CSE 373: Data Structures and Algorithms

Hash Tables

Autumn 2018

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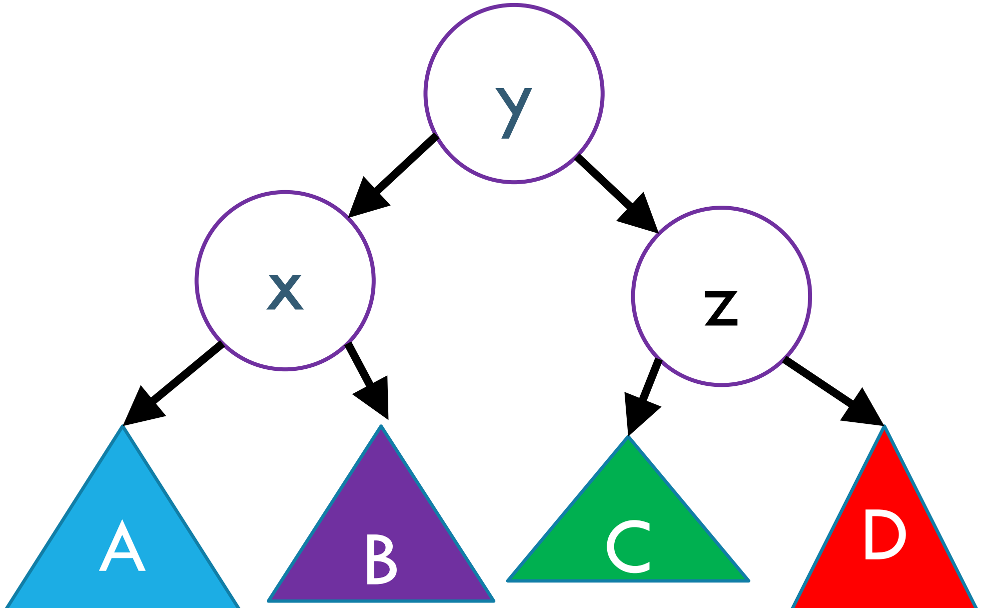
Today

- Wrap up AVL Trees
- Problem: Can we make get(k) operation on dictionaries fast: O(1)
- Motivation
- Hashing
- Separate Chaining

AVL Trees: Four cases to consider

Insert location	Case	Solution
Left subtree of left child of y (A)	Left line case	Single right rotation
Right subtree of left child of y (B)	Left kink case	Double (left-right) rotation
Left subtree of right child of y (C)	Right kink case	Double (right-left) rotation
Right subtree of right child of y (D)	Right line case	Single left rotation



