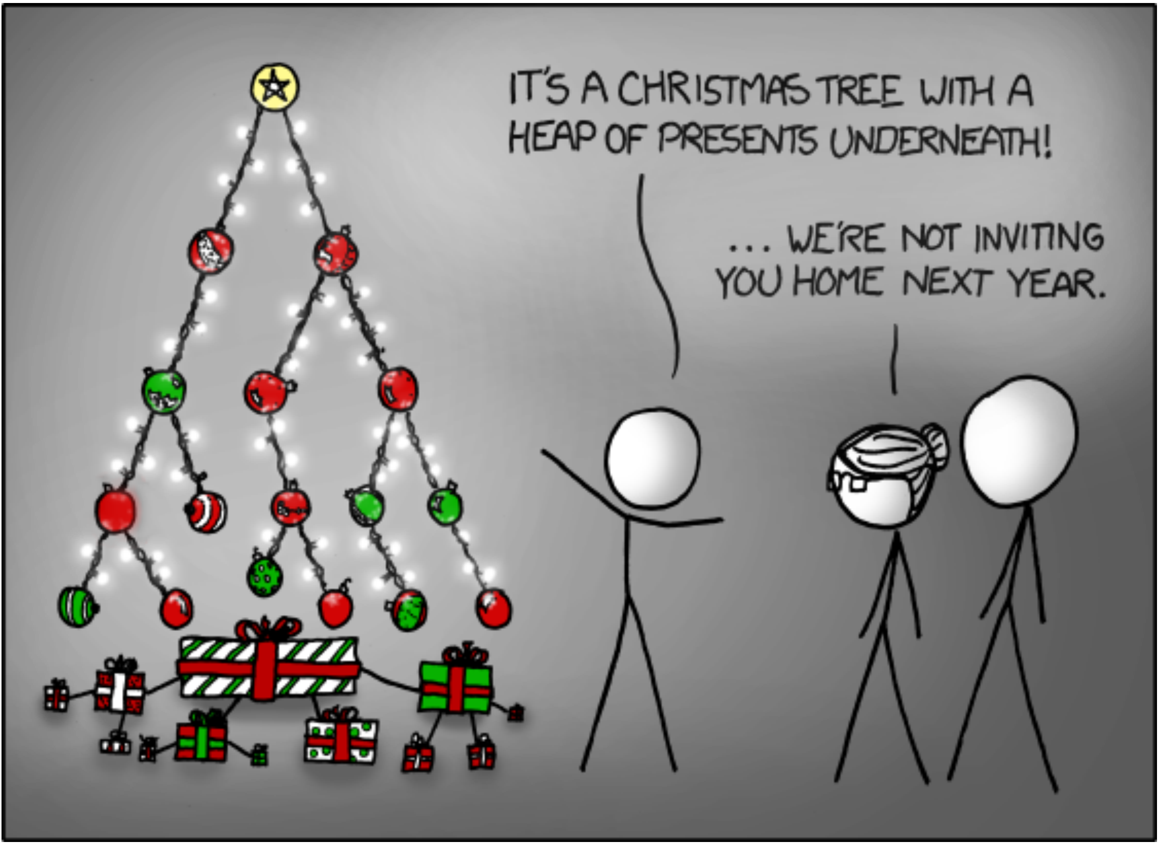
CSE 373: Data Structures and Algorithms



Binary Heaps

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Thanks to Kasey Champion, Ben Jones, Adam Blank, Michael Lee, Evan McCarty, Robbie Weber, Whitaker Brand, Zora Fung, Stuart Reges, Justin Hsia, Ruth Anderson, and many others for sample slides and materials ...

Problems

1. Merging multiple sorted arrays

- `OutArray[k] = min(Array1[i1], Array2[i2])` // for 2 sorted arrays
- `OutArray[k] = min(Array1[i1], Array2[i2], …, Arrayk[ik])` // for k sorted arrays

2. Given n 2D points, find k points which are closest to point P(x, y)

```
S = Set of k distances
For each point Q in the remaining points:
  if dist(P, Q) is less than max(S)
    removeMax from S
    Insert dist(P, Q) in S
```

3. Priority queues: Job schedulers

- Among all the job in a queue, get the job with the highest priority: removeMax(Priority Queue)

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Desired behavior: Get extreme values (min or max)

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- Among all the job in a queue, get the job with the highest priority: `removeMax(Priority Queue)`

Min Priority Queue ADT

- Collection where elements ordered based on priority.

- Behavior:
- `removeMin()`: return element with smallest priority, removes element from collection
- `peekMin()`: find, but do not remove, the element with smallest priority
- `insert(element)`: add element to the collection

- Max Priority Queue ADT:
  - Same as Min Priority Queue ADT, just returns the largest instead of the smallest

Binary heap data structure

- Invented in 1964 for sorting
- Priority Queues is one of the main applications for binary heaps
- Lots of other applications: greedy algorithms, shortest path

- Basically, min-heap (or max-heap) is ideal when you want to maintain a collection of elements where you need to add arbitrary values but need an efficient removeMin (or removeMax).

Binary heap

Binary heap is a

1. binary tree
2. that satisfies the heap property, and
3. where every heap is a “complete” tree

Binary heap: Heap property

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max-heap: Every node is larger than (or equal to) its children

Binary heap: Heap property

max-heap: Every node is larger than (or equal to) its children

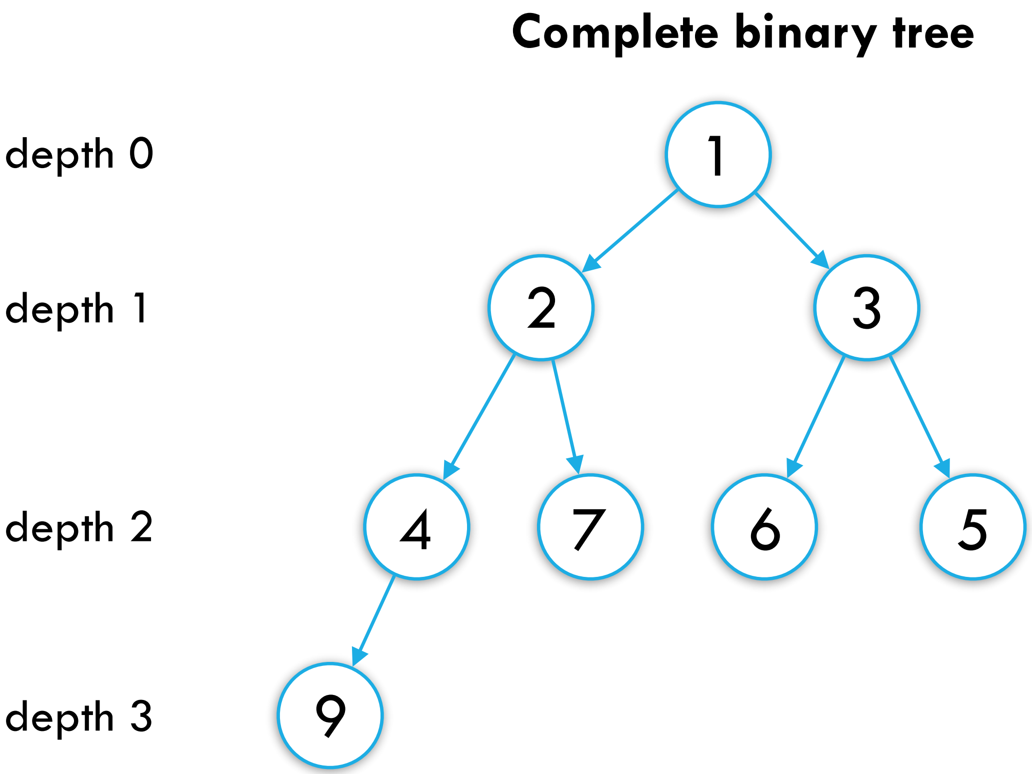
min-heap: Every node is smaller than (or equal to) its children

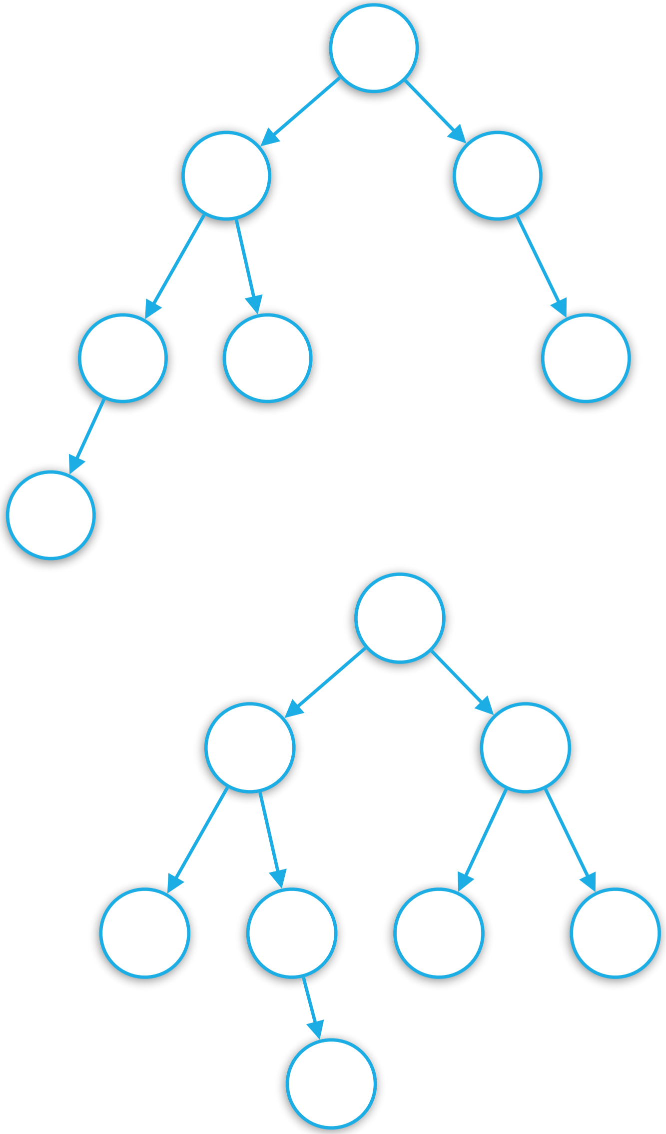
Binary heap: Complete binary tree

A complete binary tree is a binary tree in which every level, except possibly the last, is completely filled, and all nodes are as far left as possible.

There are no “gaps” in a complete tree.

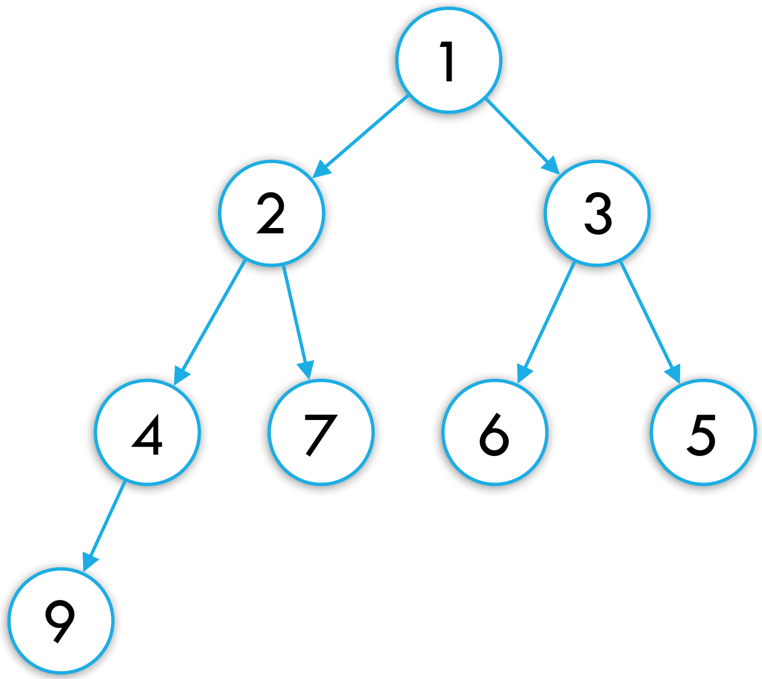
Complete binary tree





There as not complete binary trees

Question: Valid min-heap?



Complete binary tree?
Heap property satisfied?

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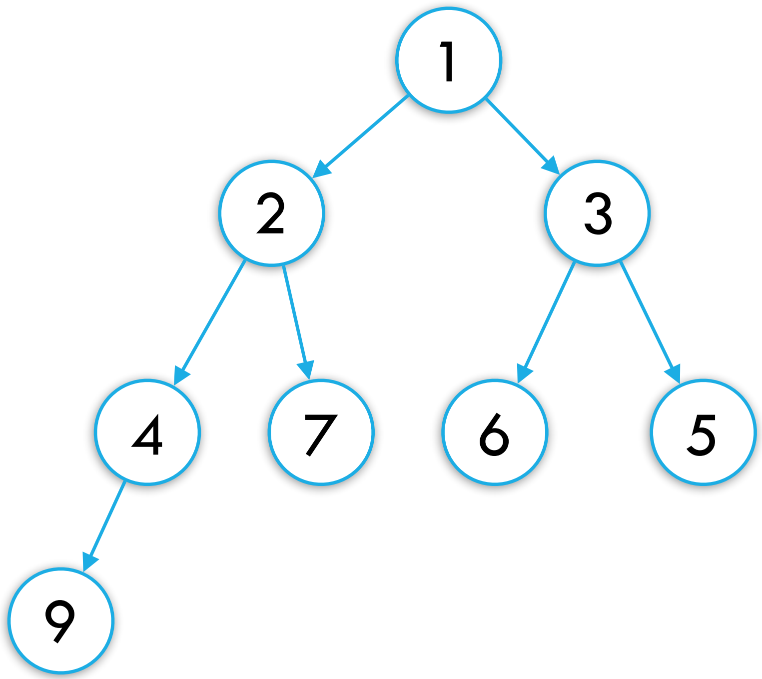

Complete binary tree? Yes!

Heap property satisfied? Yes!

