CSE 373: Data Structures and Algorithms

Floyd's buildHeap, Sorting

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Today

- Heap review and Array implementation of Heap
- Floyd's buildHeap algorithm
- Intro to Sorting
- Insertion sort
- Heap sort

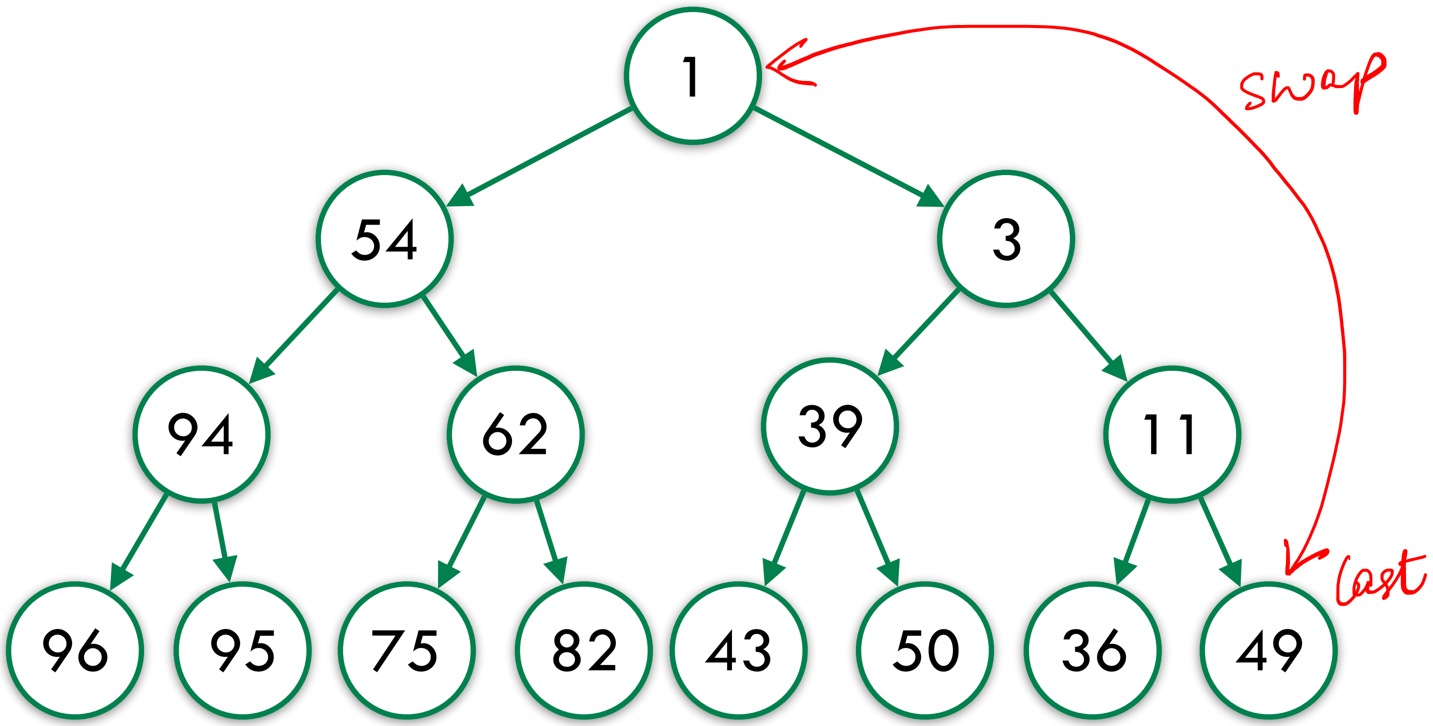
Heap review

Heap is a tree-based data structure that satisfies

- (a) structure property: it's a complete tree
- (b) heap property, which states:
  - for min-heap: $parent \leq children$
  - for max-heap: $parent \geq children$
- Operations (for min-heap):
  - removeMin()
  - peekMin()
  - insert()
- Applications: priority queue, sorting, ..

removeMin()

Calls:
removeMin()

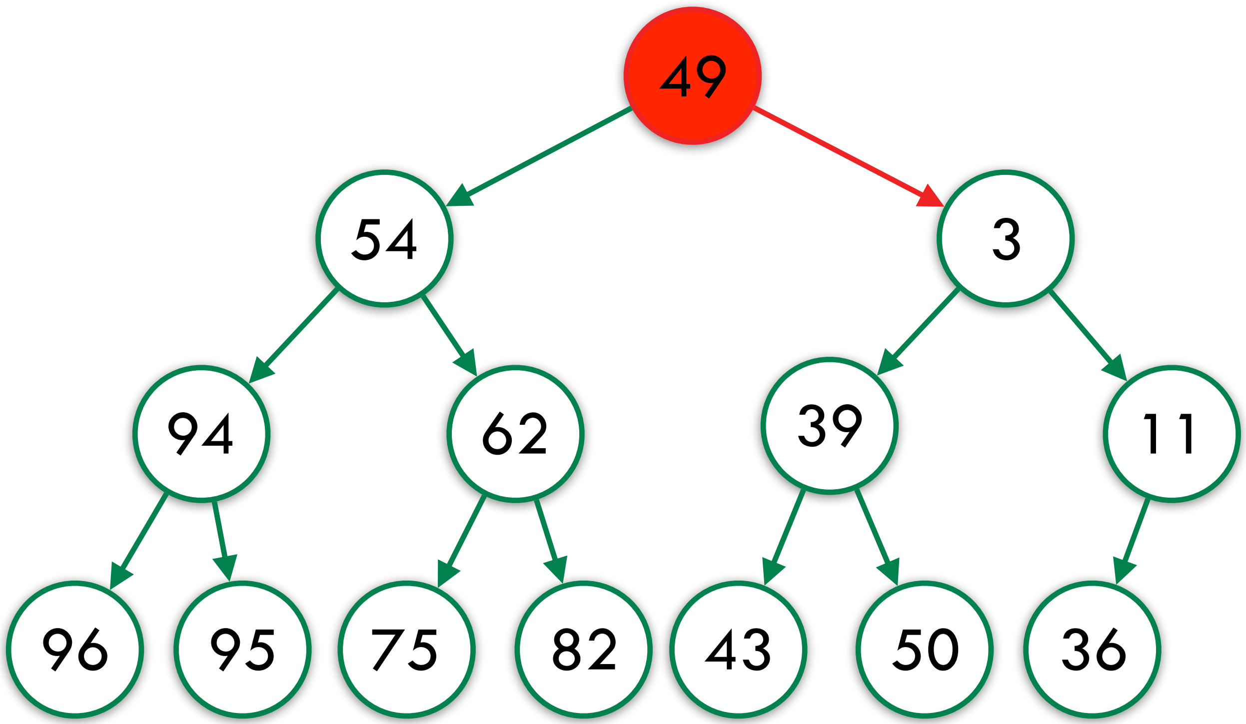


min-heap

function removeMin
  last = last node in the tree
  minvalue = *root*
  swap *root* with *last*
  percolateDown(*root*)
  return *minvalue*
end function

removeMin()