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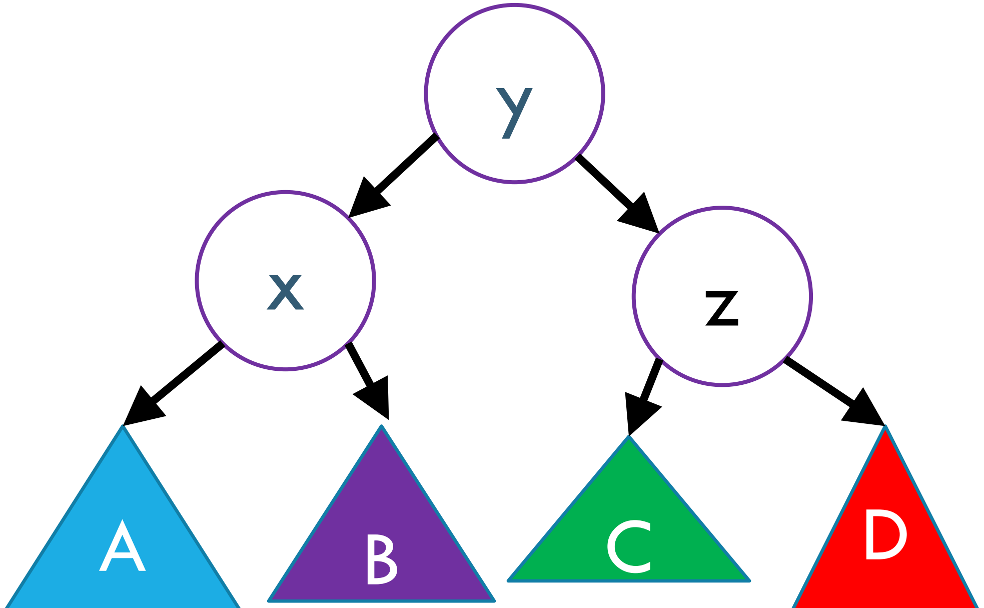
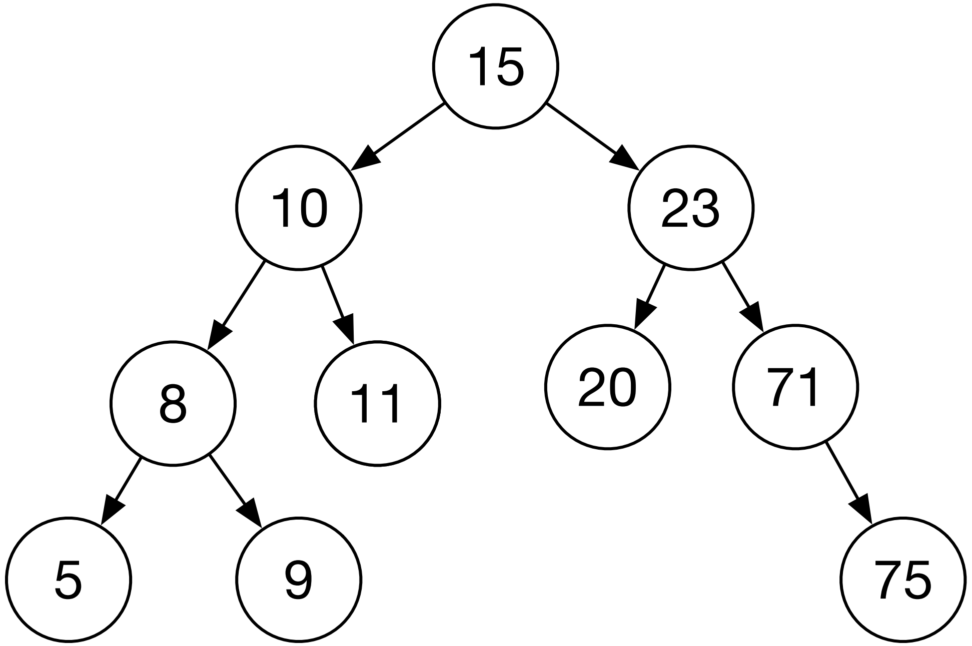
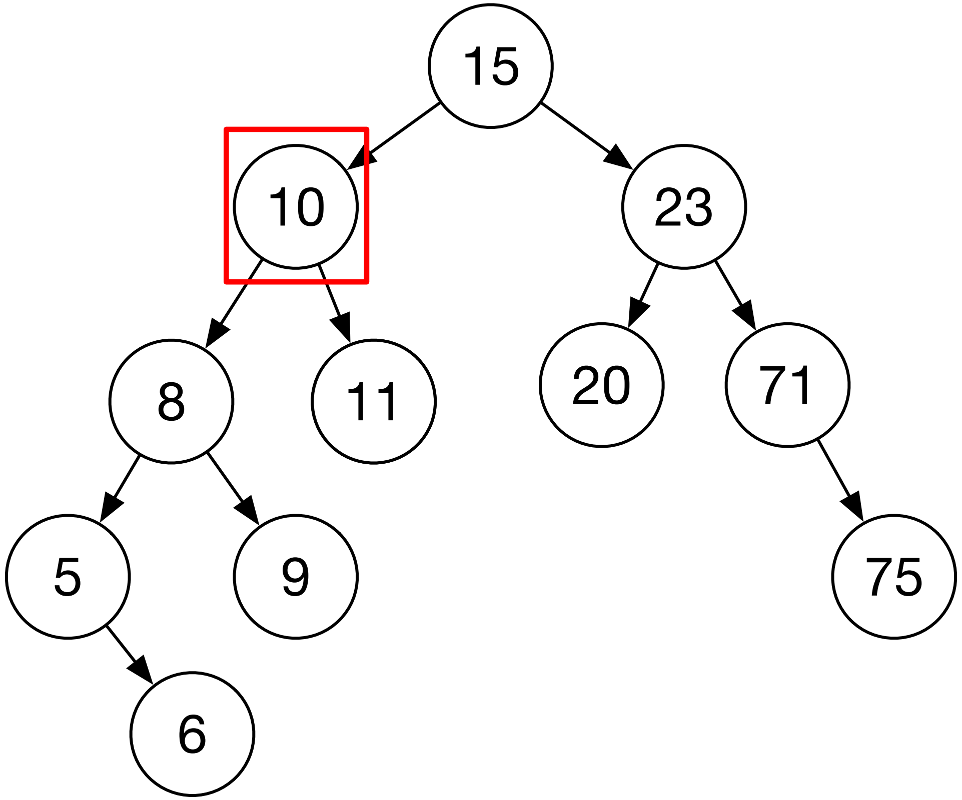


AVL Tree: Practice. Insert(6)