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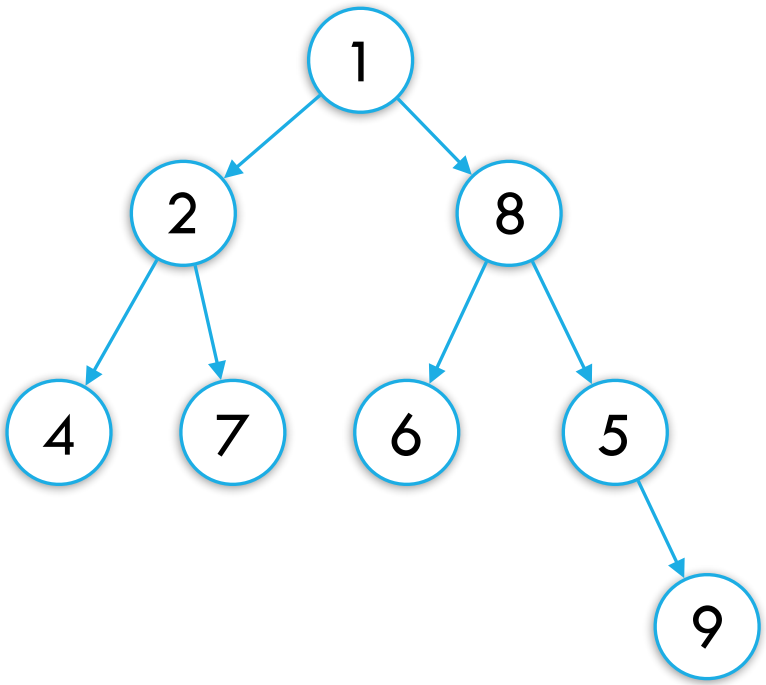

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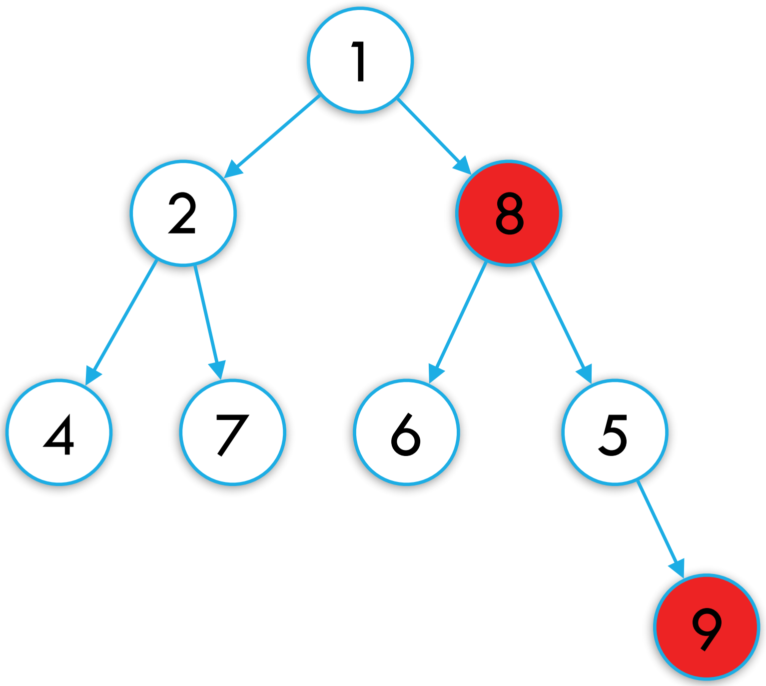
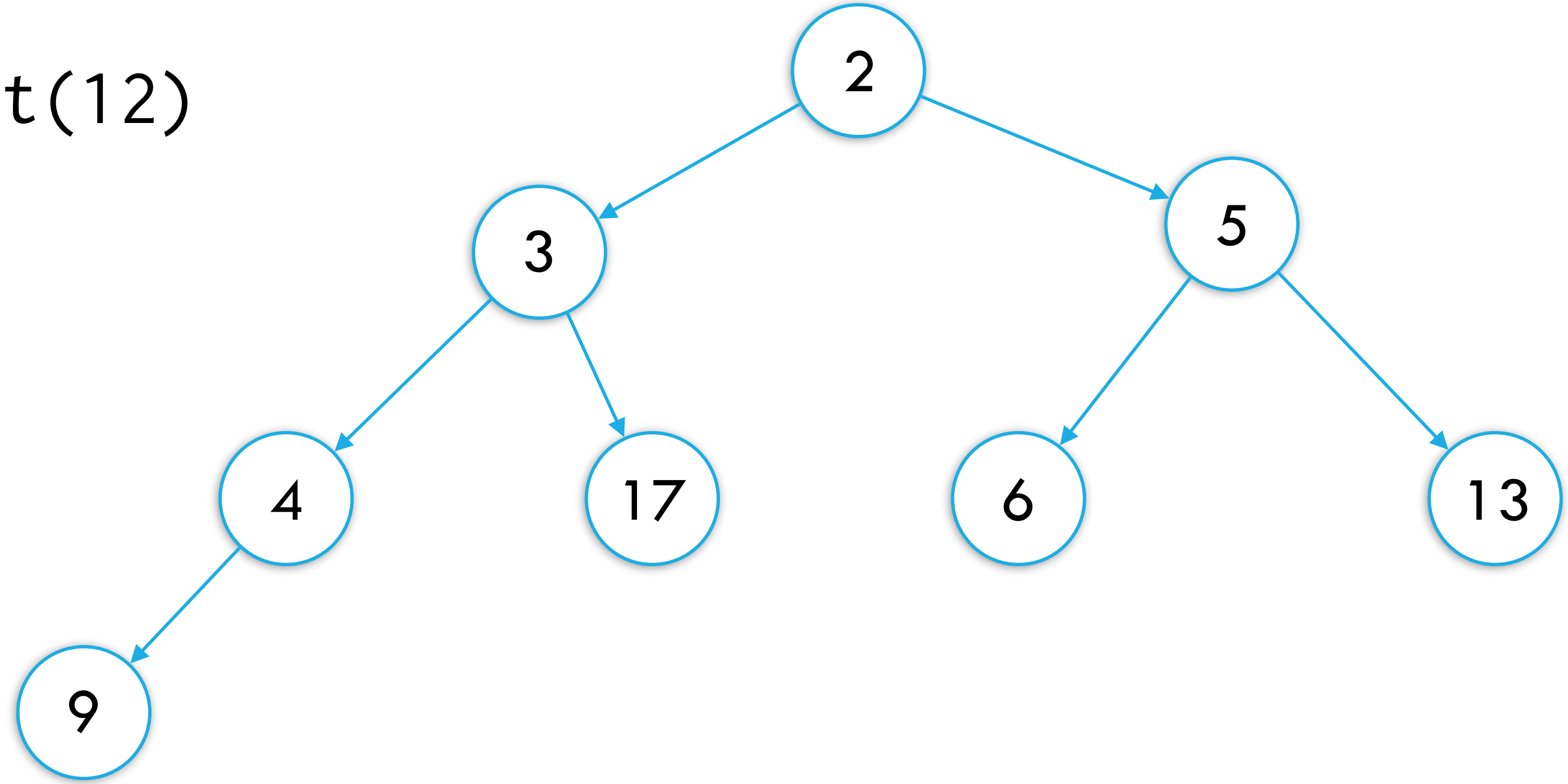


Complete binary tree? No!
Heap property satisfied? No!

Binary heap insert

`insert(12)`



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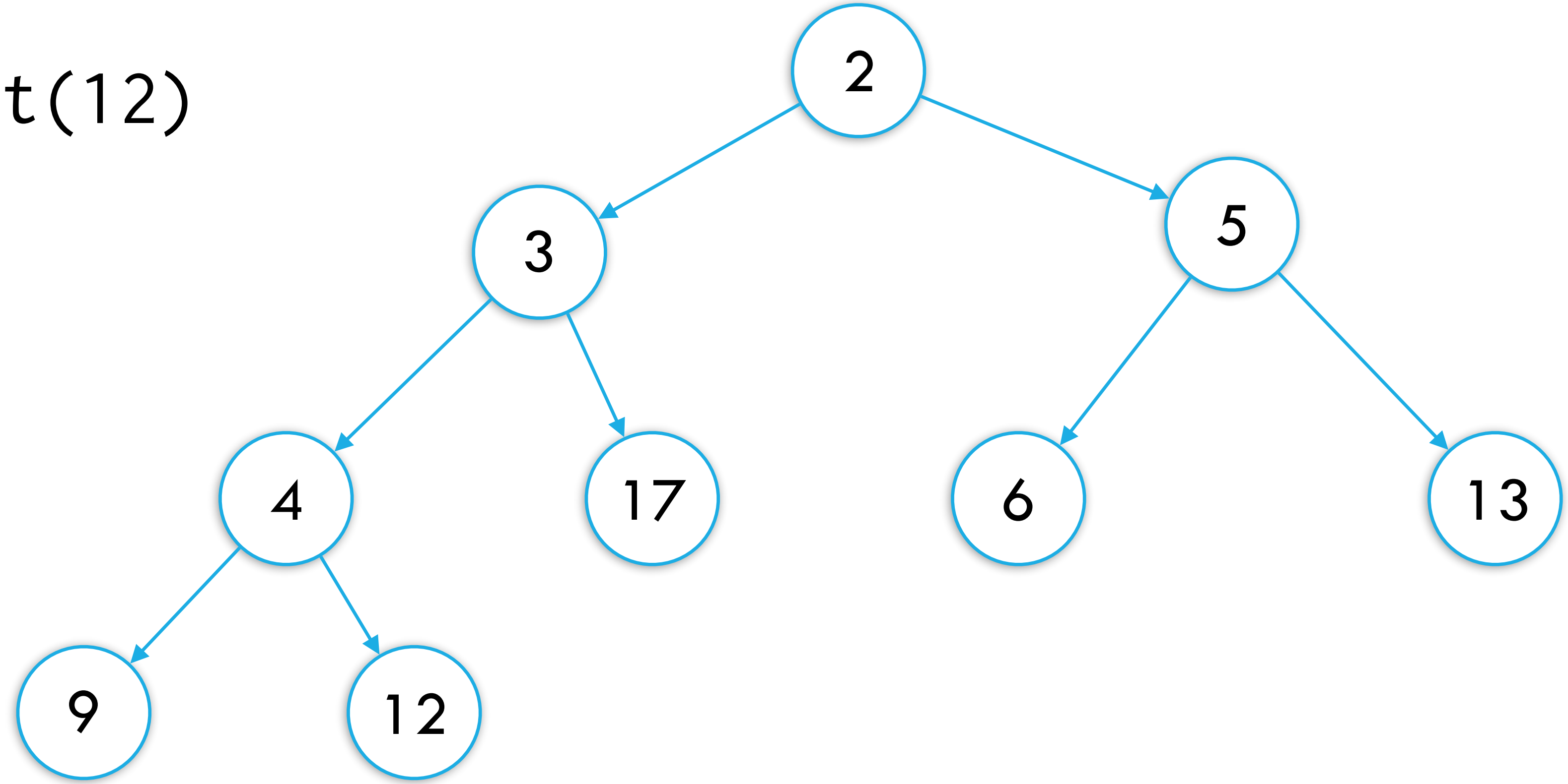
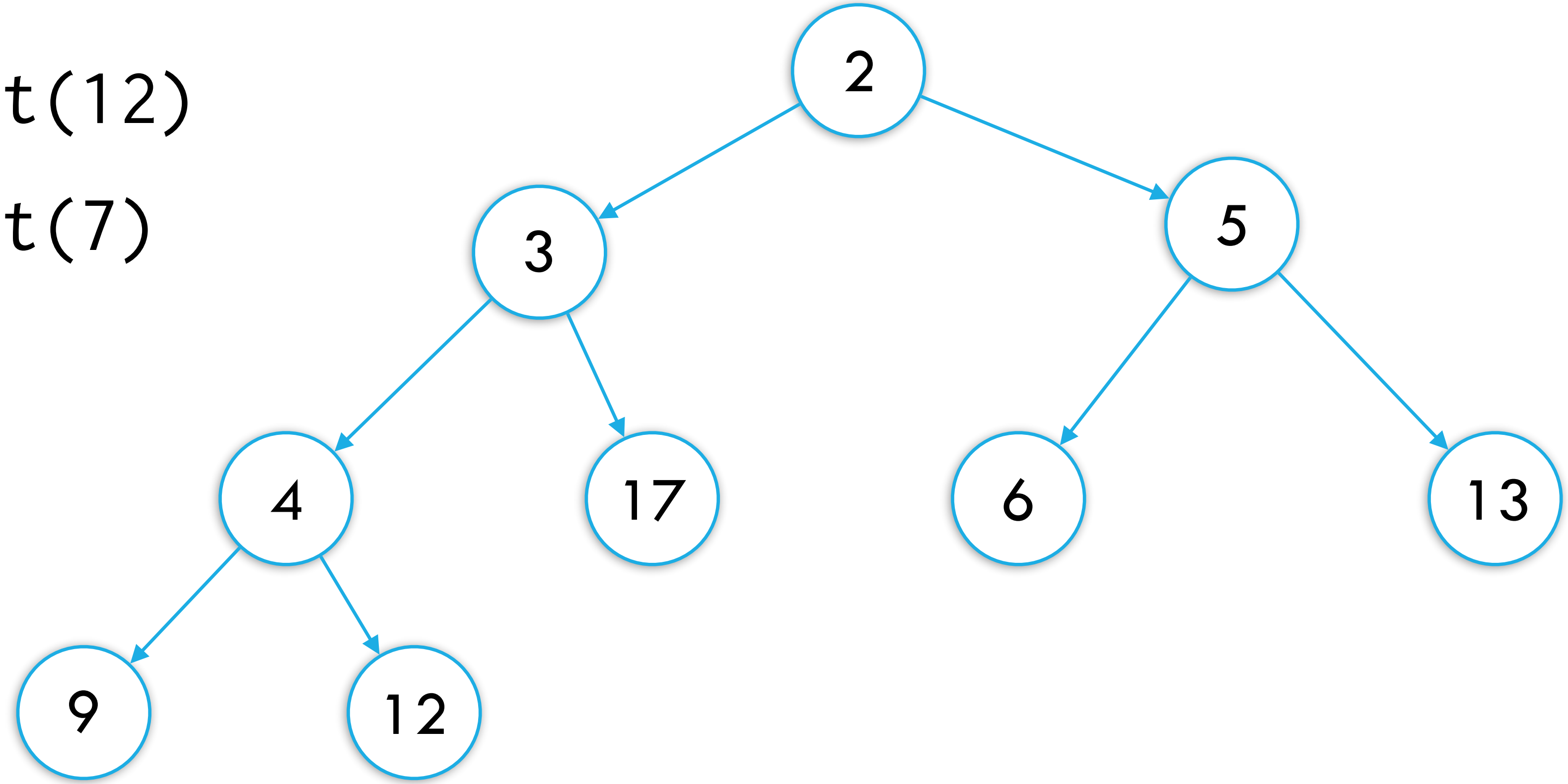