Calls:
removeMin() 1

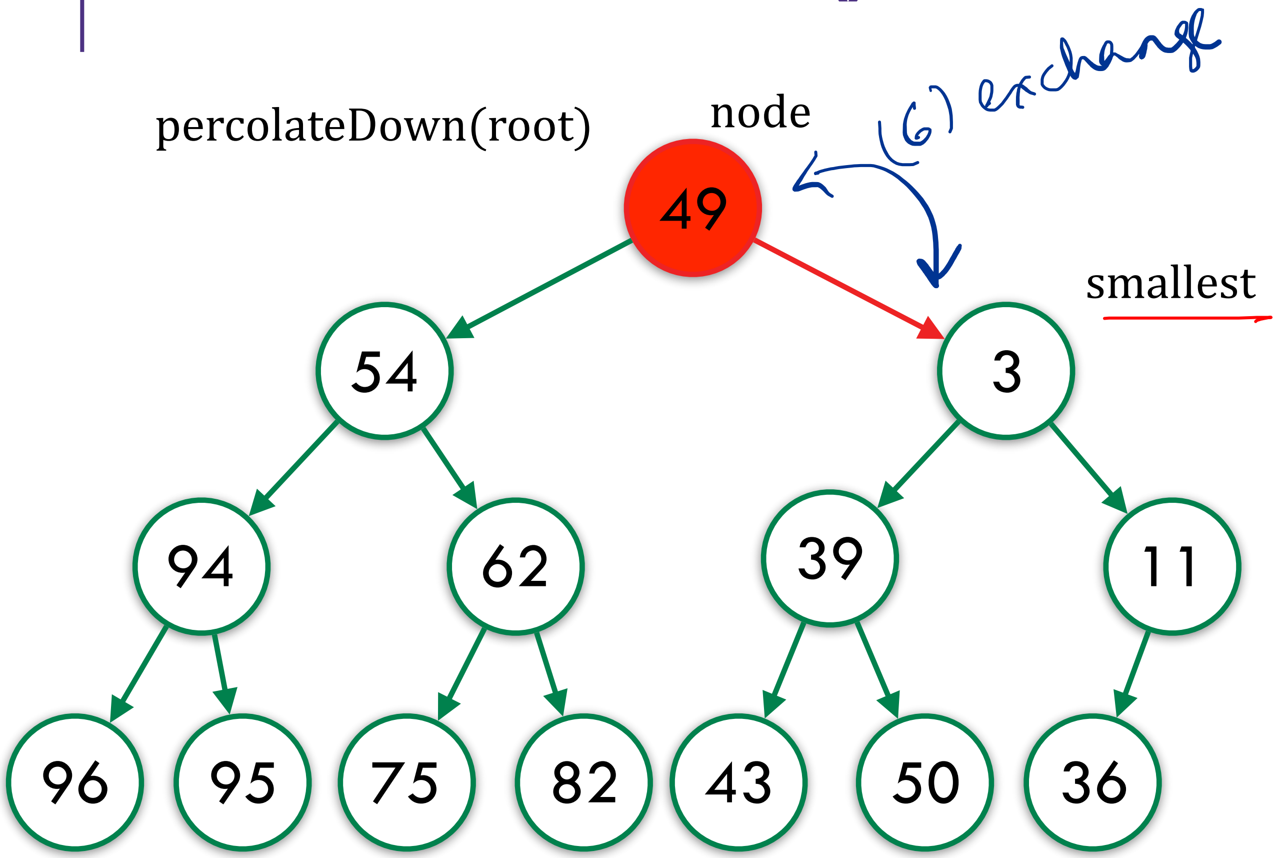


Broken heap

function removeMin
$last$ = last node in the tree
$minvalue = root$
swap $root$ with $last$
percolateDown($root$)
return $minvalue$
end function

removeMin()

Calls:
removeMin() 1

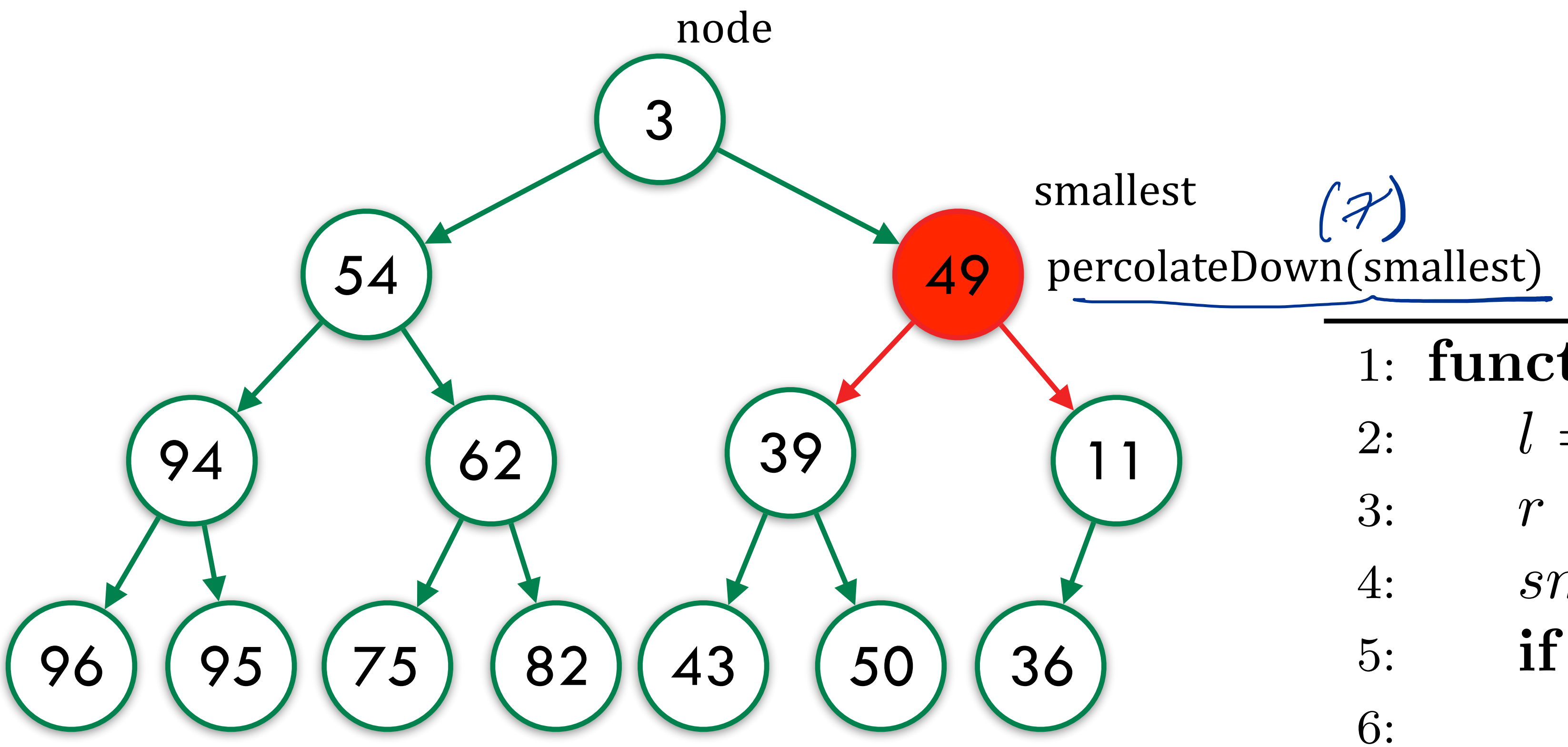


Broken heap

1: **function** percolateDown($node$)
2: l = left child of $node$
3: r = right child of $node$
4: $smallest$ = smallest in $\{l, r, node\}$
5: **if** $smallest \neq node$ **then**
6: exchange $node$ with $smallest$
7: percolate($smallest$)
8: **end if**
9: **end function**

removeMin()

