across graph

Can you move in a different pattern? Can you use a different data structure?

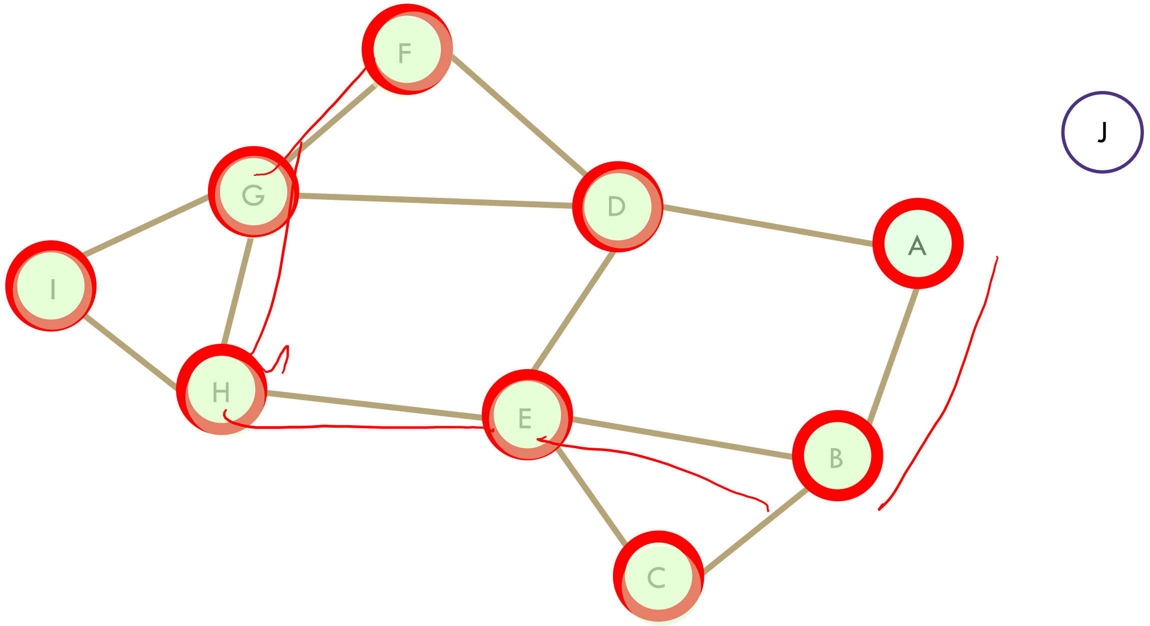
What if you used a stack instead?

```
bfs(graph)
   toVisit.enqueue(first vertex)
   mark first vertex as visited
   while(toVisit is not empty)
      current = toVisit.dequeue()
      for (V : current.neighbors())
         if (v is not visited)
            toVisit.enqueue(v)
            mark v as visited
      finished.add(current)
```

```
dfs(graph)
   toVisit.push(first vertex)
   mark first vertex as visited
   while(toVisit is not empty)
      current = toVisit.pop()
      for (V : current.neighbors())
         if (V is not visited)
            toVisit.push(v)
            mark v as visited
      finished.add(current)
```

Depth First Search

```
dfs(graph)
   toVisit.push(first vertex)
   mark first vertex as visited
   while(toVisit is not empty)
      current = toVisit.pop()
      for (V : current.neighbors())
         if (V is not visited)
            toVisit.push(v)
            mark v as visited
      finished.add(current)
```



Current node: D

Stack: D C EI HG

Finished: A B E H G F I C D

How many times do you visit each node? 1 time each

How many times do you traverse each edge? Max 2 times each

- Putting them into toVisit
- Checking if they're visited

Runtime? $O(V + 2E) = O(V + E)$ "graph linear"