Binary heap insert

```
insert(12)

insert(7)
```



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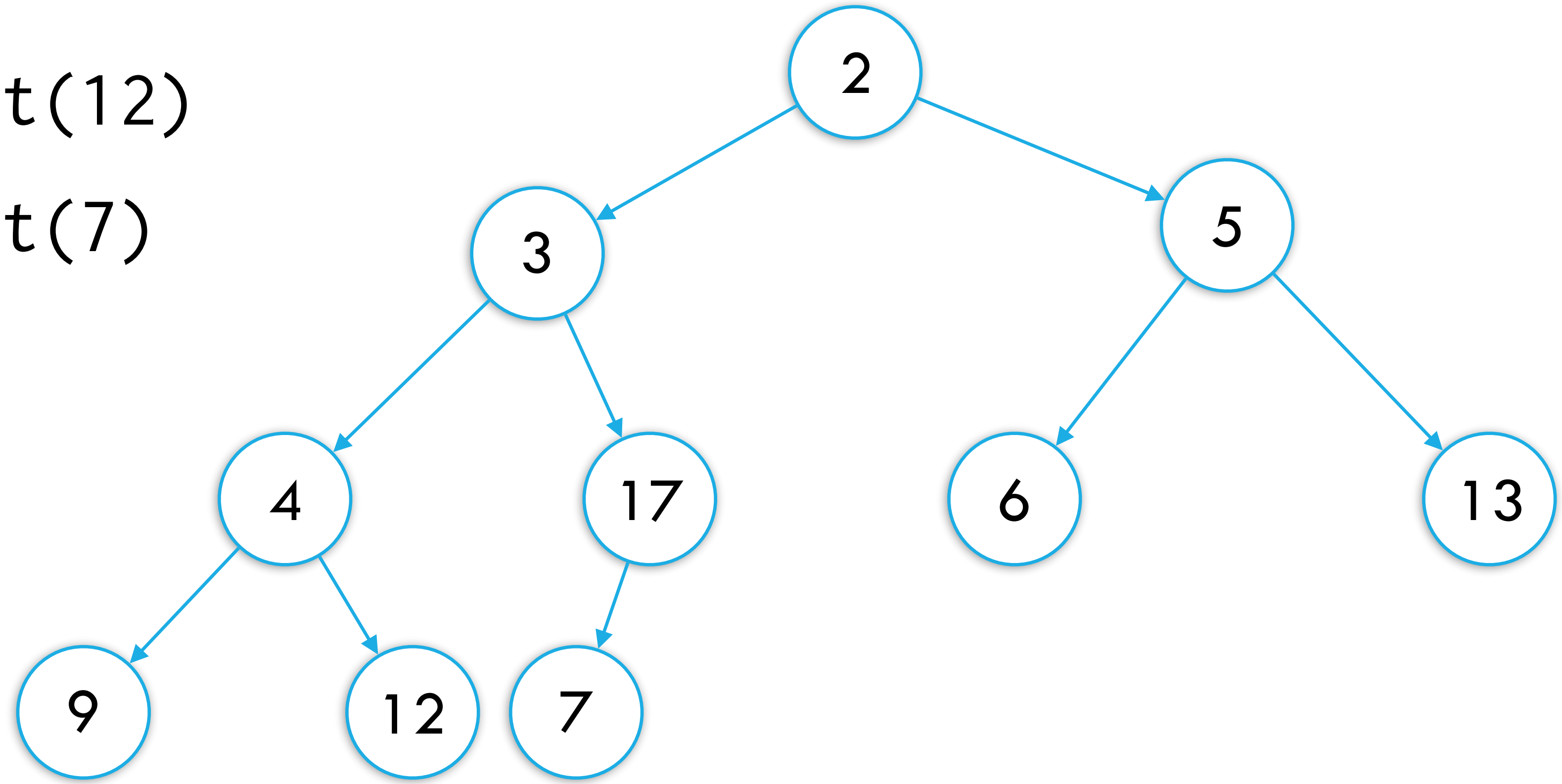

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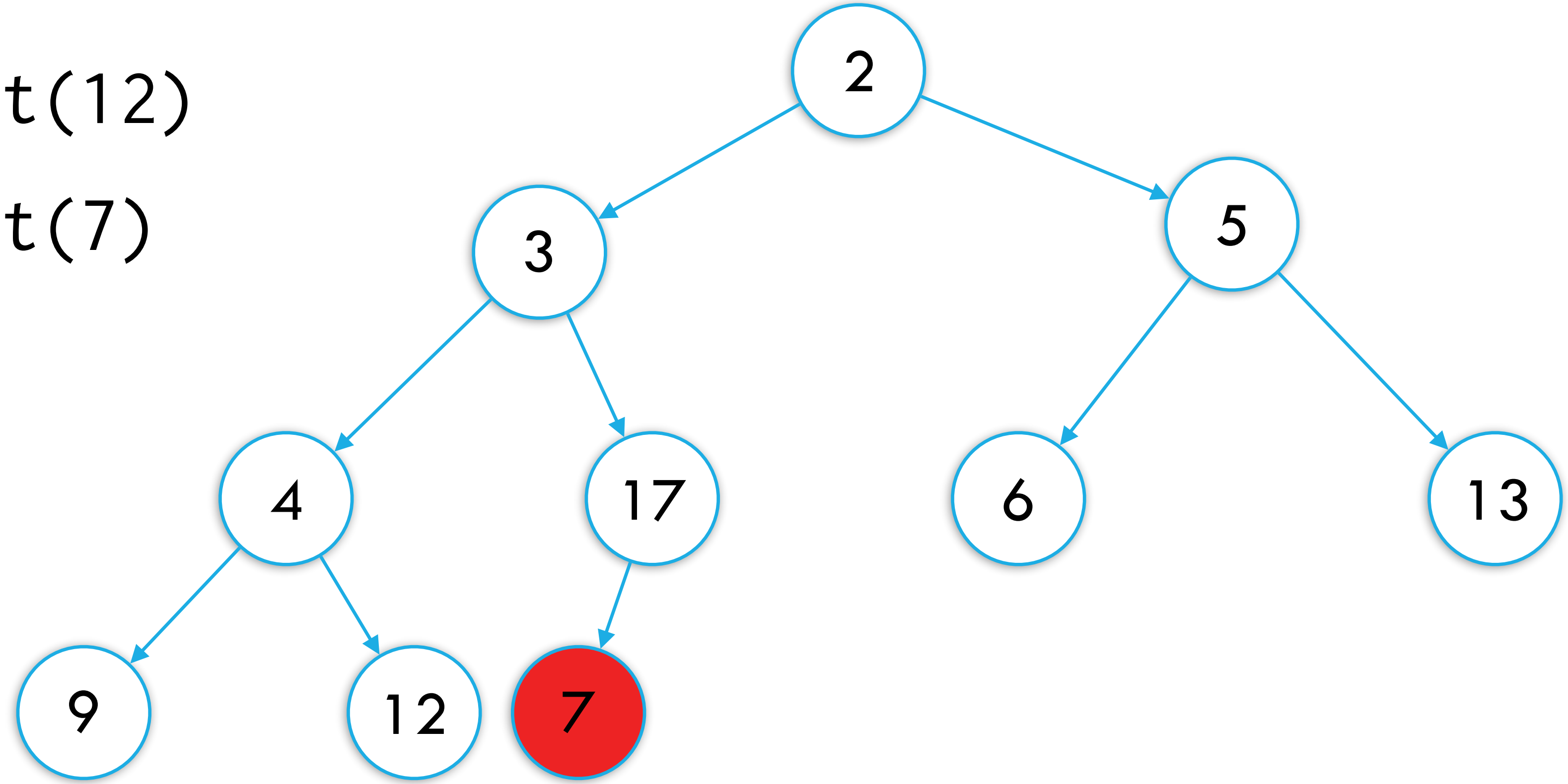

Heap broken

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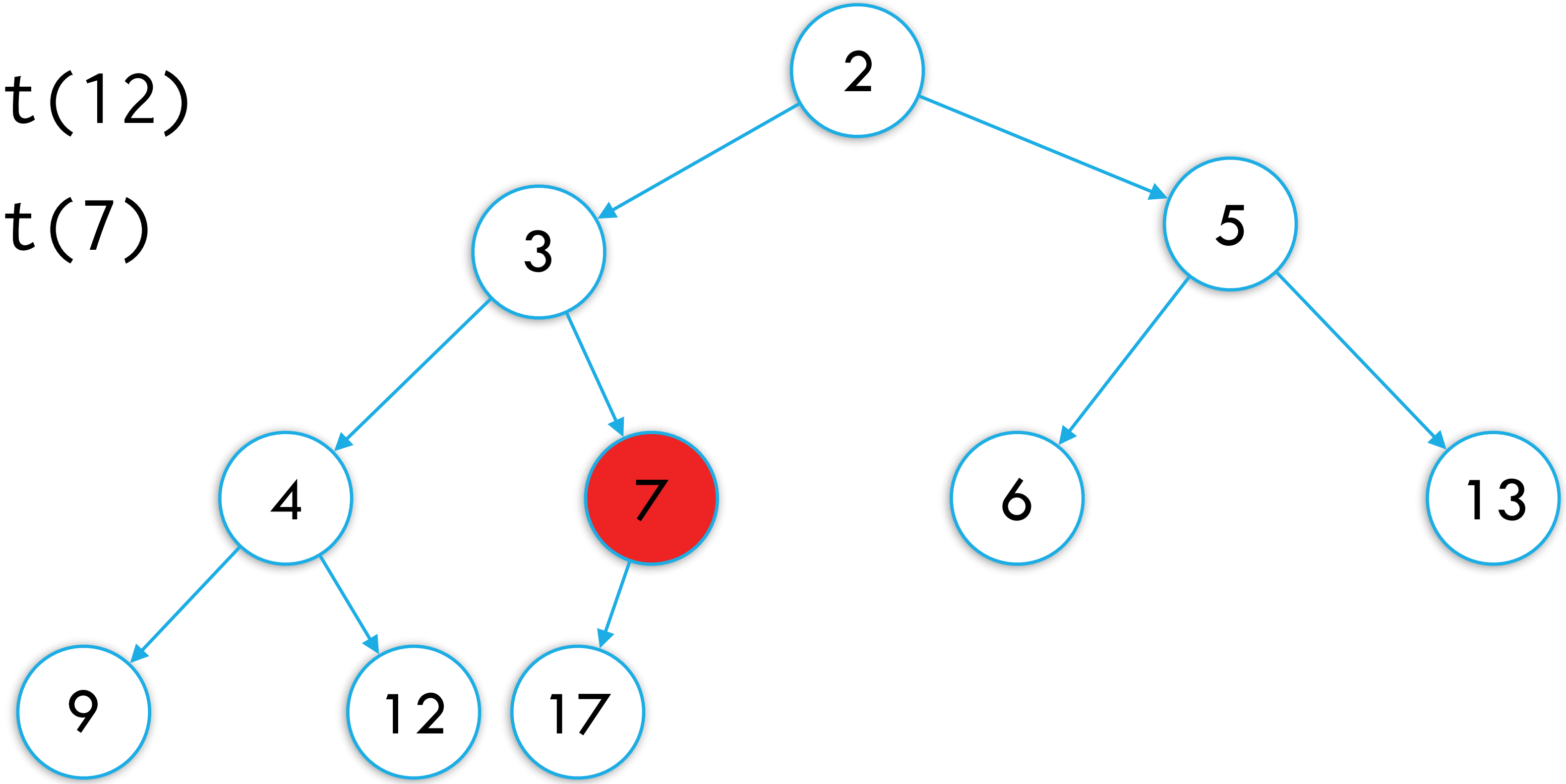

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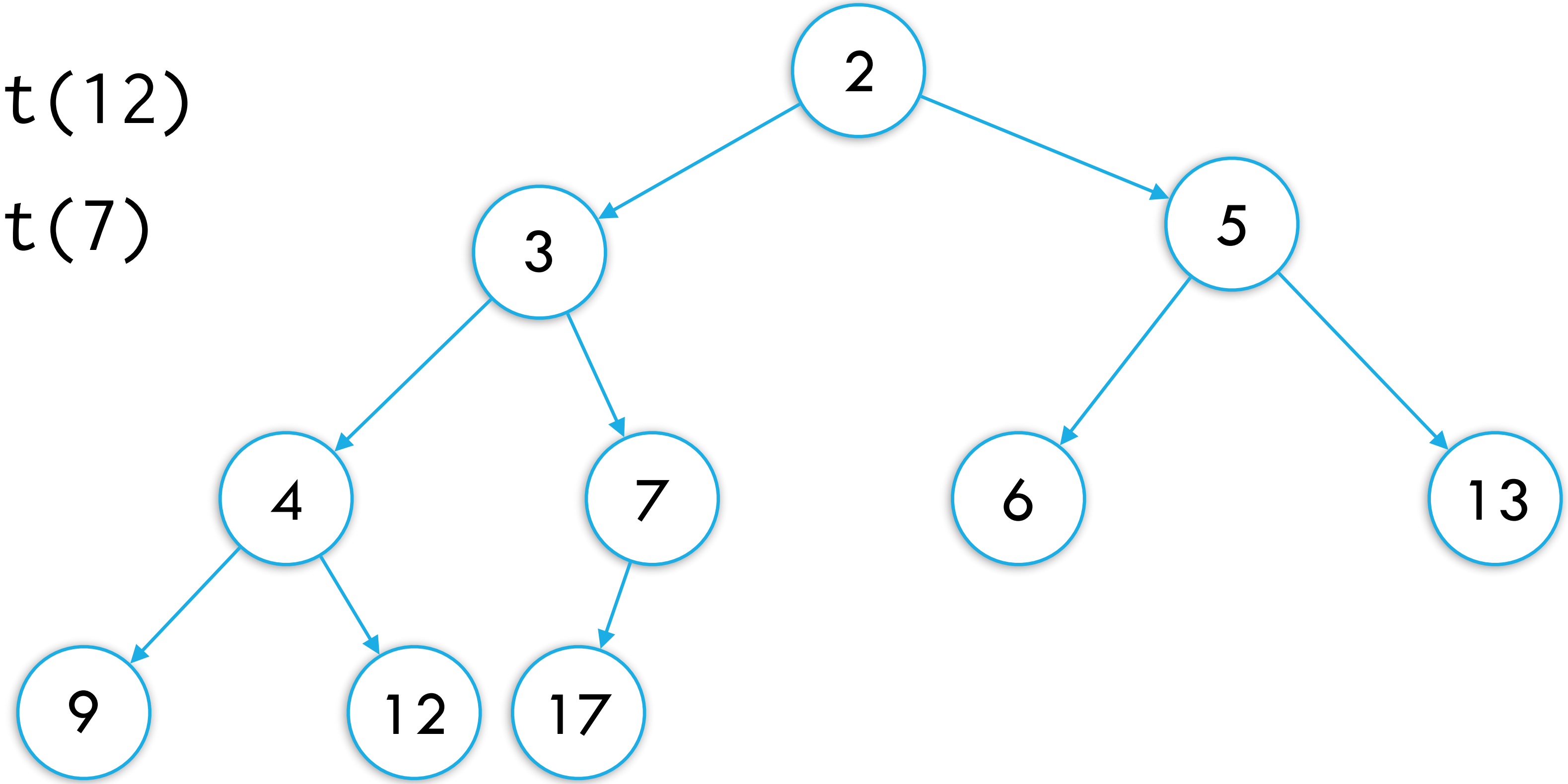