Calls:
removeMin() 1

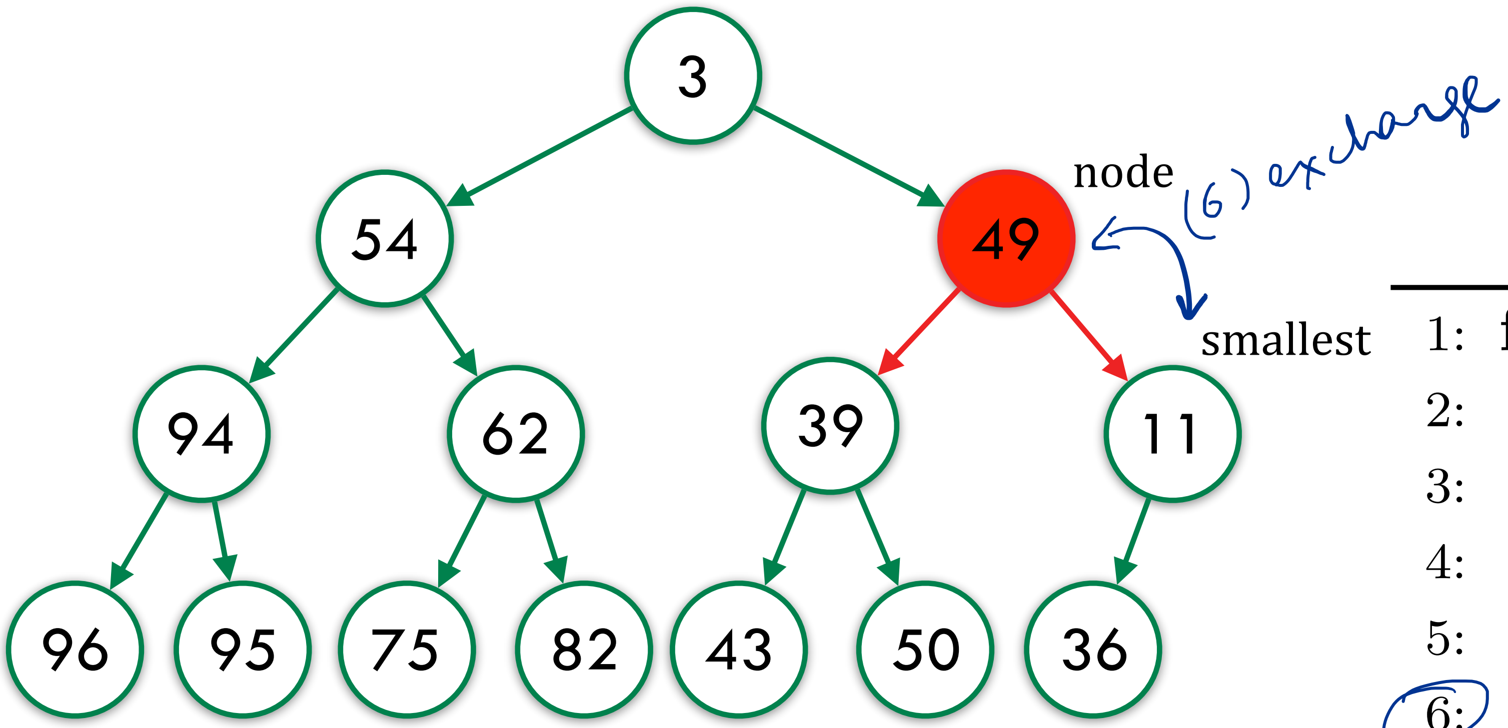


Broken heap

```
function percolateDown(node)
    l = left child of node
    r = right child of node
    smallest = smallest in {l, r, node}
    if smallest ≠ node then
        exchange node with smallest
        percolate(smallest)
    end if
end function
```

removeMin()

Calls:
removeMin() 1



Broken heap

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1: function percolateDown(node)
2:     l = left child of node
3:     r = right child of node
4:     smallest = smallest in {l, r, node}
5:     if smallest ≠ node then
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removeMin()

Calls:
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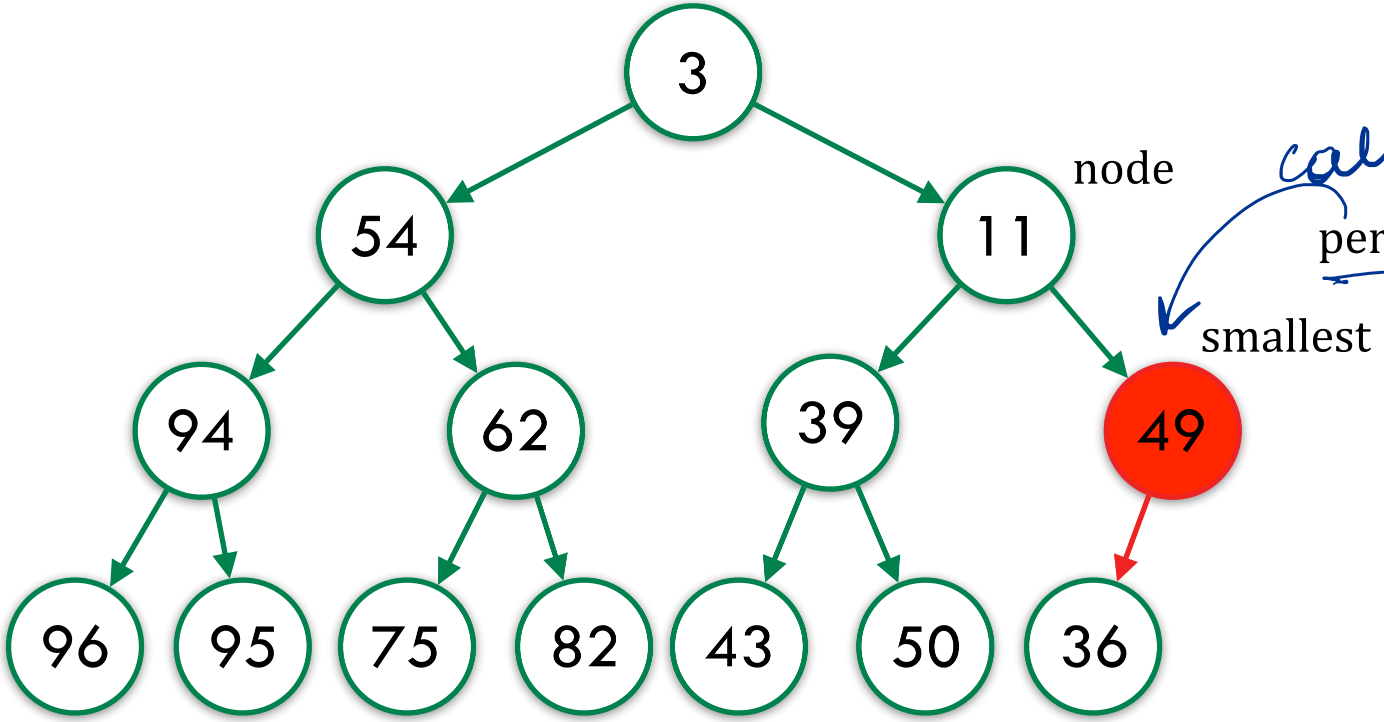

Broken heap

called on this node (7)

percolateDown(smallest)

```
1: function percolateDown(node)
2:     l = left child of node
3:     r = right child of node
4:     smallest = smallest in {l, r, node}
5:     if smallest ≠ node then
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7:         percolate(smallest)
8:     end if
9: end function
```

removeMin()