AVL Tree: Practice



Unbalanced

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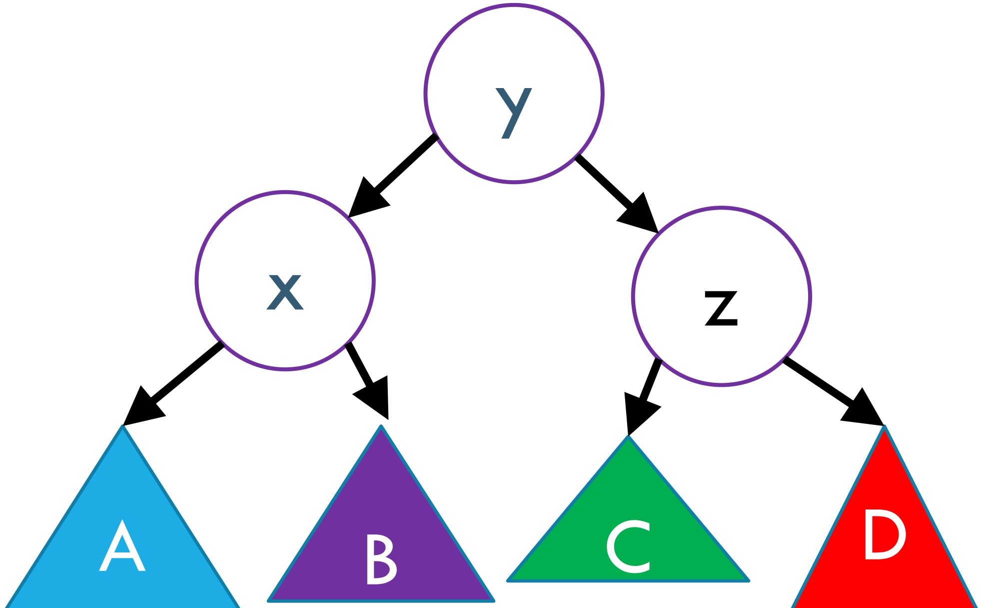
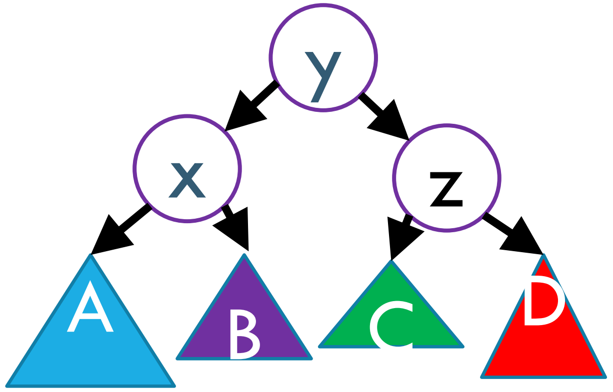
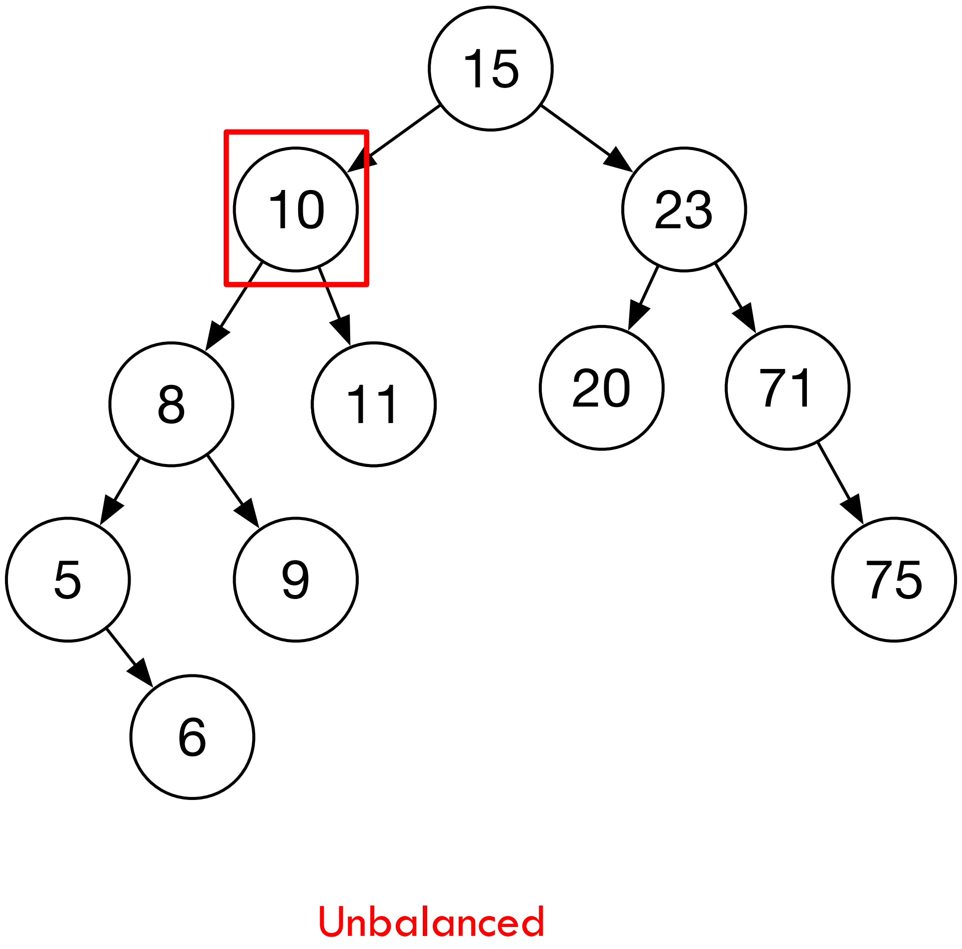