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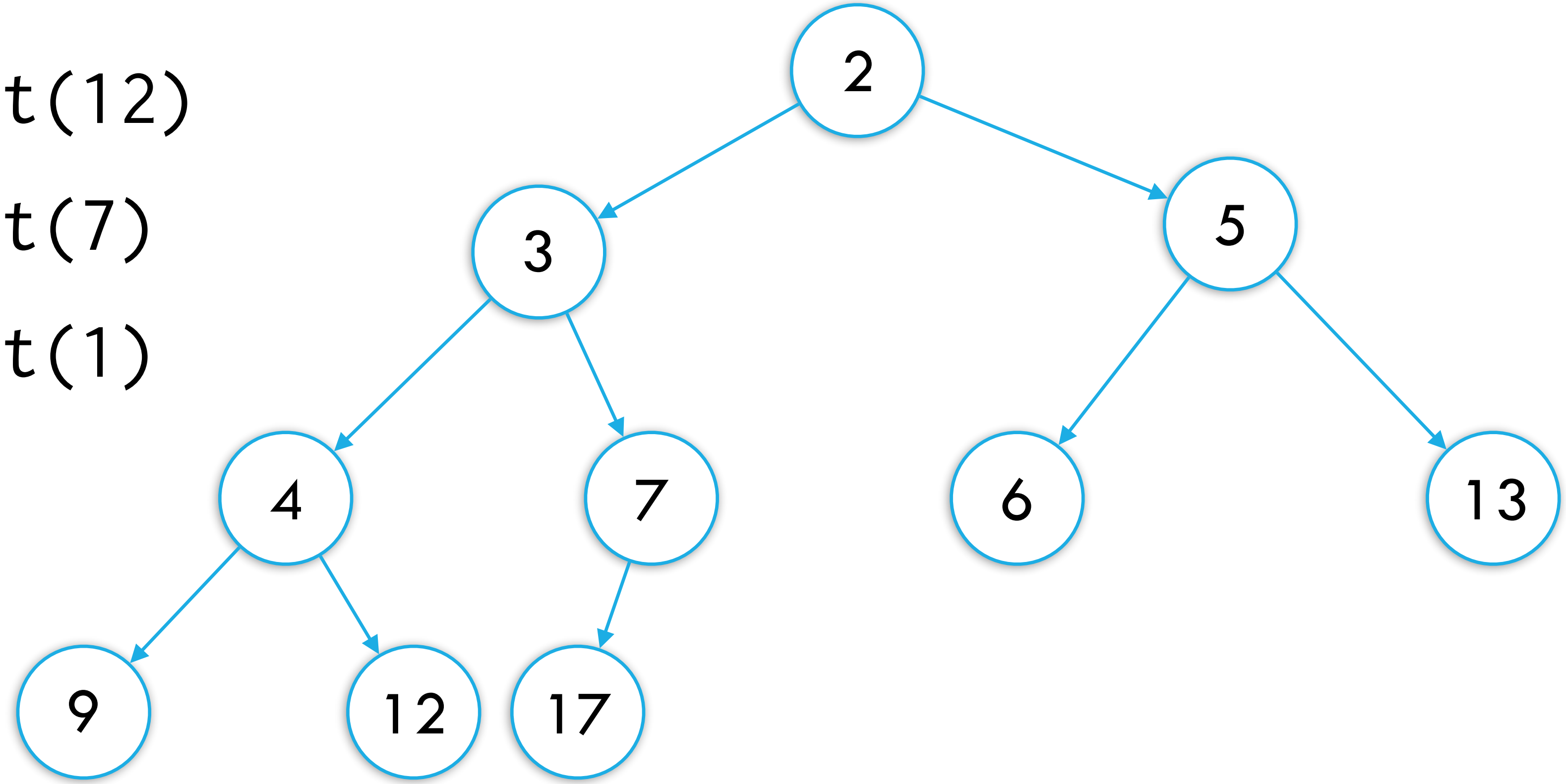

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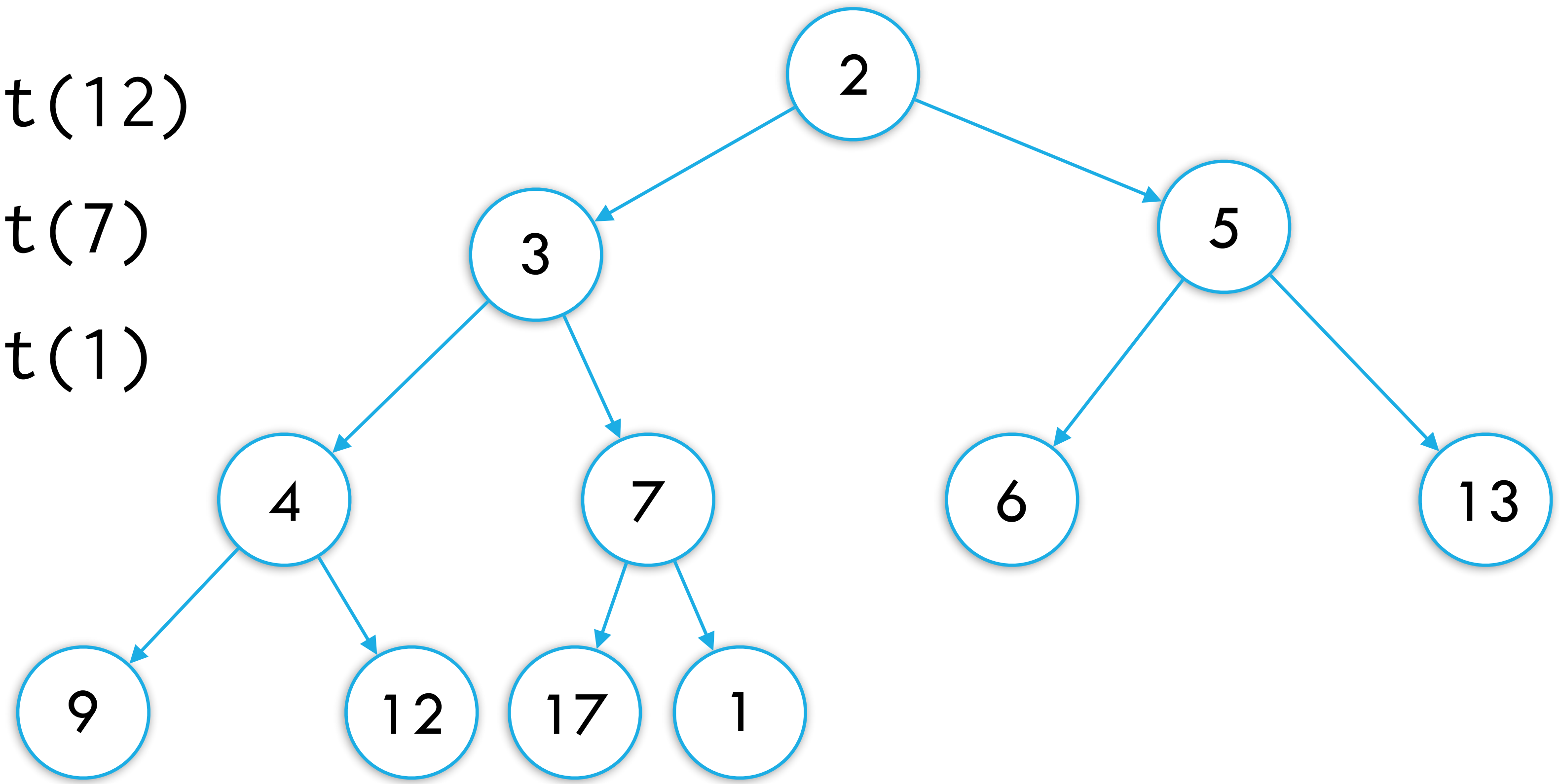

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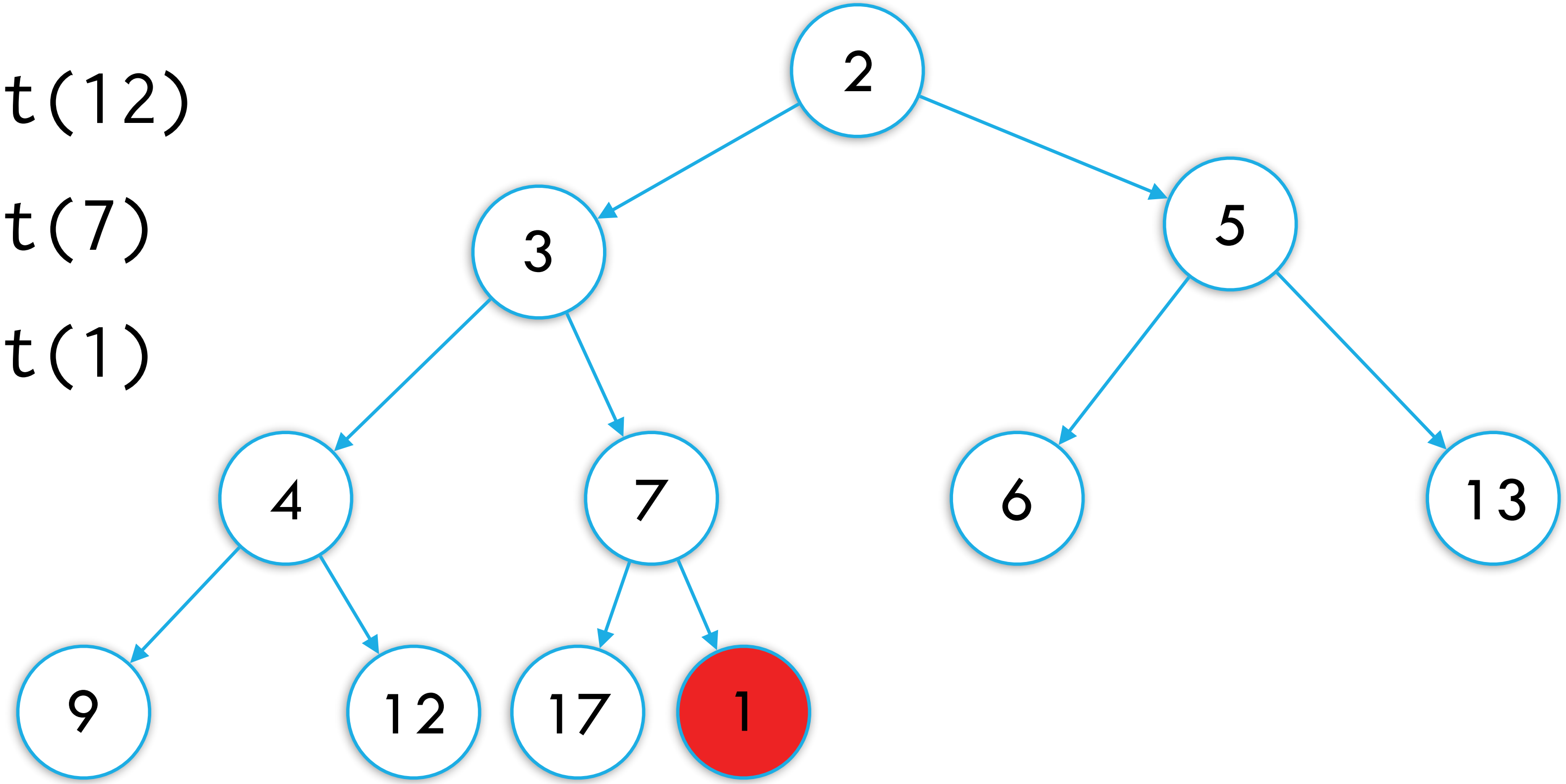

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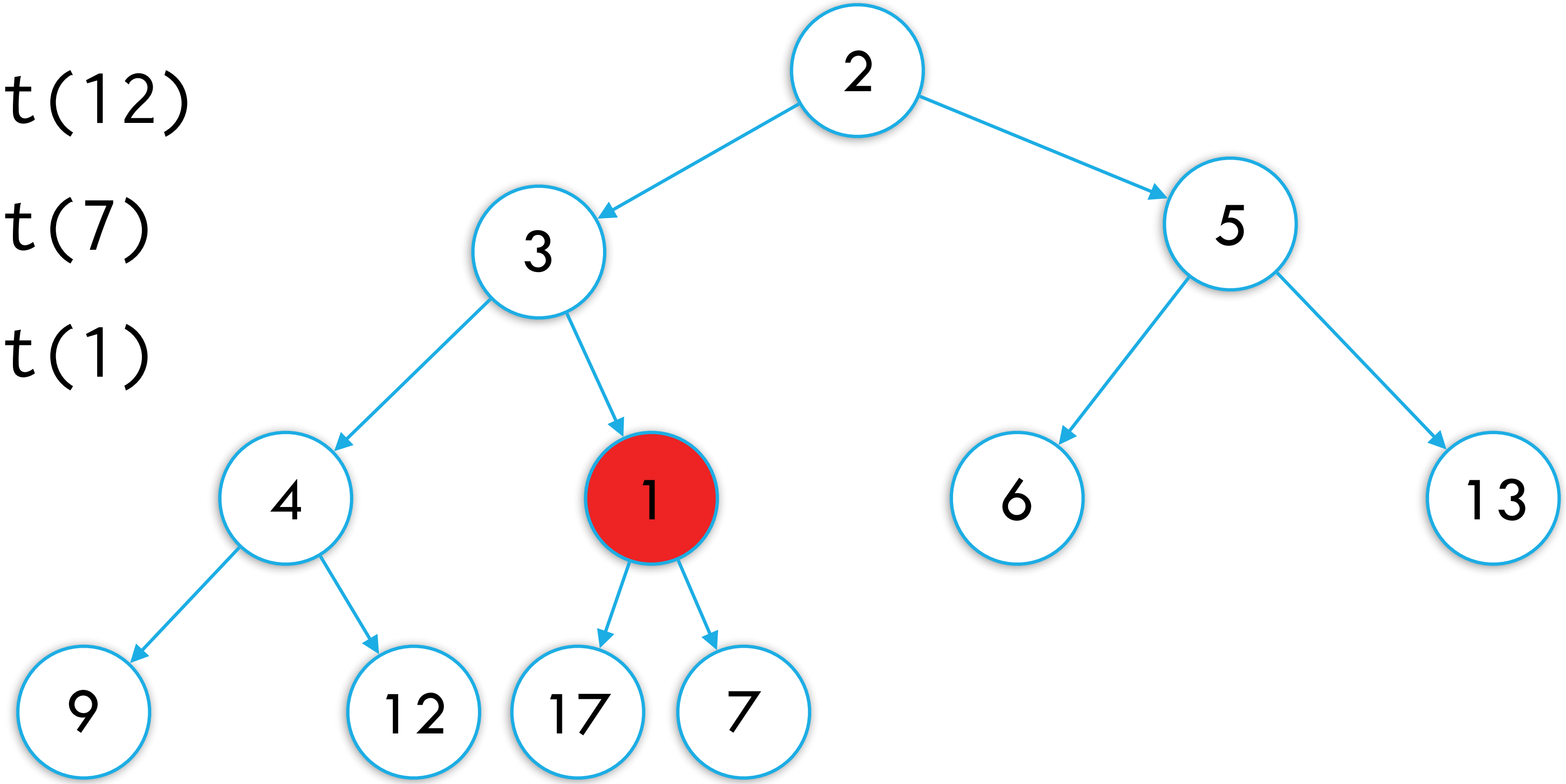