Calls:
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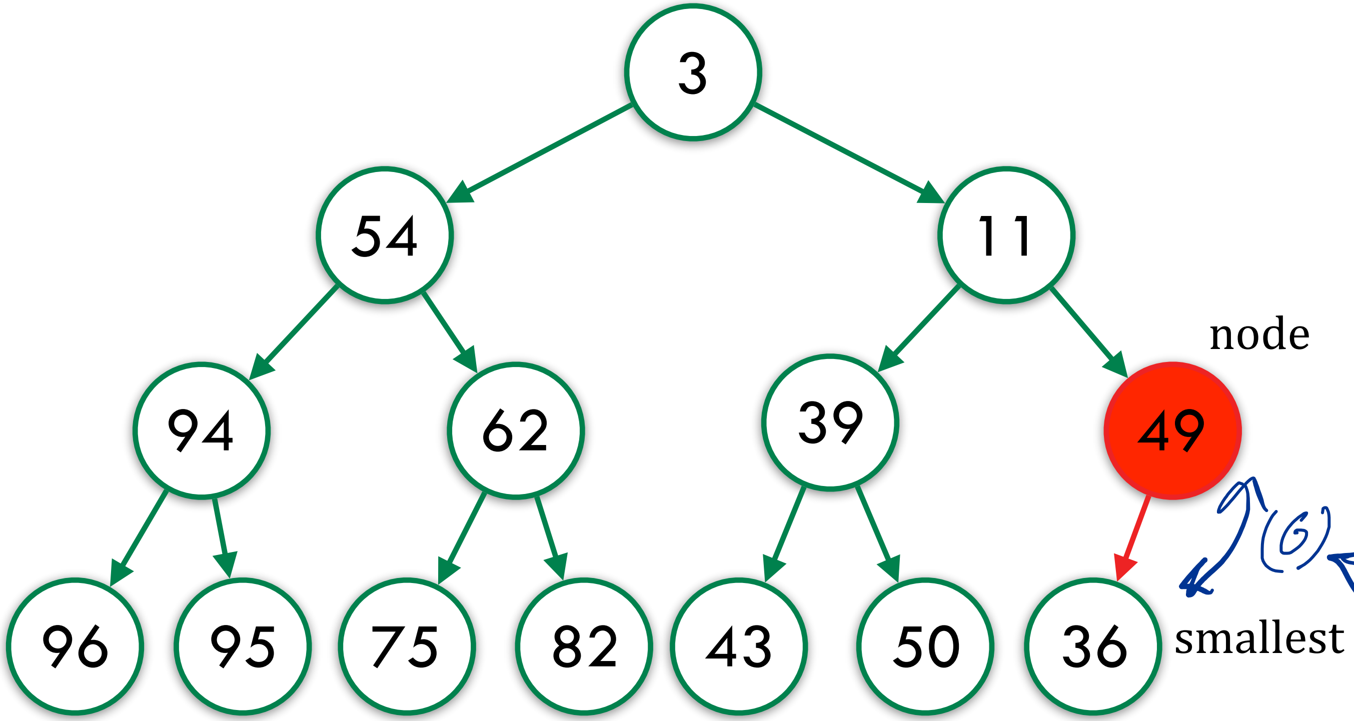

Broken heap

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function percolateDown(node)
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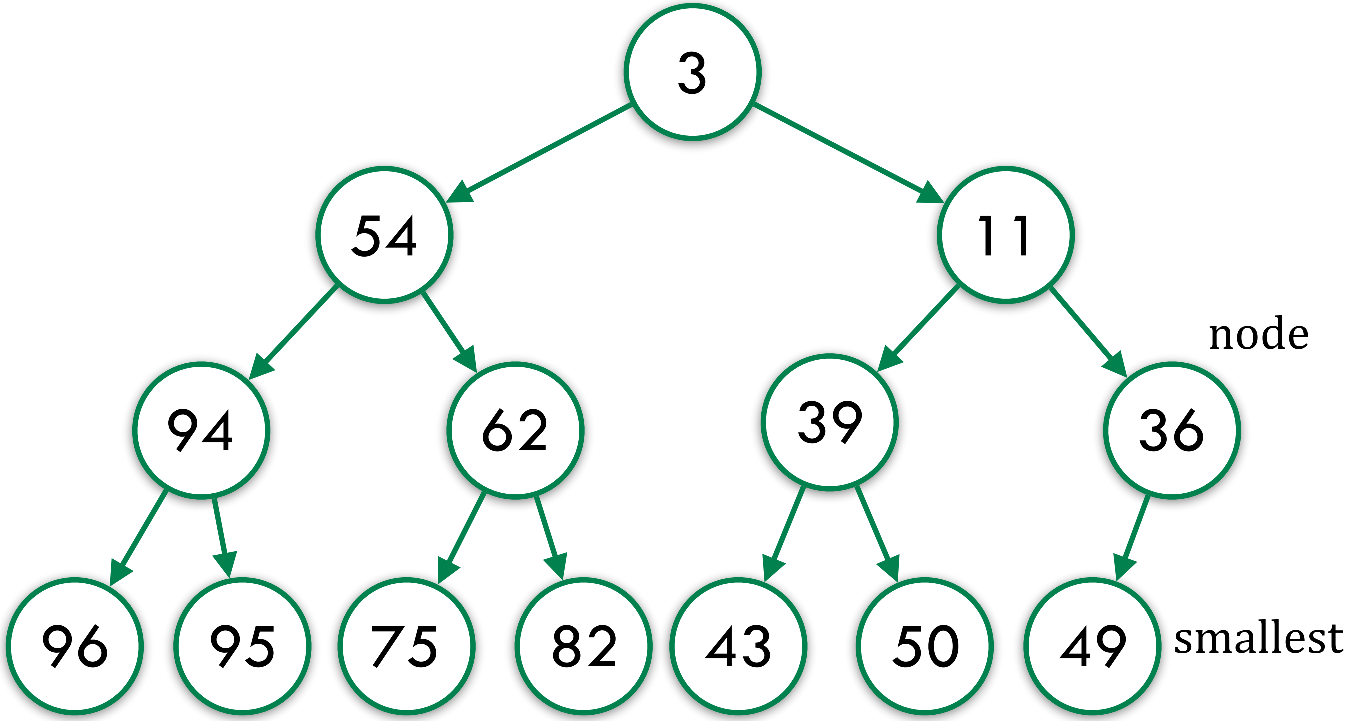
    end if
end function
```

removeMin()

Calls:
removeMin() 1



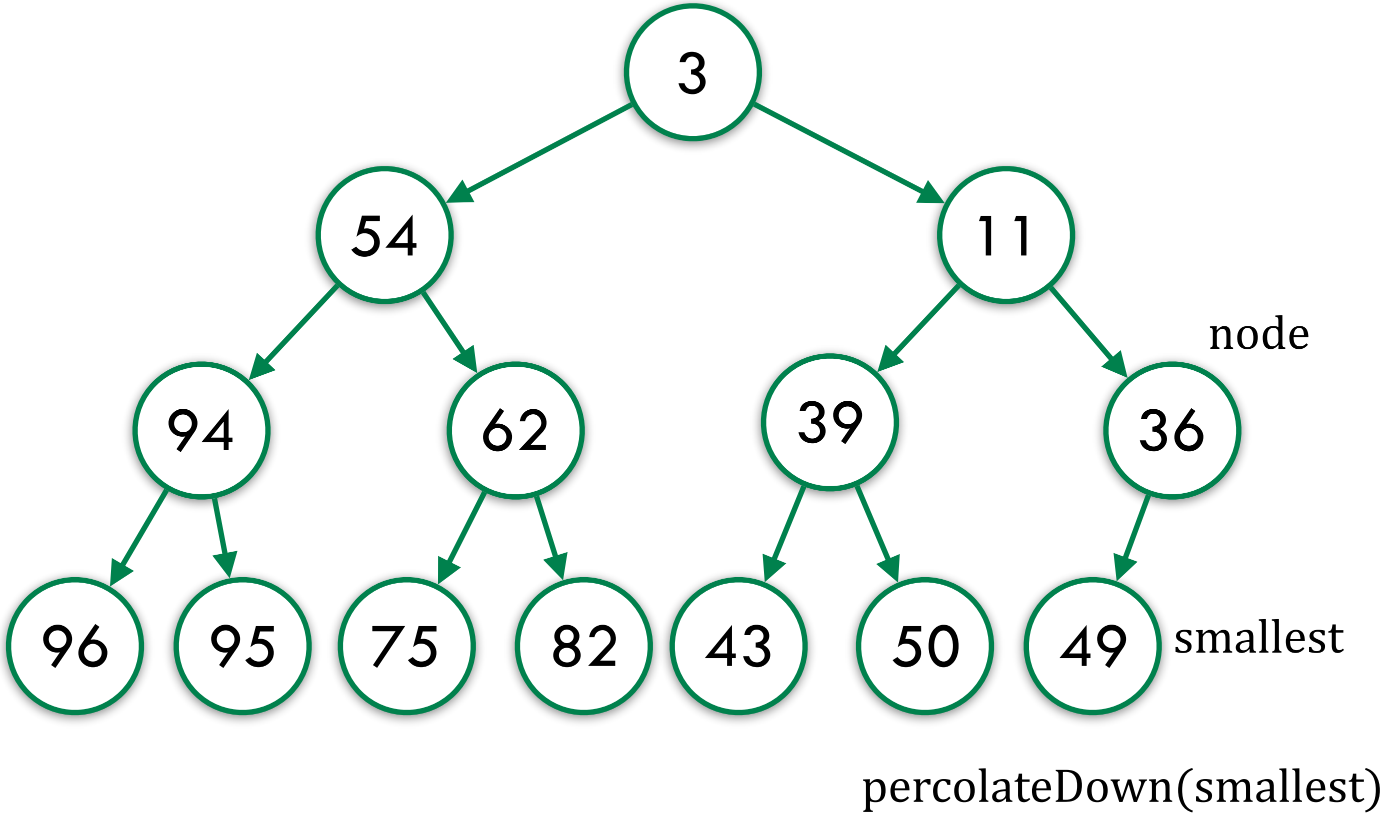
percolateDown(smallest)

final recursive call

min-heap