AVL Tree: Practice





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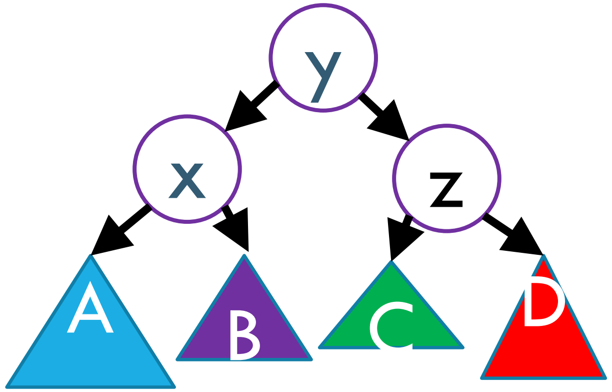
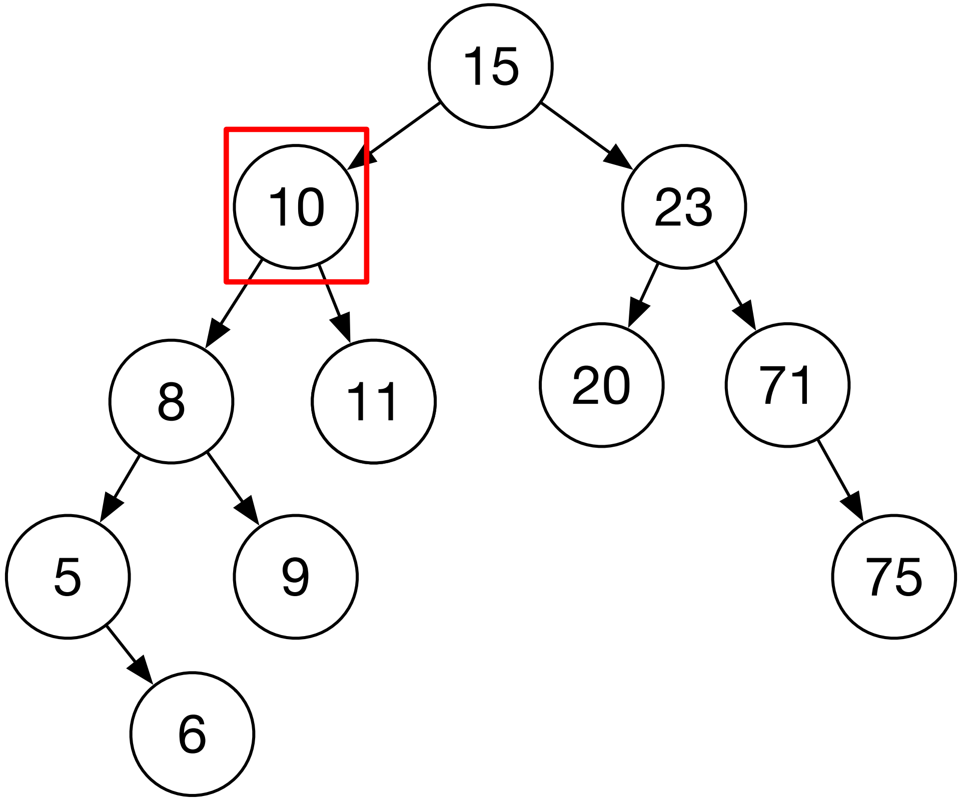
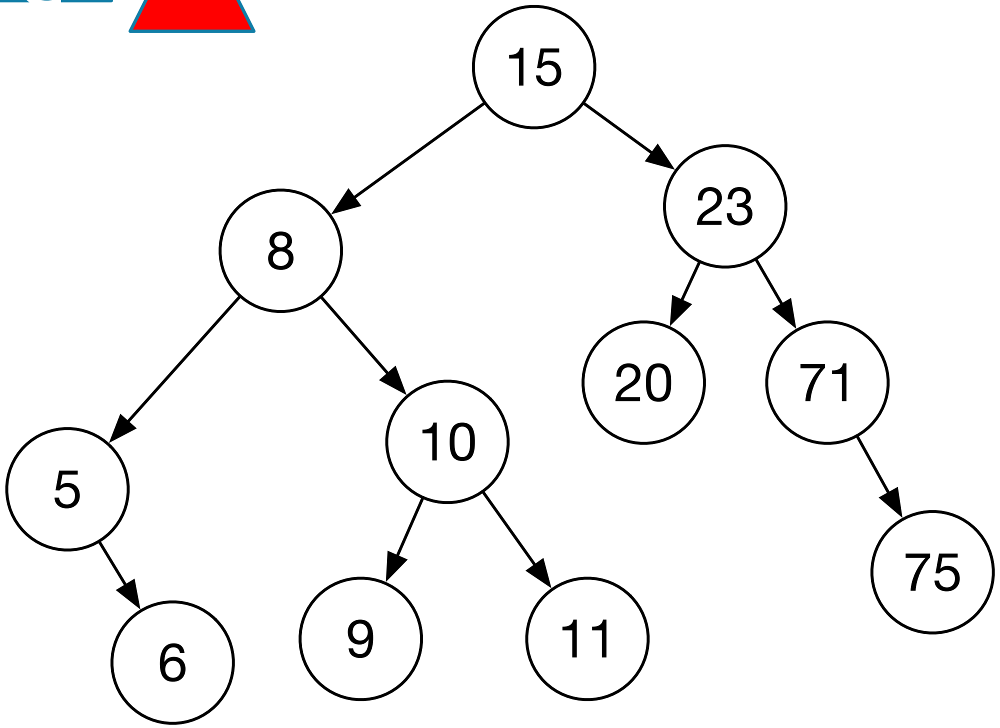