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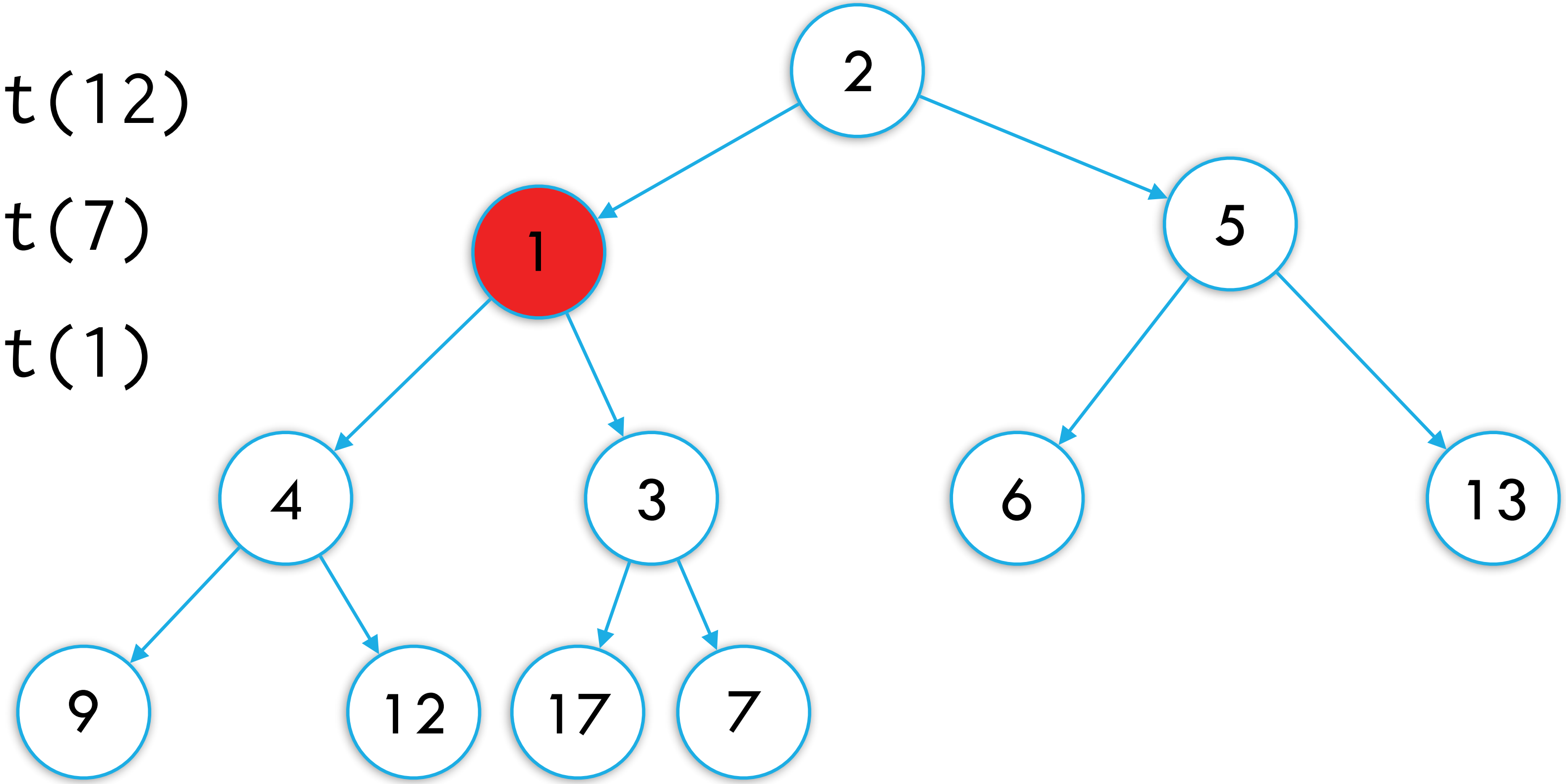

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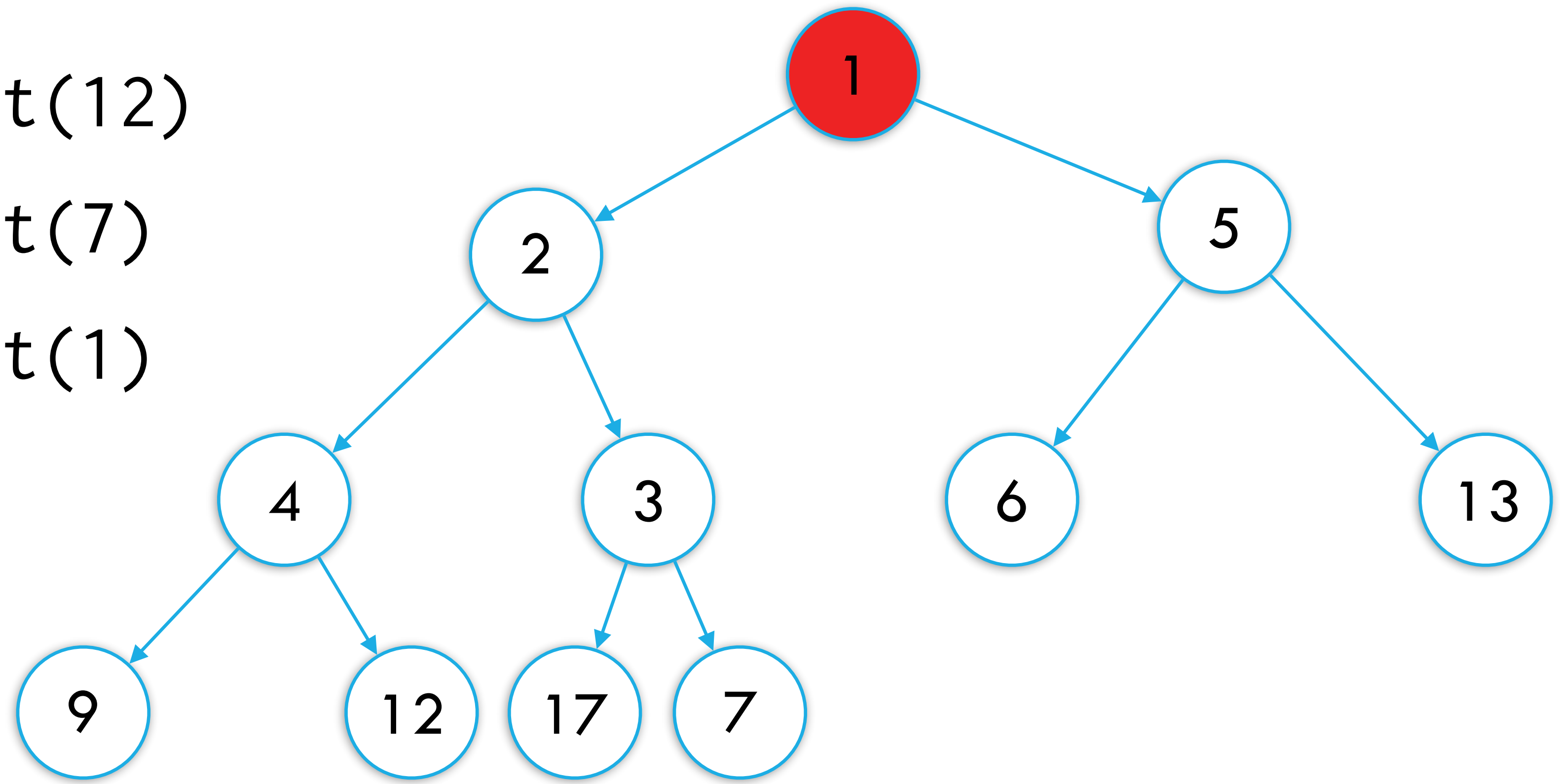

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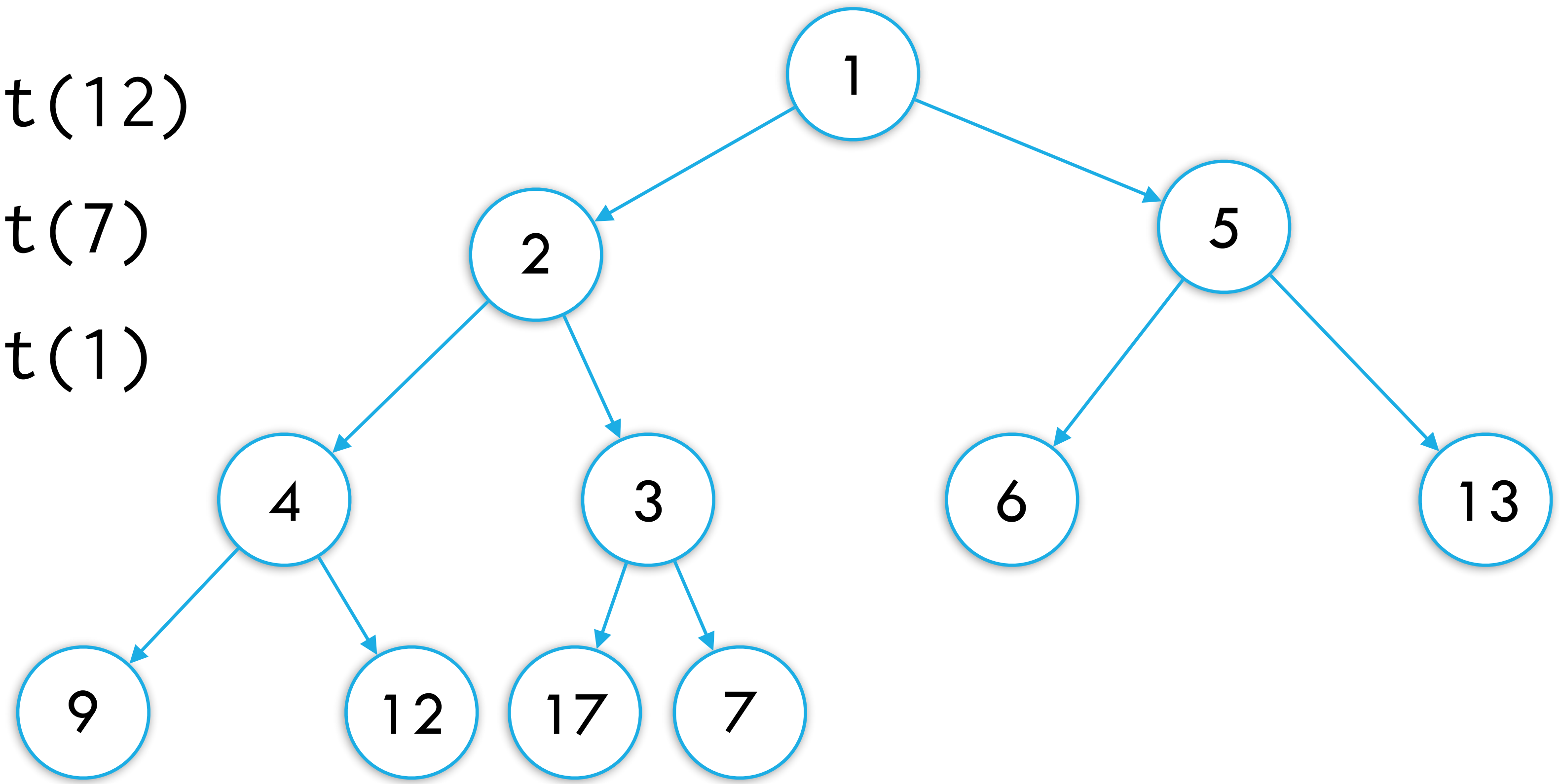

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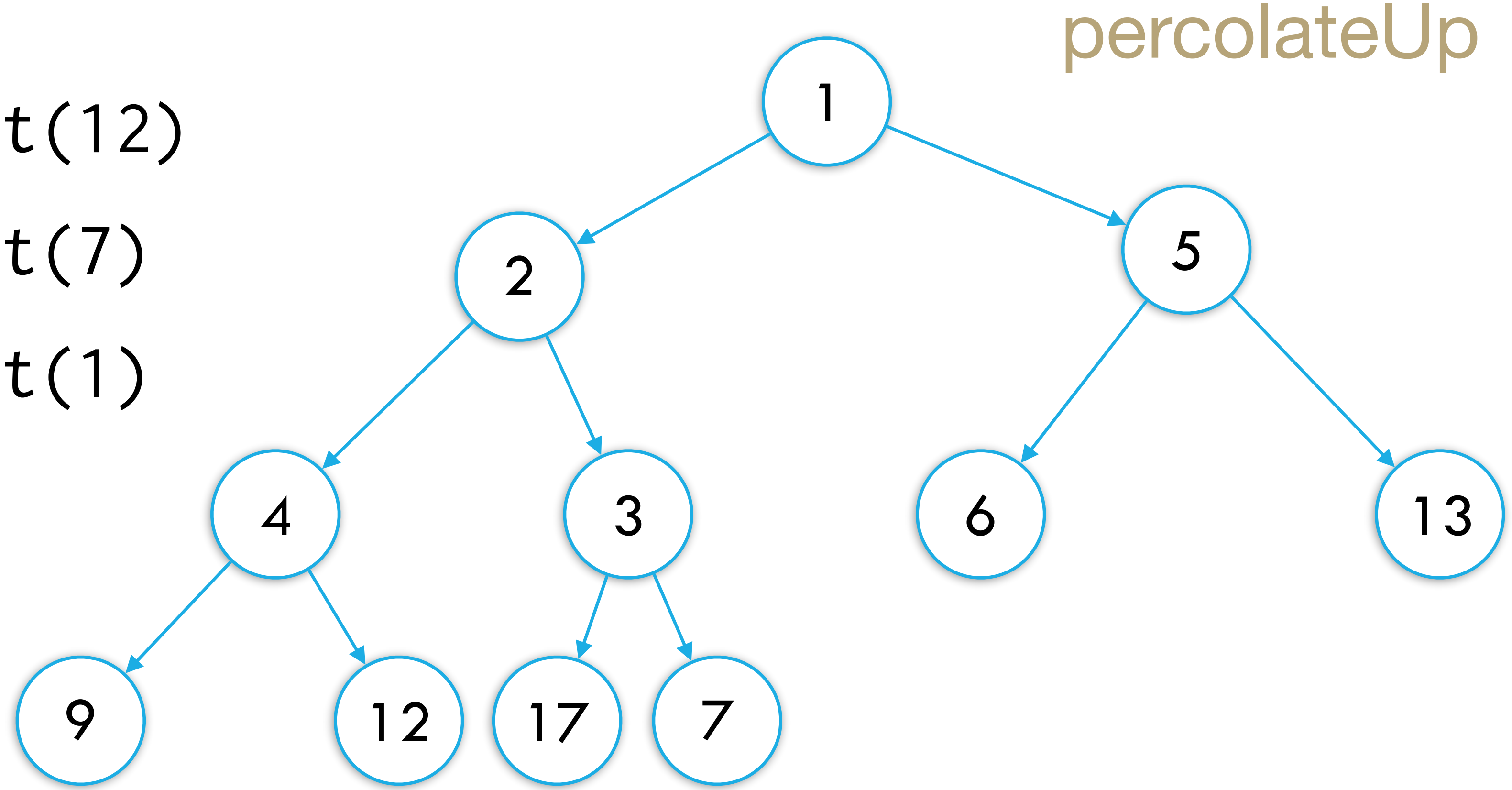
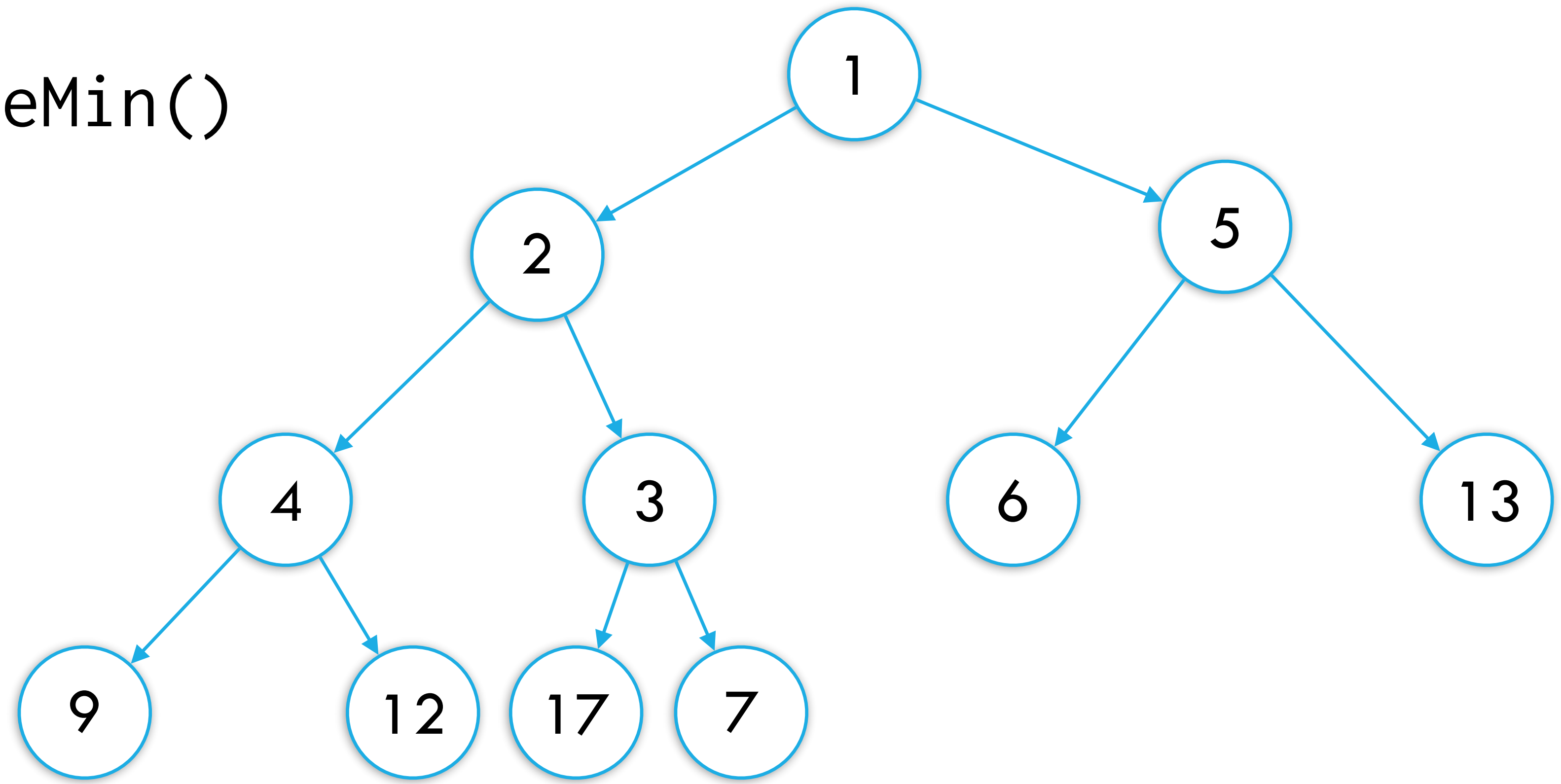