```
function percolateDown(node)
    l = left child of node
    r = right child of node
    smallest = smallest in {l, r, node}
    if smallest ≠ node then
        exchange node with smallest
        percolate(smallest)
    end if
end function
```

removeMin()



min-heap

---

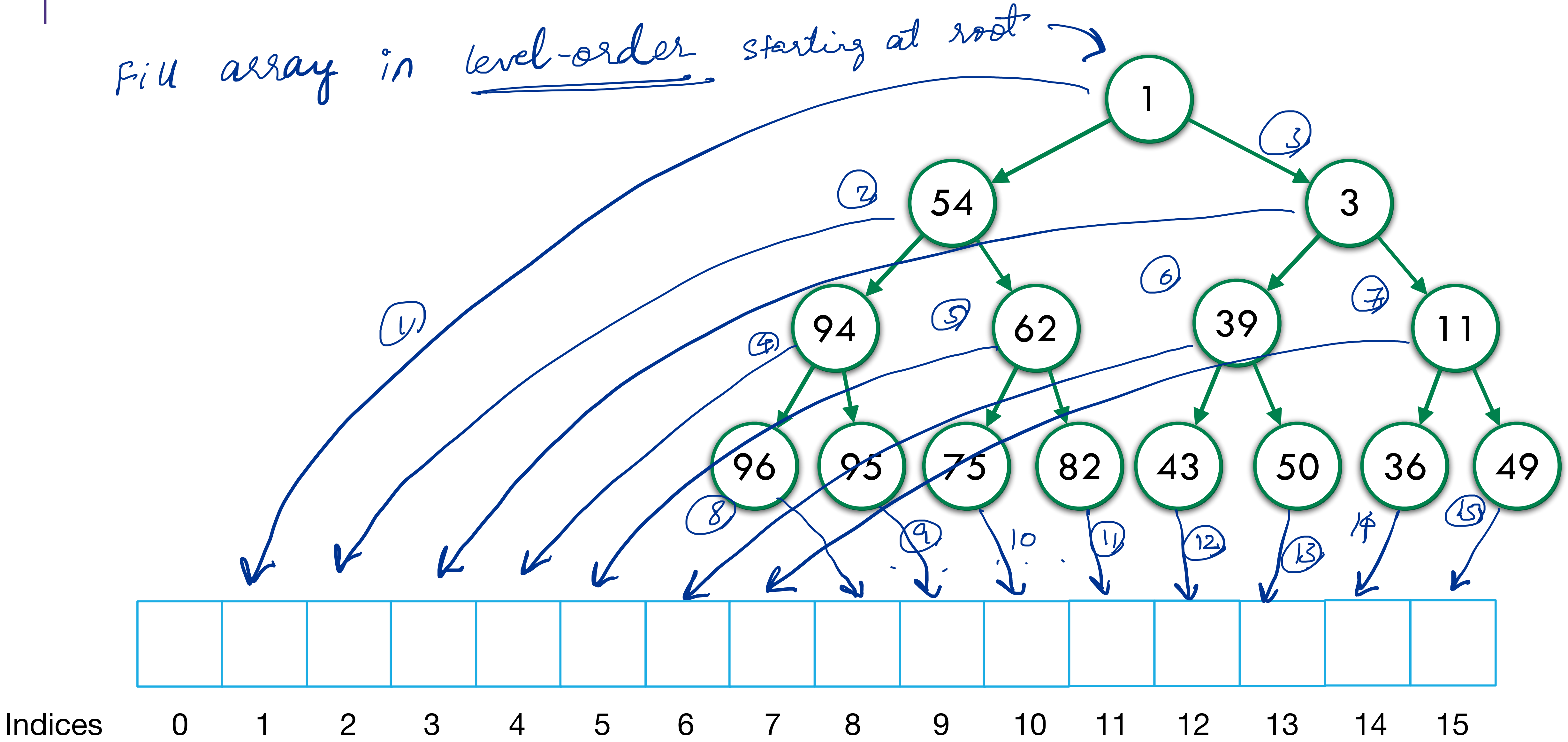
function removeMin
    last = last node in the tree
    minvalue = *root*
    swap *root* with *last*
    percolateDown(*root*)
    return *minvalue*
end function

Runtime of removeMin():

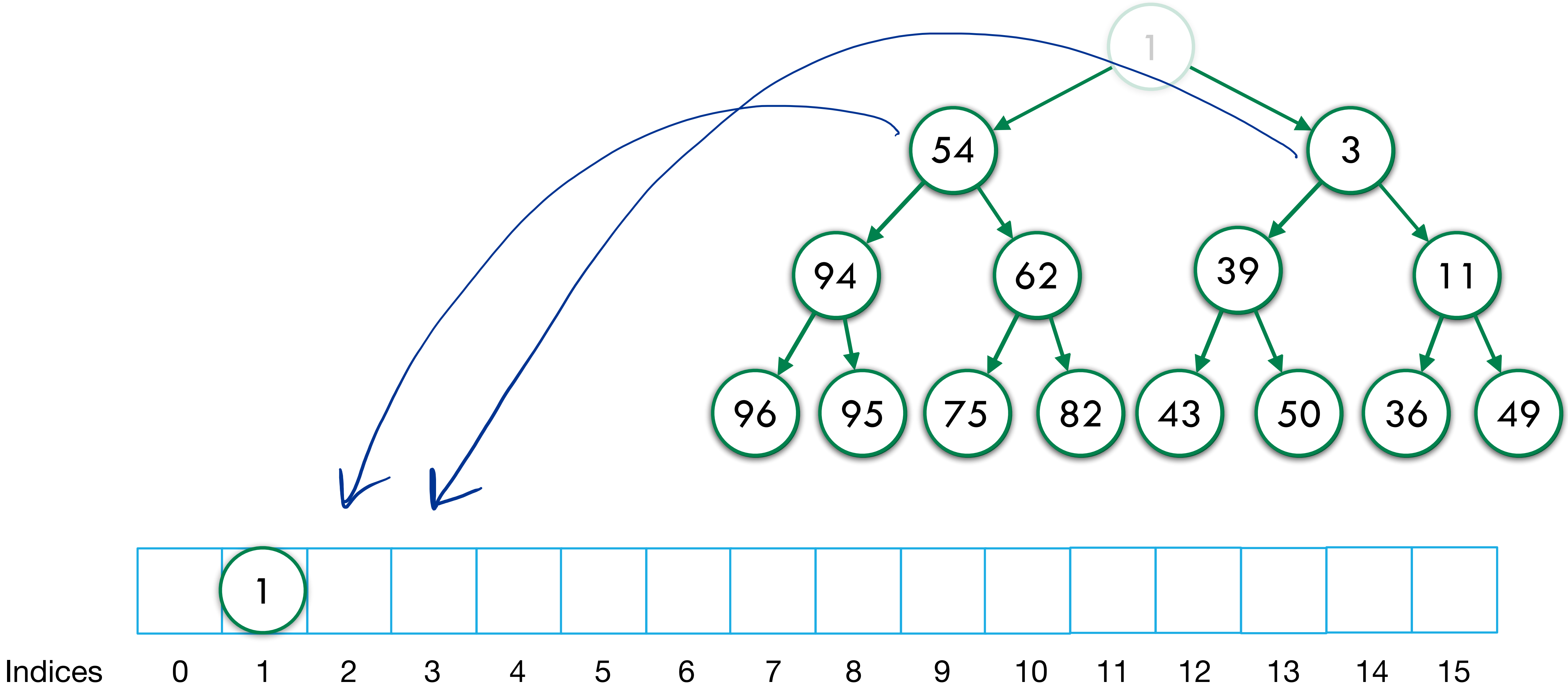
$$O(n) + c_1 + O(n \log n) = O(n)$$

How can we do better?

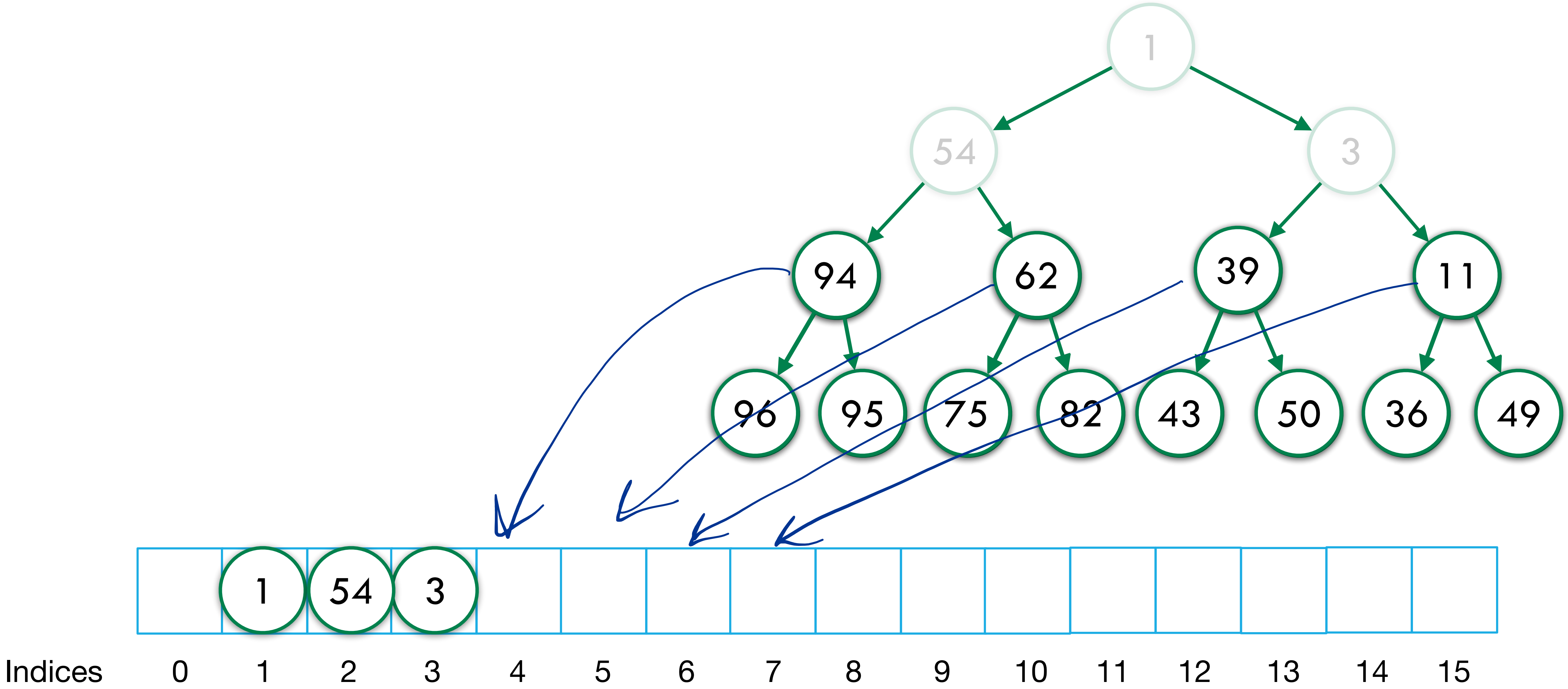