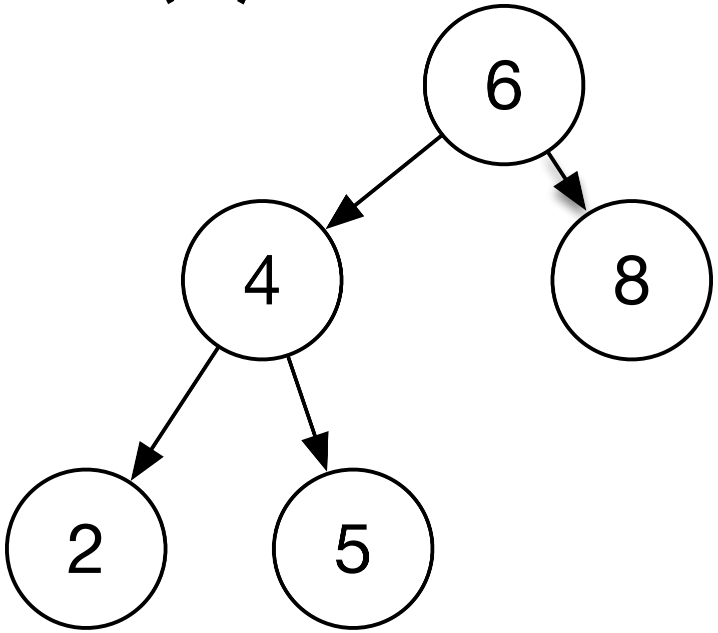


Unbalanced



Worksheet Q1

insert(1)



Case: Line or Kink?

Worksheet Q1

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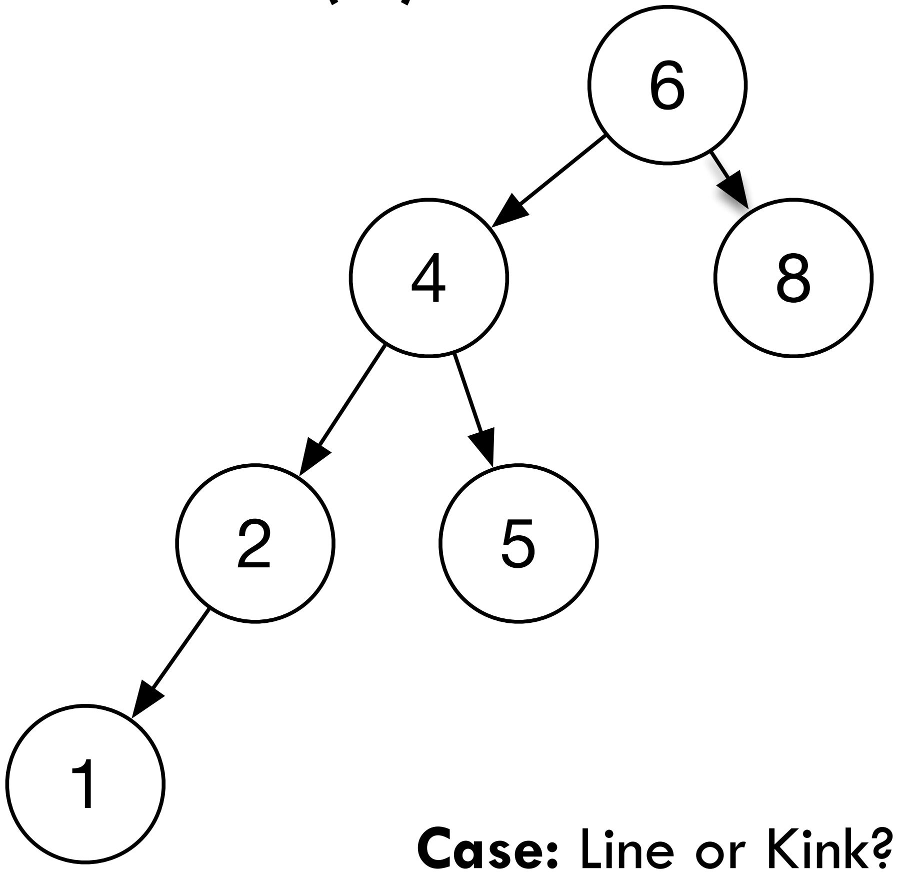