Binary heap removeMin

`removeMin()`



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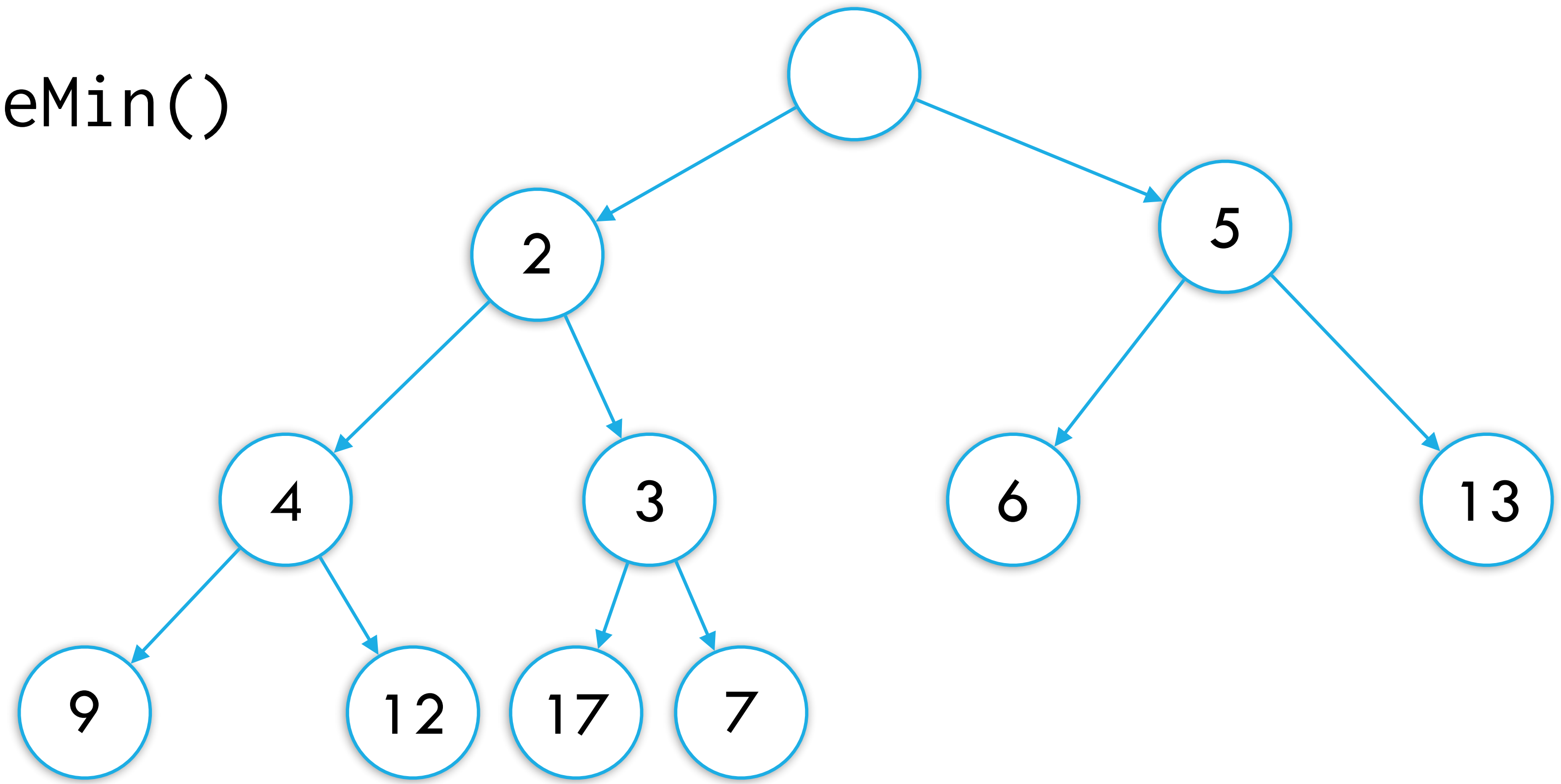

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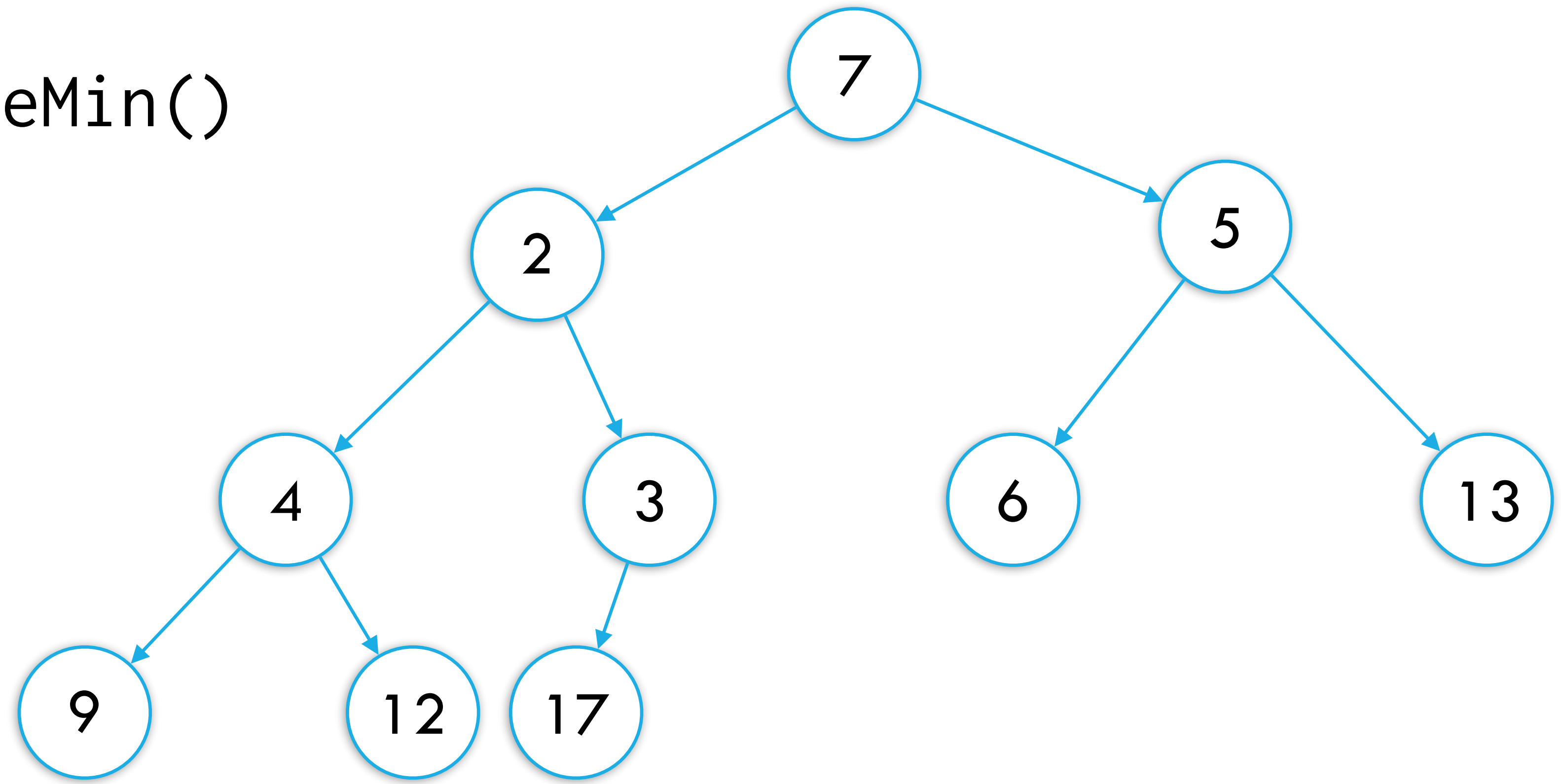

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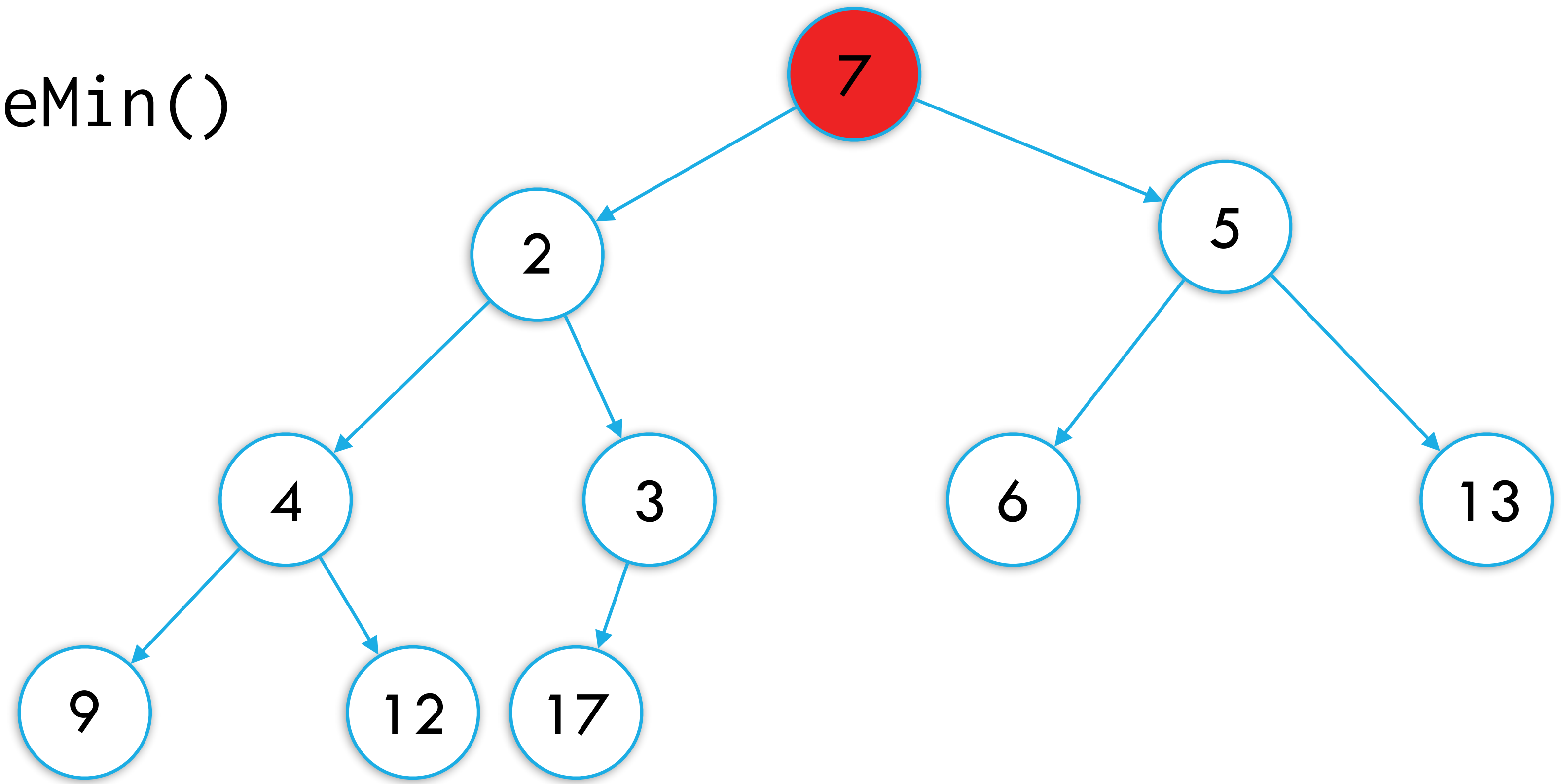

