

CSE 373: Data Structures and Algorithms

Hash Tables

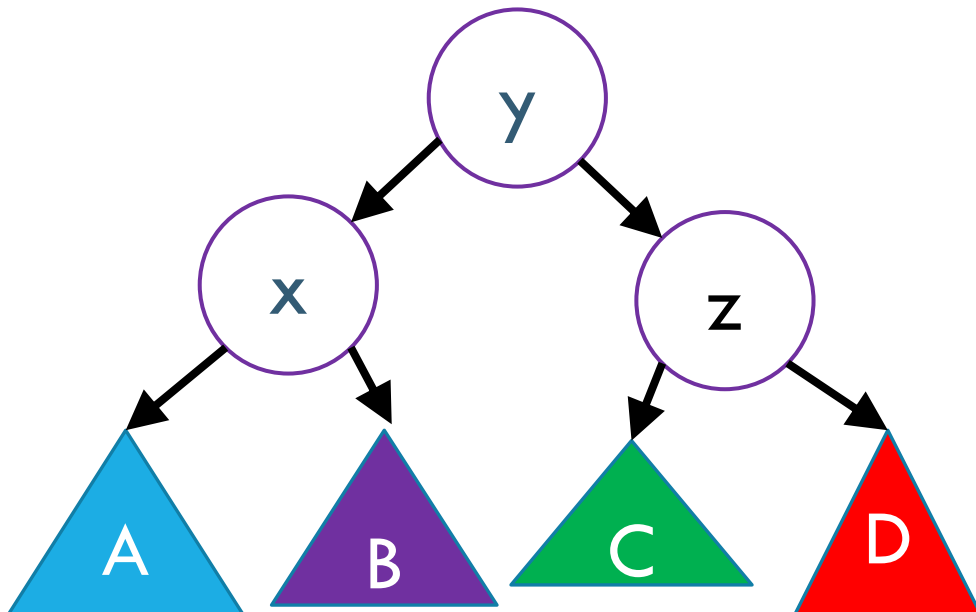
Autumn 2018

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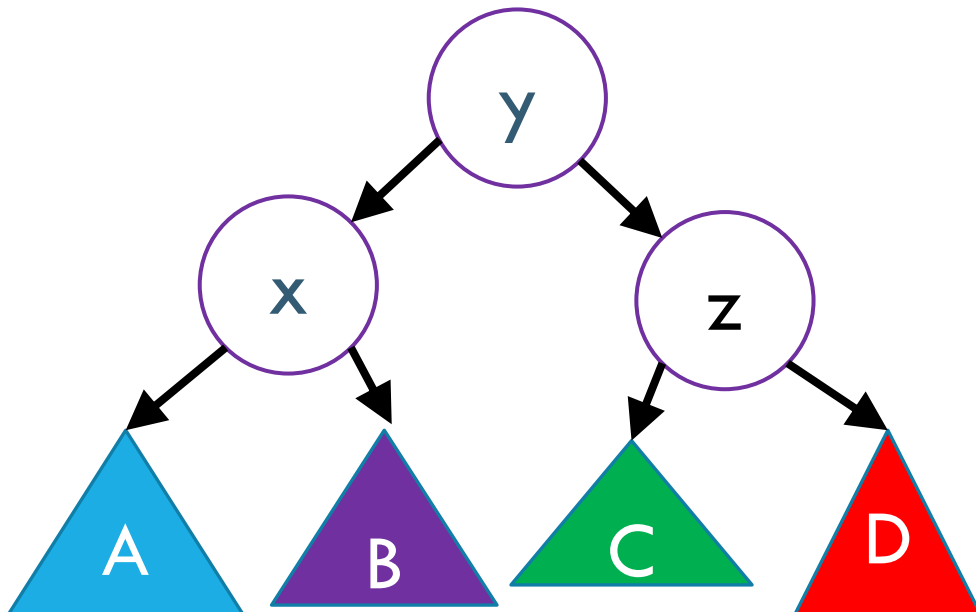
AVL Trees: Four cases to consider

Insert location	Case	Solution
Left subtree of left child of y (A)	Left line case	Single right rotation
Right subtree of left child of y (B)	Left kink case	Double (left-right) rotation
Left subtree of right child of y (C)	Right kink case	Double (right-left) rotation
Right subtree of right child of y (D)	Right line case	Single left rotation