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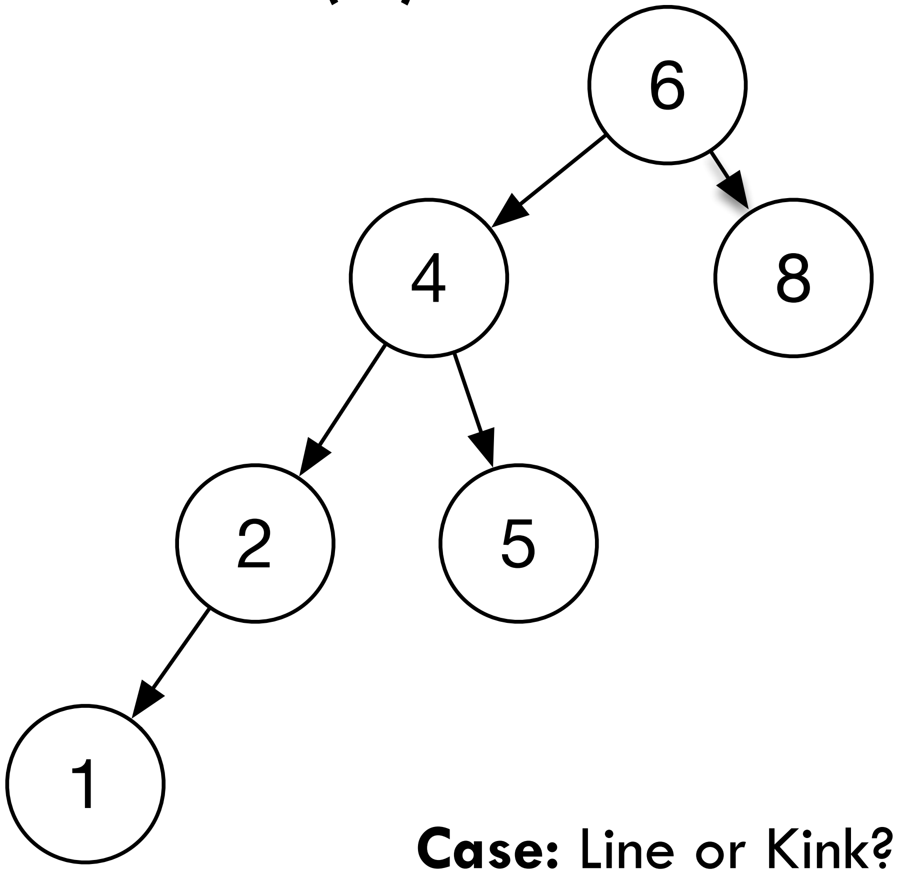

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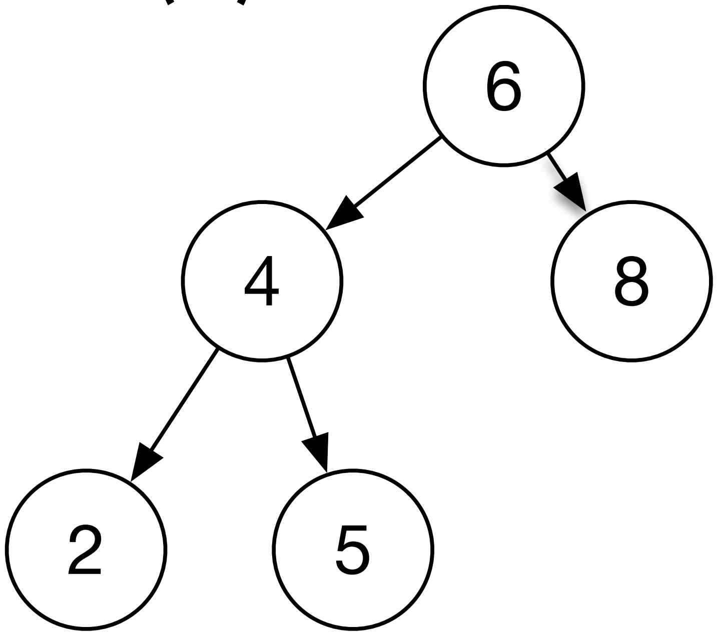