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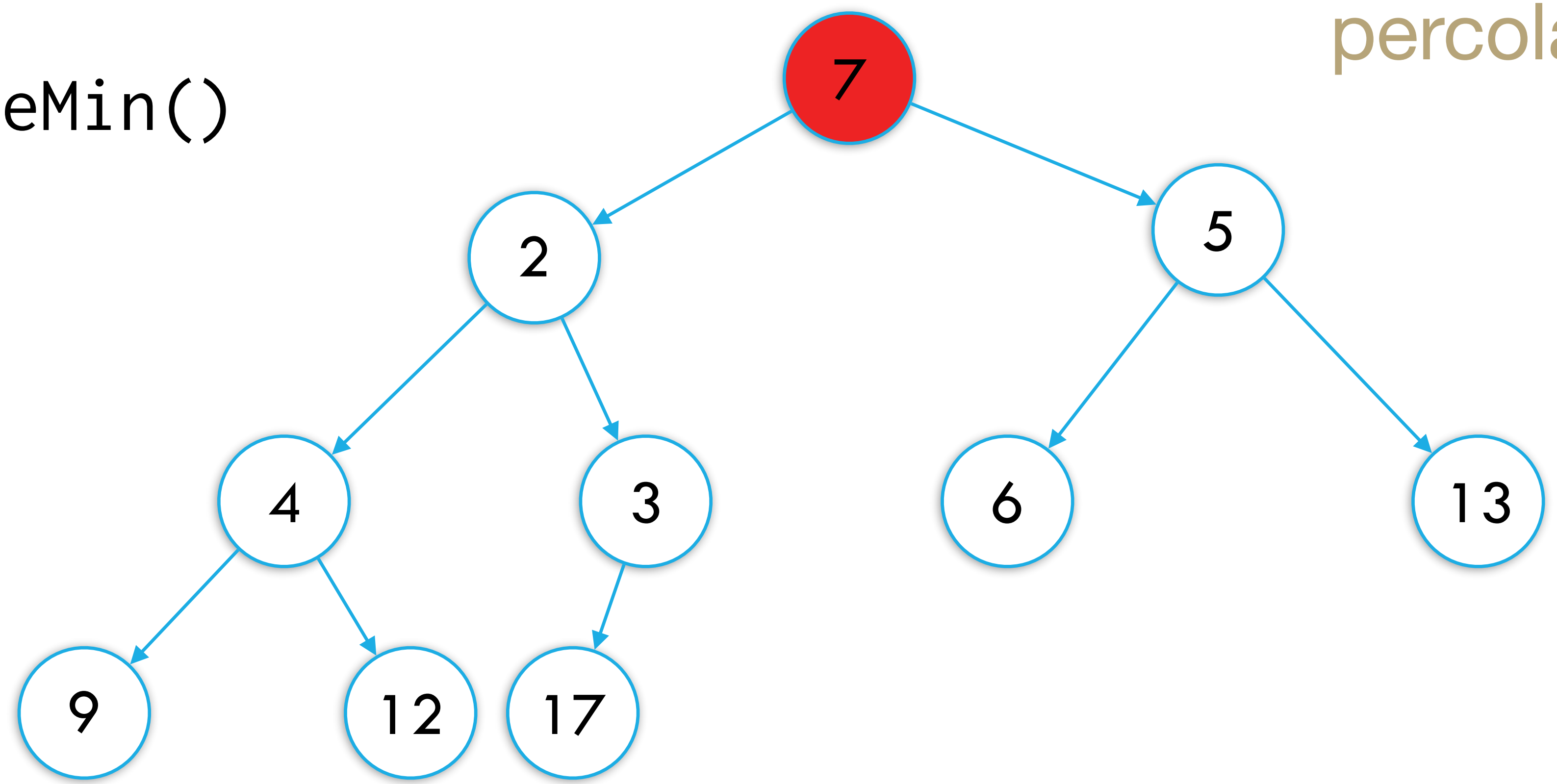
percolateDown



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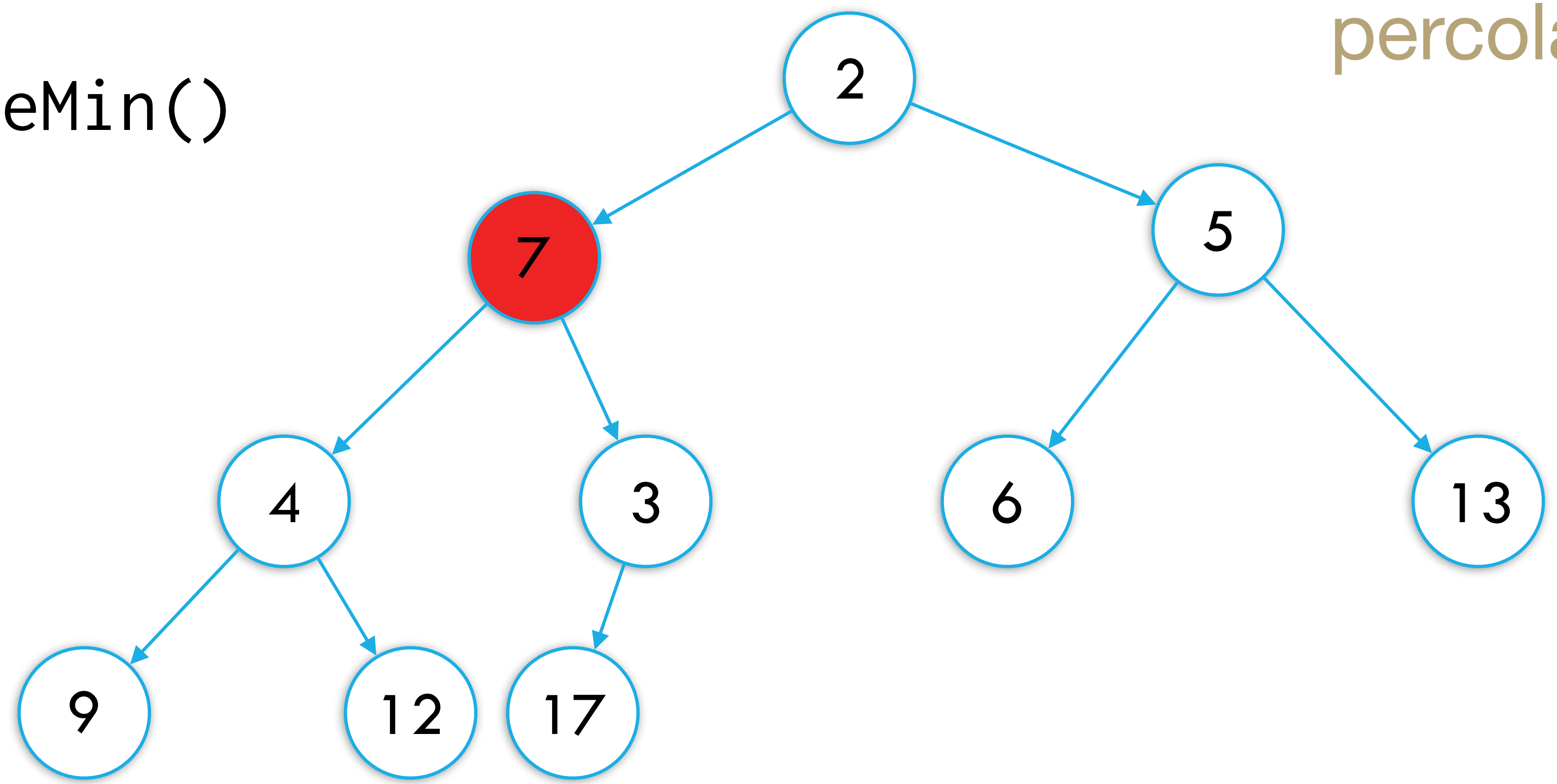

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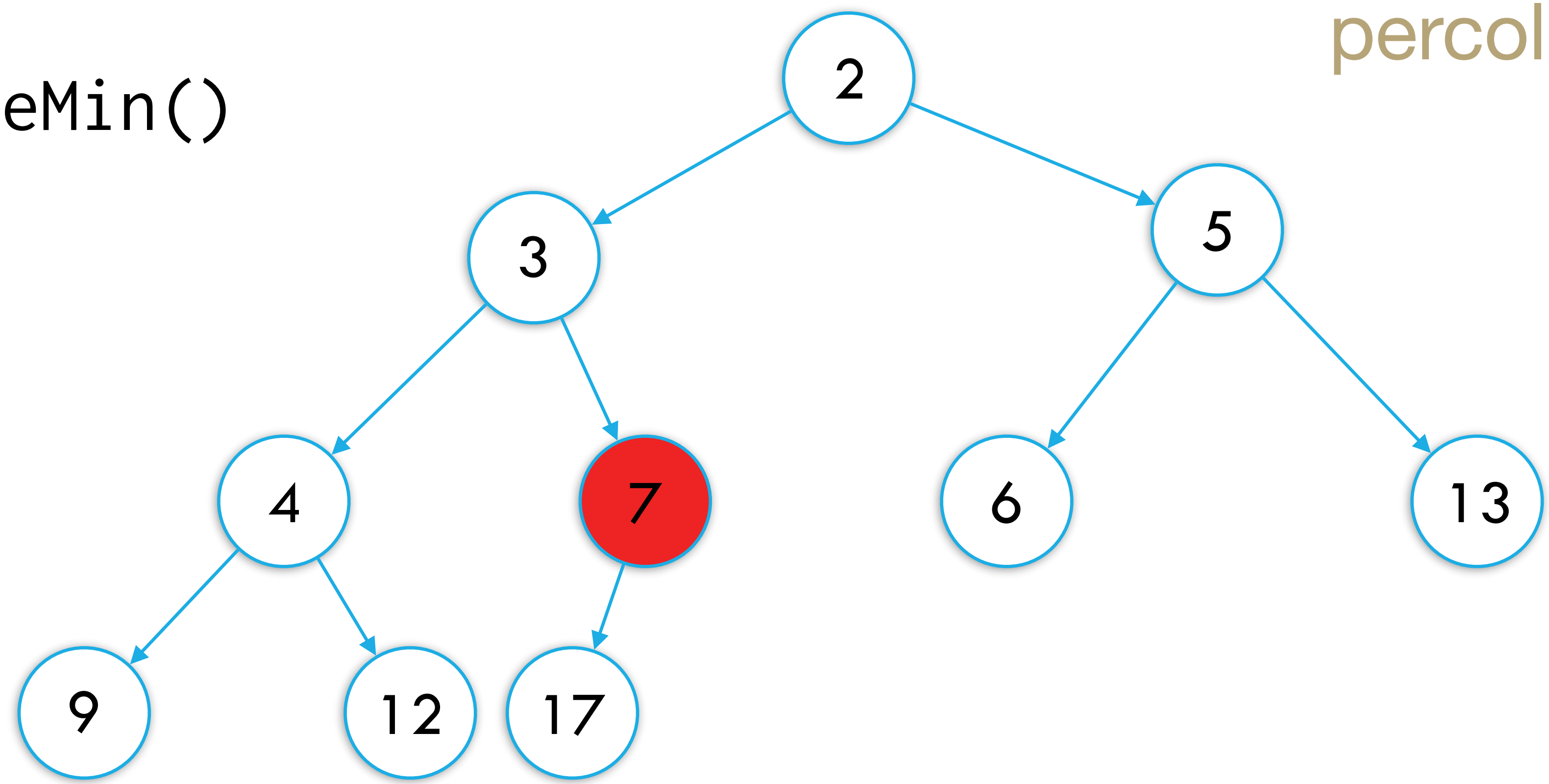

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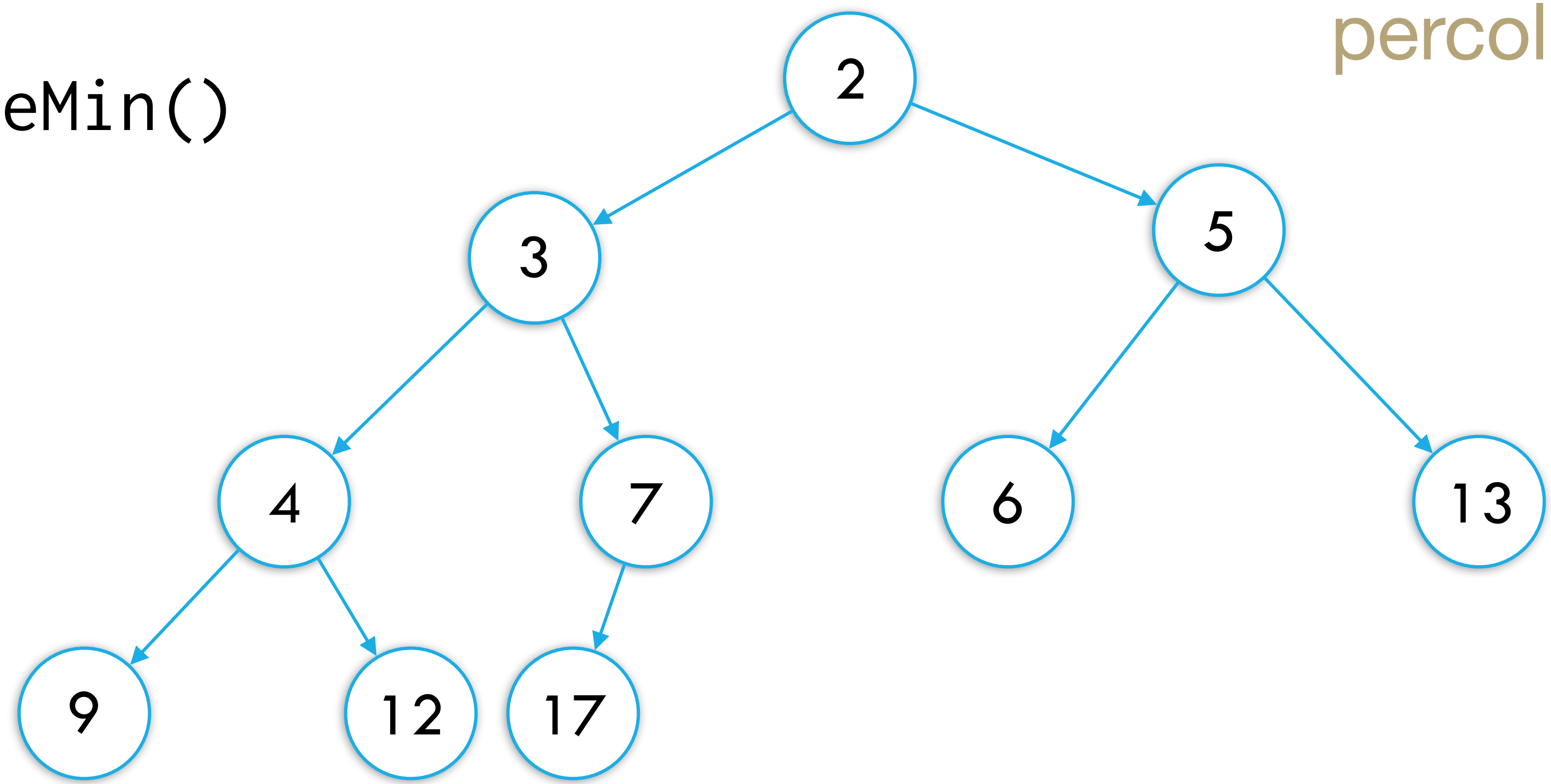

Binary heap: removeMin() runTime

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findLastNodeTime + removeRootTime + numOfSwaps * swapTime

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n + 1 + log(n) * 1 = O (n)

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$n + 1 + \log(n) * 1 = O(n)$

How can we do better?