Binary heap: Array implementation



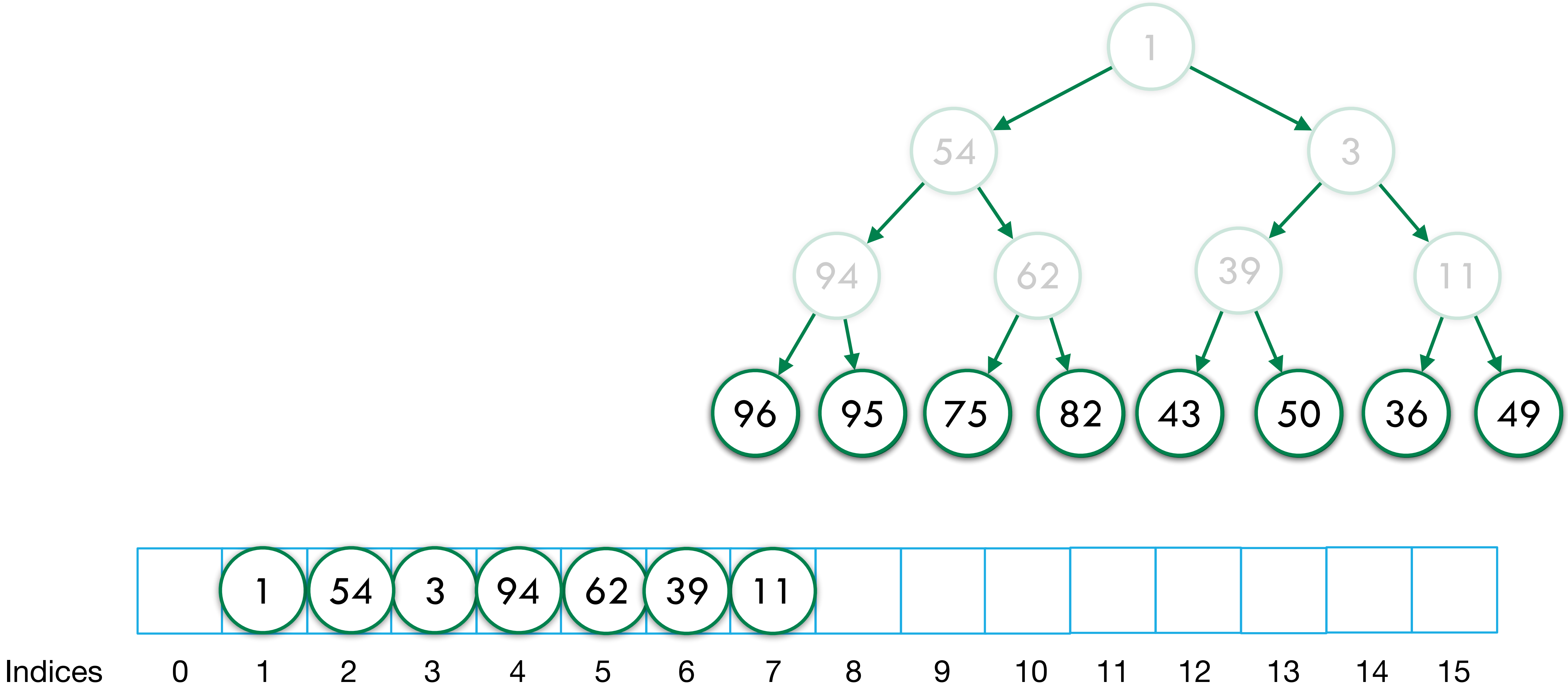
Binary heap: Array implementation



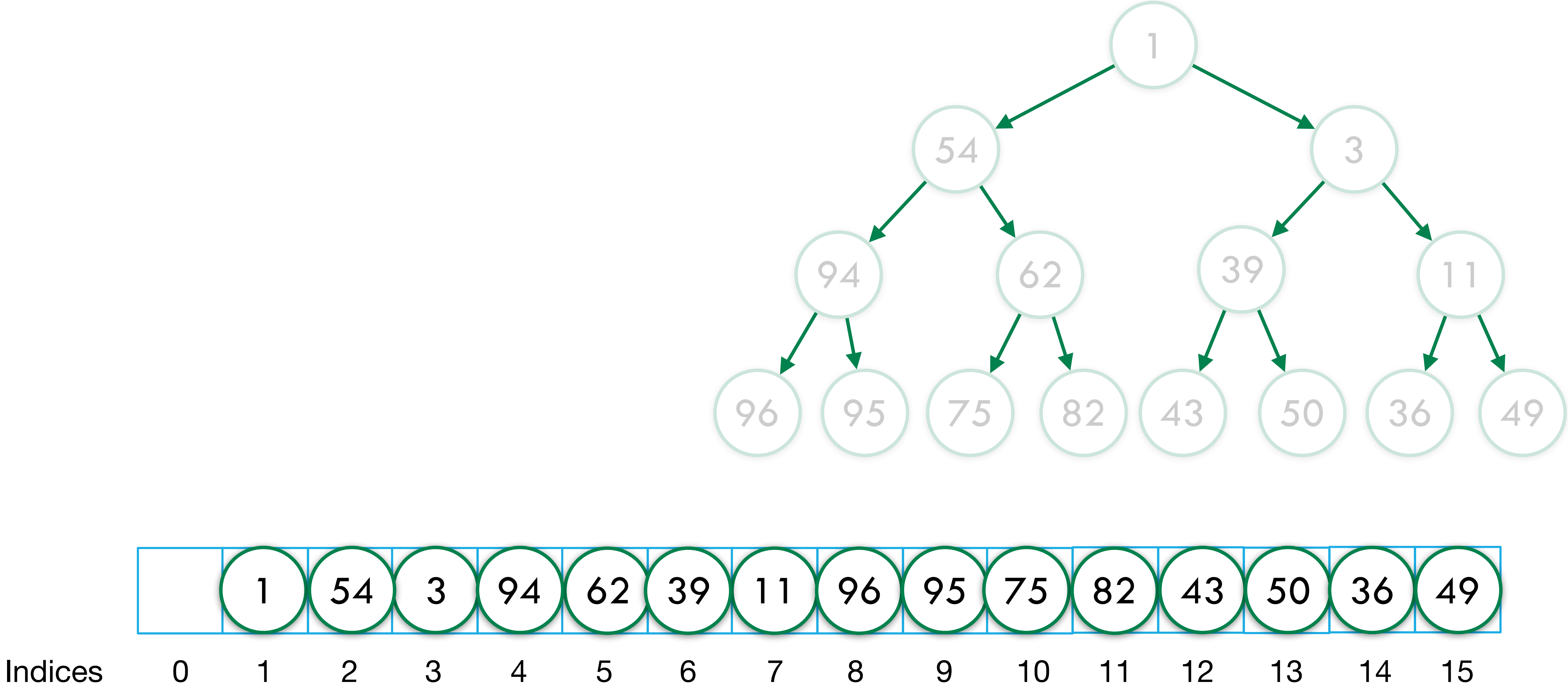
Binary heap: Array implementation



Binary heap: Array implementation



Binary heap: Array implementation

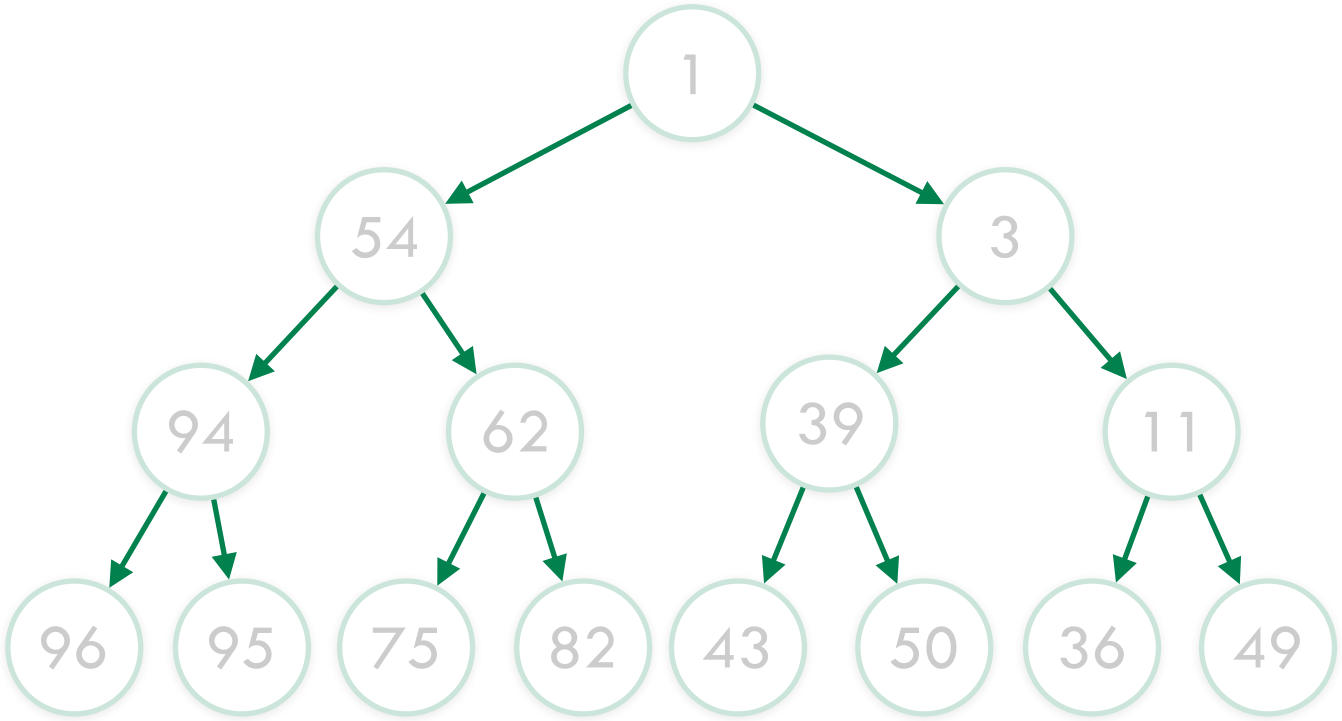


Binary heap: Array implementation

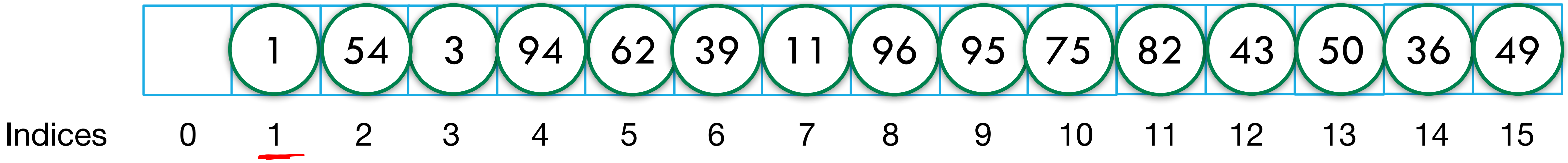
$$leftChild(i) = 2i$$

$$rightChild(i) = 2i + 1$$

$$parent(i) = \left\lfloor \frac{i}{2} \right\rfloor$$



With array starting at index 1

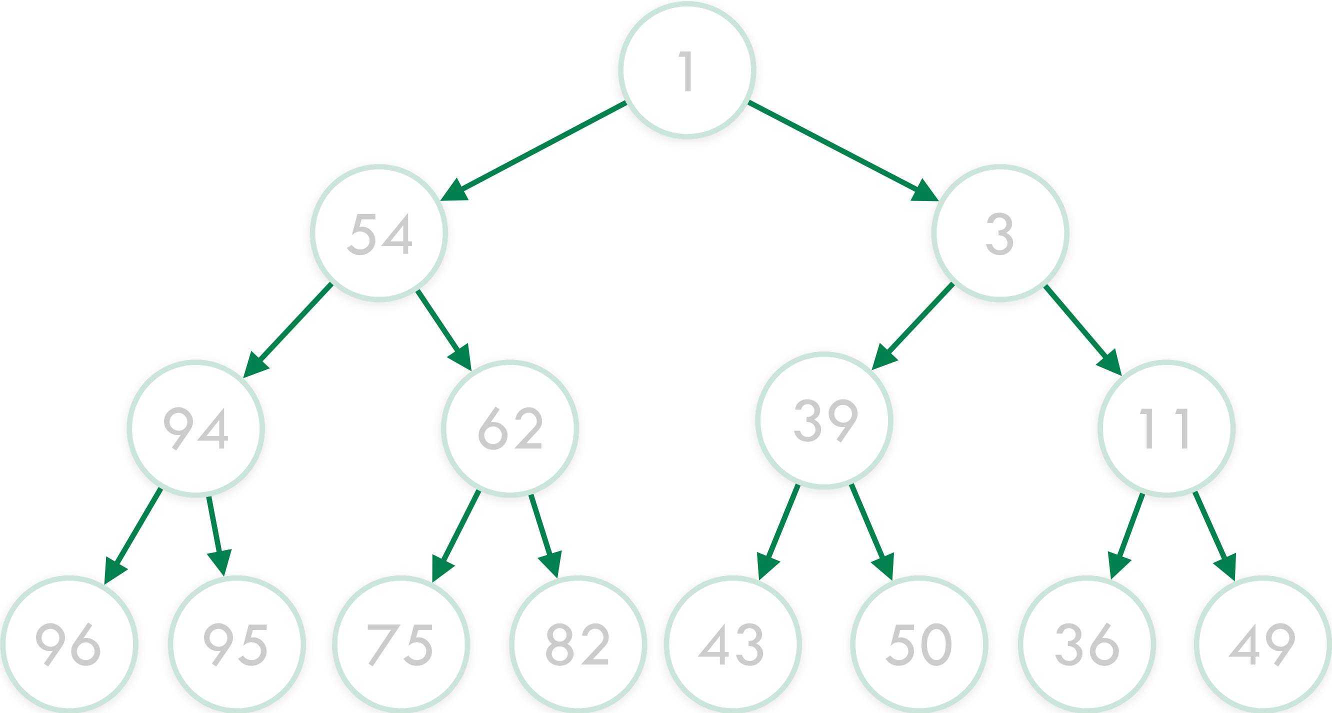


Binary heap: Array implementation