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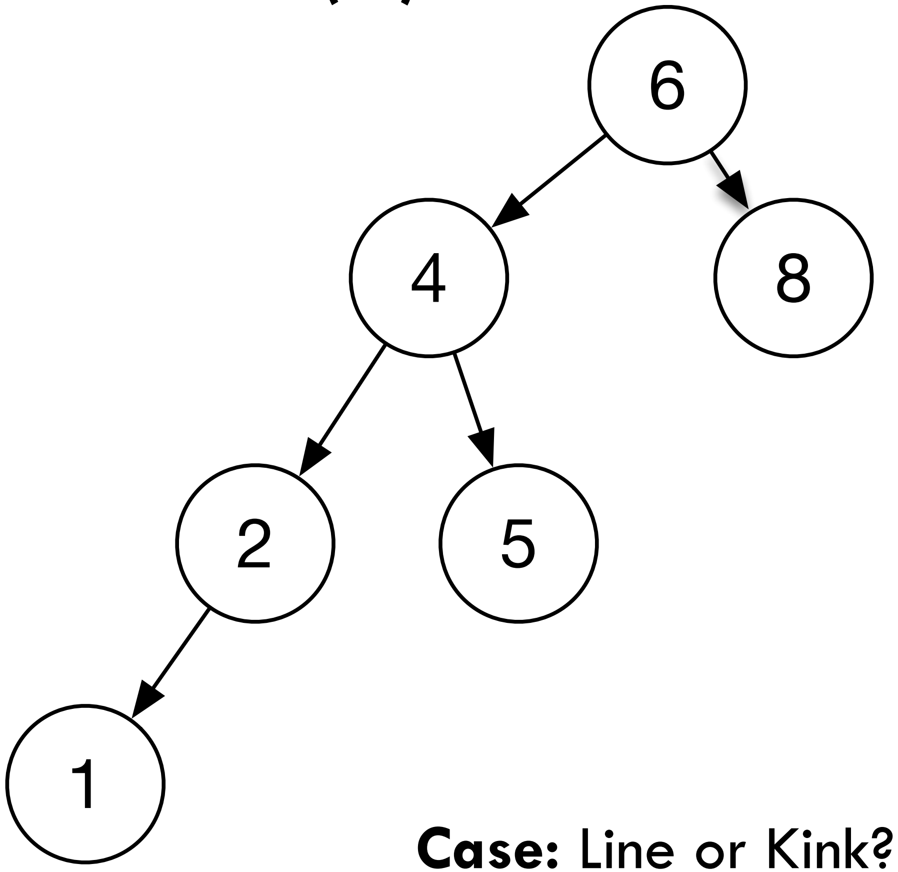

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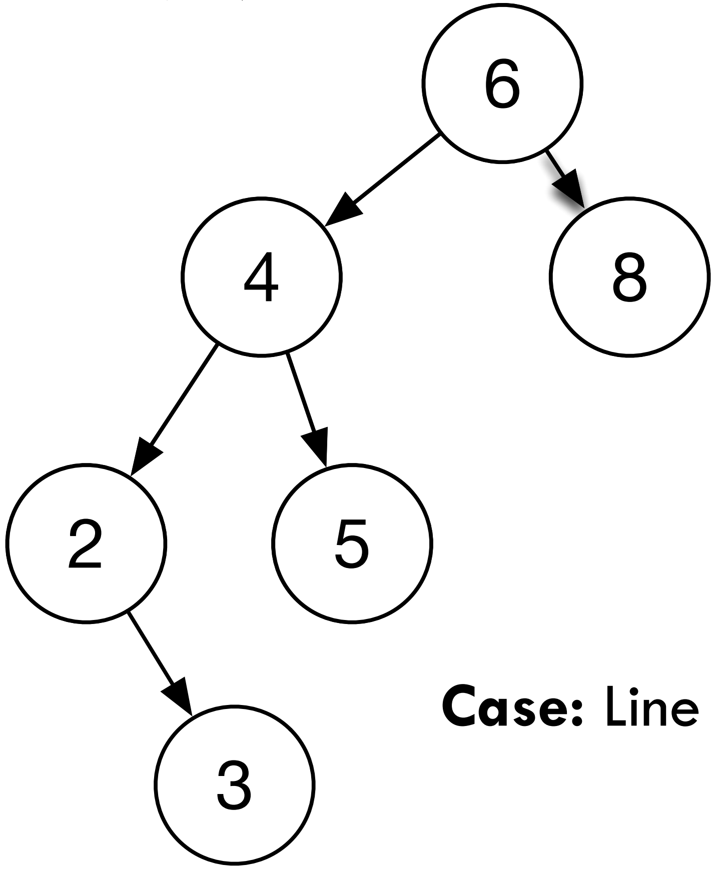

Case: Line or Kink?