Binary heap: removeMin() runTime

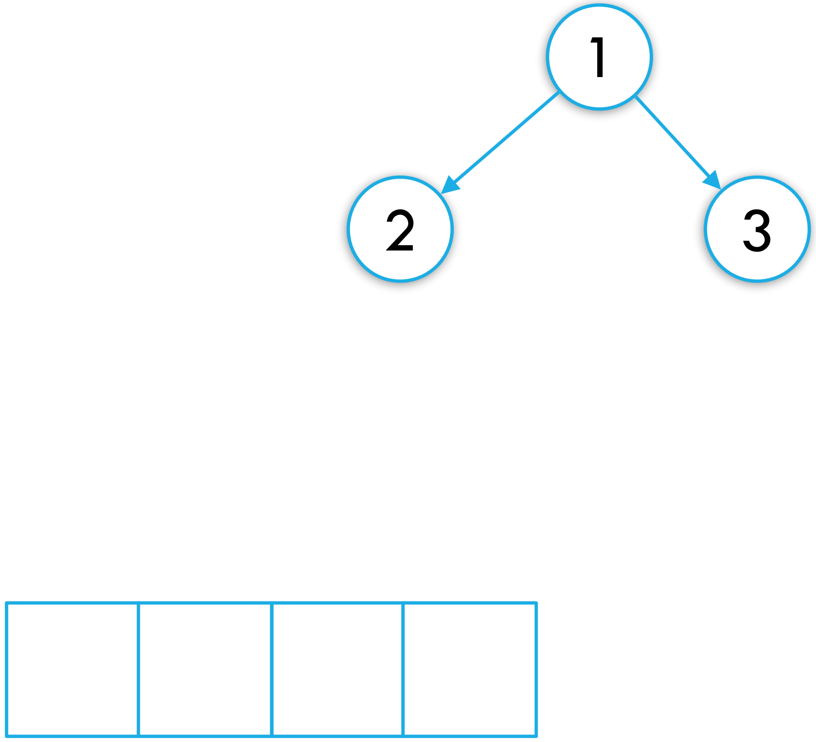
findLastNodeTime + removeRootTime + numOfSwaps * swapTime

$$n + 1 + \log(n) * 1 = O(n)$$

How can we do better?

What do we make efficient in the above expression?

Array implementation of a complete binary tree



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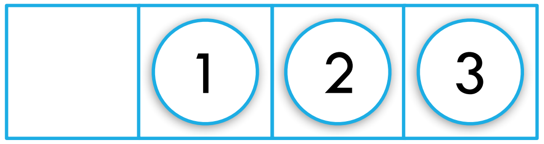

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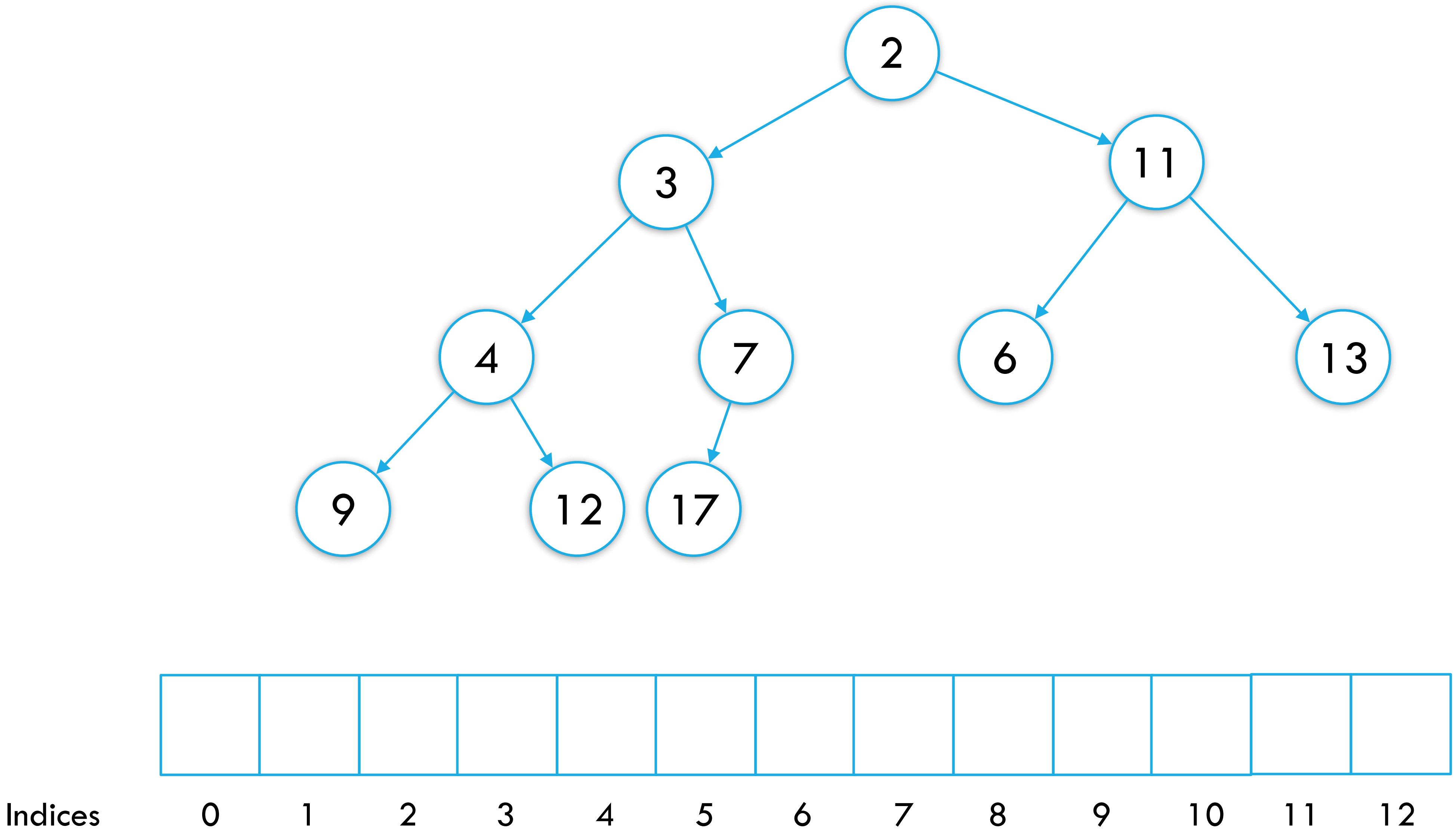

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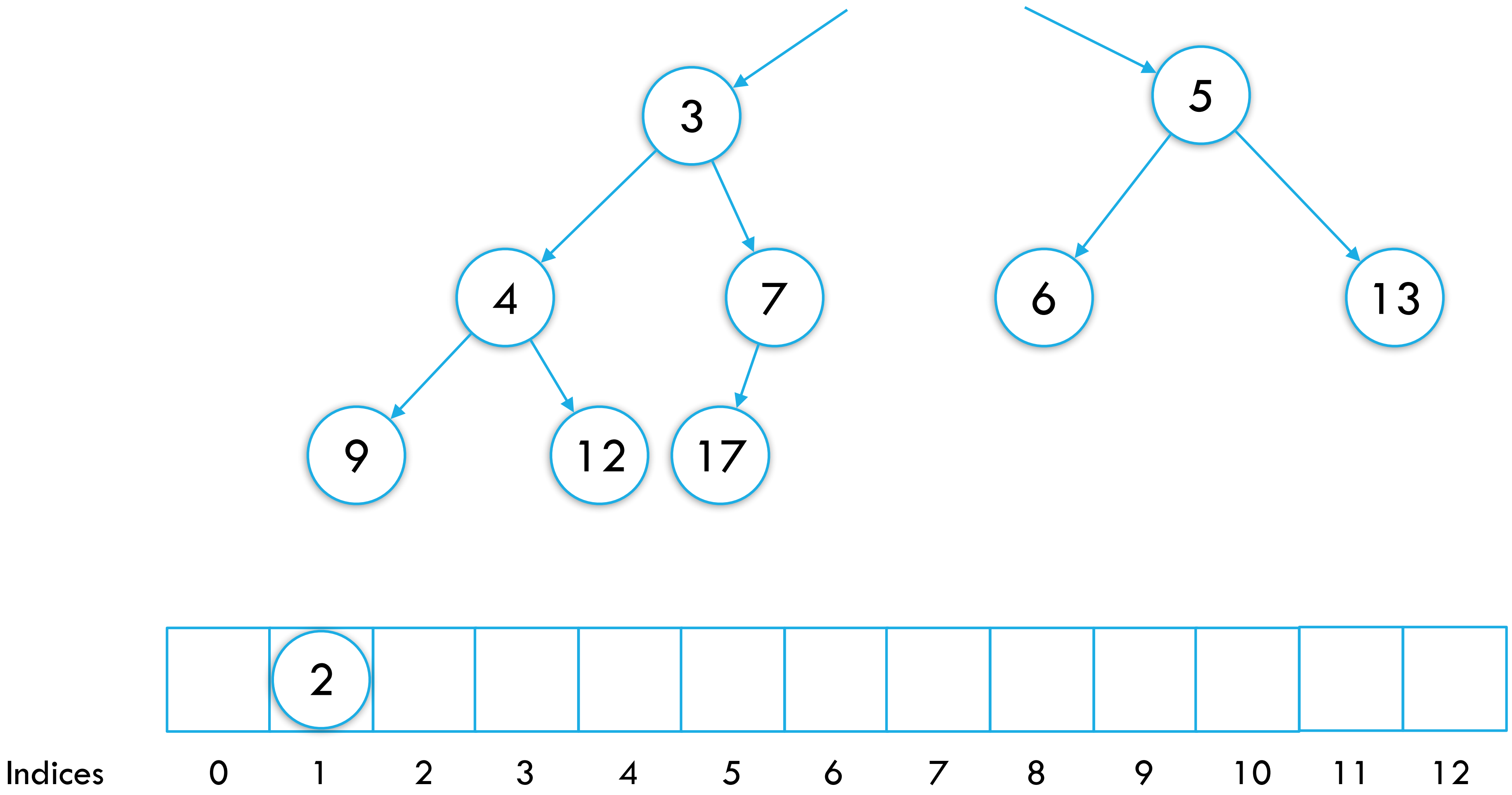

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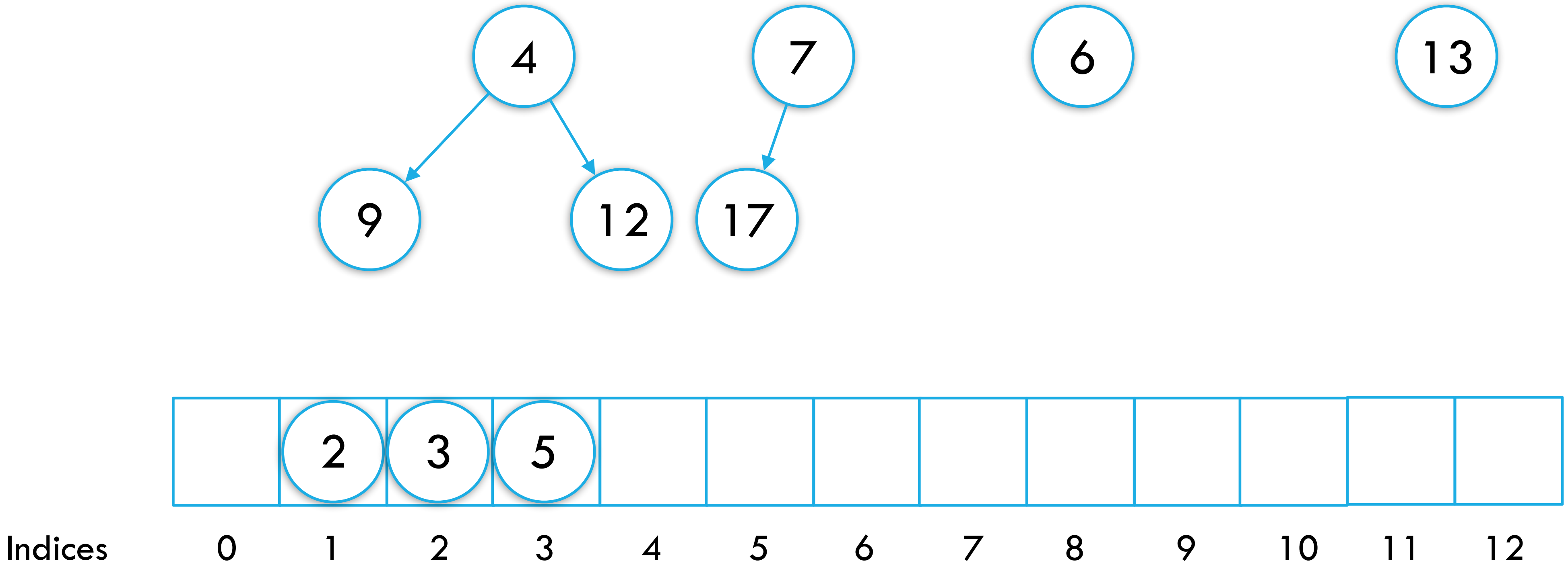

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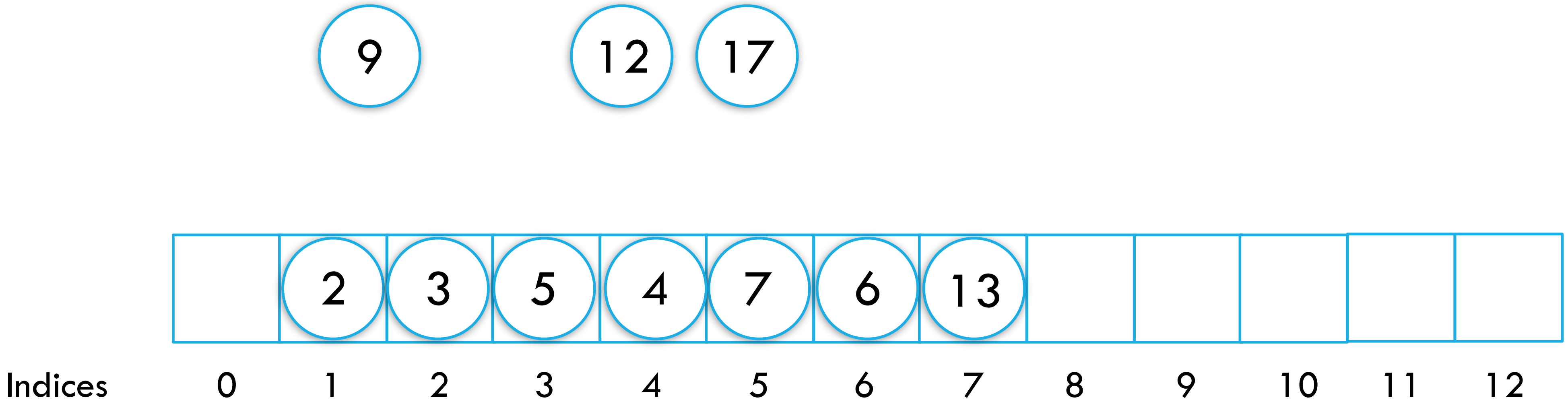

Binary heap: Array implementation

		2	3	5	4	7	6	13	9	12	17		
Indices	0	1	2	3	4	5	6	7	8	9	10	11	12

Binary heap: Array implementation

```
leftChild(i) = 2i

rightChild(i) = 2i + 1

parent(i) = i / 2
```

Indices	0	1	2	3	4	5	6	7	8	9	10	11	12
		2	3	5	4	7	6	13	9	12	17		