$$leftChild(i) = 2i + 1$$

$$rightChild(i) = 2i + 2$$

$$parent(i) = \left\lfloor \frac{i-1}{2} \right\rfloor$$



With array starting at index 0

	1	54	3	94	62	39	11	96	95	75	82	43	50	36	49	
Indices	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Sorting

Sorting

- - Problem: Arrange items in a collection in a specified order.
- - Lots of applications:
  - lookup / search
  - merging sequences
  - data processing
- Lots of sorting algorithms out there
- Why study sorting?

Types of sorting algorithms

1. Comparison Sorts
   - Order of elements determined by comparing them
   - Fastest comparison sort:
   - Elements should support compareTo()

$O(n \log n)$

2. Non-comparison Sorts
   - Order of elements determined by leveraging properties of input
   - Typical runtime:
   - Also called as Niche Sorts aka “linear sorts”

$O(n)$

In this class we’ll focus on comparison sorts

Insertion sort

0	1	2	3	4	5	6	7	8	9
2	3	6	7	5	1	4	10	2	8

https://visualgo.net/en/sorting

Insertion sort

0	1	2	3	4	5	6	7	8	9
2	3	6	7	5	1	4	10	2	8

- Runtime:
- Stable:
- In-place:

for i = 0 to n **do**
  $current = A[i]$
  $j = findNewIndex(current, i)$
  shift elements from j to $i - 1$ by 1
  $A[j] = current$
end for

Heap sort

- 1. Run Floyd’s buildHeap
- 2. removeMin() n times