AVL Tree insertions

1. Do a BST insert – insert a node as you would in a BST.
2. Check balance condition at each node in the path from the inserted node to the root.
3. If balance condition is not true at a node, identify the case
4. Do the corresponding rotation for the case

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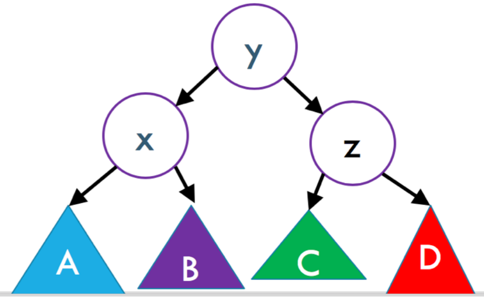

Worksheet Q2

Draw the AVL tree that results from inserting the keys 1, 3, 7, 5, 6, 9 in that order into an initially empty AVL tree. (*Hint:* Drawing intermediate trees as you insert each key can help.)

How long does AVL insert take?

AVL insert time = BST insert time + time it takes to rebalance the tree

= O(log n) + time it takes to rebalance the tree

How long does rebalancing take?

- Assume we store in each node the height of its subtree.
- How long to find an unbalanced node:
  - Just go back up the tree from where we inserted.
- How many rotations might we have to do?
  - Just a single or double rotation on the lowest unbalanced node.

AVL insert time = O(log n)+ O(log n) + O(1) = O(log n)

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- How many rotations might we have to do?
  - Just a single or double rotation on the lowest unbalanced node. ← O(1)

AVL insert time = O(log n)+ O(log n) + O(1) = O(log n)

AVL wrap up

Pros:

- $O(\log n)$ worst case for `find`, `insert`, and `delete` operations.
- Reliable running times than regular BSTs (because trees are balanced)

Cons:

- Difficult to program & debug [but done once in a library!]
- (Slightly) more space than BSTs to store node heights.

Lots of cool Self-Balancing BSTs out there!

Popular self-balancing BSTs include:

AVL tree

Splay tree

2-3 tree

AA tree

Red-black tree

Scapegoat tree

Treap

(Not covered in this class, but several are in the textbook and all of them are online!)

(From https://en.wikipedia.org/wiki/Self-balancing_binary_search_tree#Implementations)

Announcements

- HW3 out. Due this Friday (10/26) at **Noon** (not at the usual time 11:59pm)
- HW4 out later today (or latest by tomorrow morning). Due next Tuesday (10/30)

- Midterm coming up – Nov 2, 2:30-3:20pm, here in the class
- If you can't take the midterm on Nov 2, let me know ASAP.
- Midterm practice material will be posted on the website tomorrow
- Midterm review next Wednesday

Hash tables

Revisiting Dictionaries

- data = (key, value)
- operations: `put(key, value);` `get(key);` `remove(key)`

- O(n) with Arrays and Linked List
- O(log n) with BST and AVL trees.

- Can we do better? Can we do this in O(1) ?

Motivation

Why we are so obsessed with making dictionaries fast?

Dictionaries are extremely most common data structures.

- Databases
- Network router tables
- Compilers and Interpreters
- Faster than O(log n) search in certain cases
- Data type in most high level programming languages

Question

How would you implement at dictionary such that dictionary operations are O(1)?

(Assume all keys are non-zero integers)

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Problems?

1. ?

2. ?

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(Assume all keys are non-zero integers)

Idea: Create a giant array and use keys as indices

Problems?

1. Can only work with integer keys?
2. Too much wasted space

Idea 2: Can we convert the key space into a smaller set that would take much less memory

Solve problem: Too much wasted space

Review: Integer remainder with %

The `%` operator computes the remainder from integer division.

`14 % 4` is `2` `218 % 5` **is** `3`

```
      3            43
4 ) 14       5 ) 218
    12           20
     2            18
                  15
                   3
```

Applications of `%` operator:

- Obtain last digit of a number: `230857 % 10` is `7`
- See whether a number is odd: `7 % 2` is `1`, `42 % 2` is `0`
- Limit integers to specific range: `8 % 12` is `8`, `18 % 12` is `6`

Implement Direct Access Map

```
public V get(int key) {
   // input validation
   return this.array[key].value;
}

public void put(int key, V value) {
    this.array[key] = value;
}

public void remove(int key) {
   // input validation
   this.array[key] = null;
}
```

Implement First Hash Function

```
public V get(int key) {
   // input validation
   int newKey = key % this.array.length;
   return this.array[newkey].value;
}

public void put(int key, V value) {
    this.array[key % this.array.length] = value;
}
public void remove(int key) {
   // input validation
   int newKey = key % this.array.length;
   this.array[newKey] = null;
}
```

First Hash Function: % table size

indices	0	1	2	3	4	5	6	7	8	9
elements	"foo"	"biz"				"bar"			"bop"	

"poo"

```
put(0, "foo");  0 % 10 = 0
put(5, "bar");  5 % 10 = 5
put(11, "biz")  11 % 10 = 1
put(18, "bop"); 18 % 10 = 8
put(20, "poo"); 20 % 10 = 0
```

→ Collision!

Today

√ Wrap up AVL Trees

√ Problem: Can we make get(k) operation on dictionaries fast: O(1)

√ Motivation

√ Hashing

- Separate Chaining

Hashing: Separate Chaining