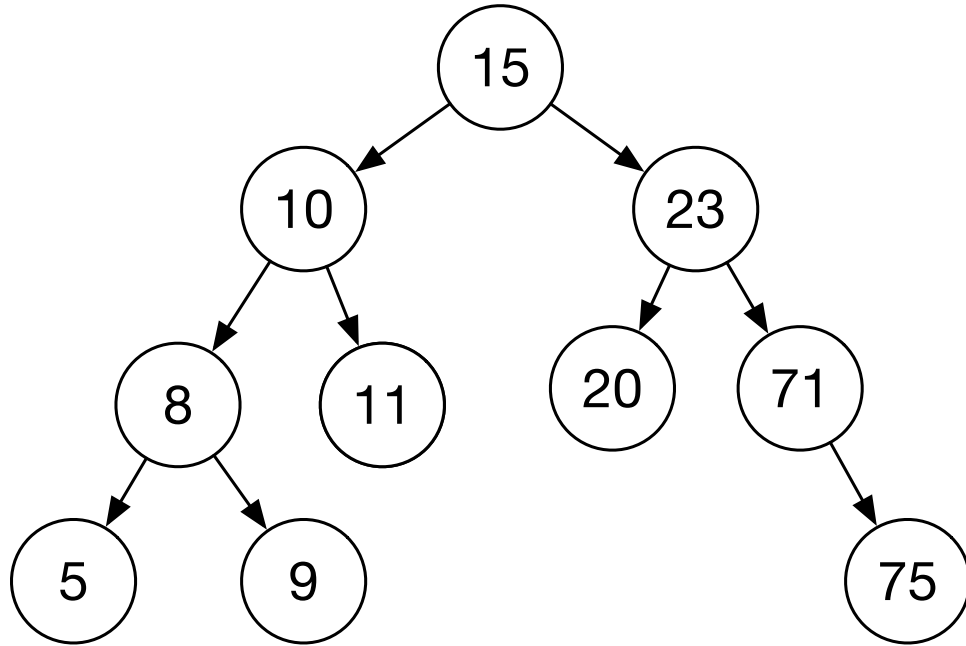


AVL Trees: Four cases to consider

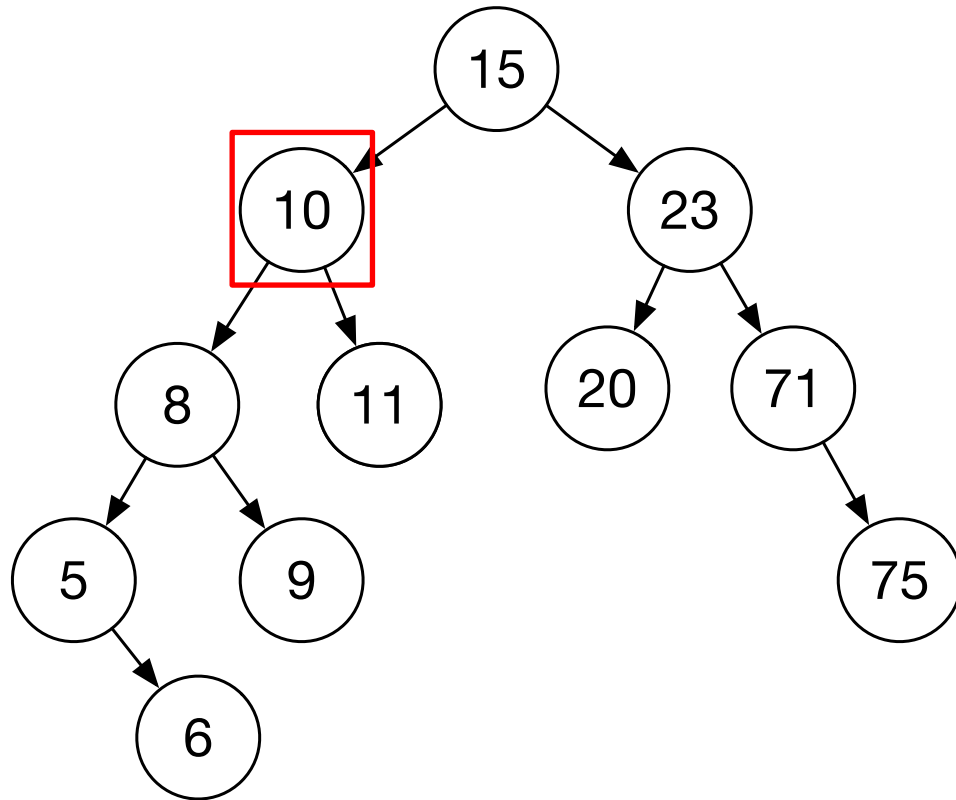
Insert location	Case (also called as)	Solution
Left subtree of left child of y (A)	Left line case (case 1)	Single right rotation
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Left subtree of right child of y (C)	Right kink case (case 3)	Double (right-left) rotation
Right subtree of right child of y (D)	Right line case (case 4)	Single left rotation



AVL Tree: Practice. Insert(6)



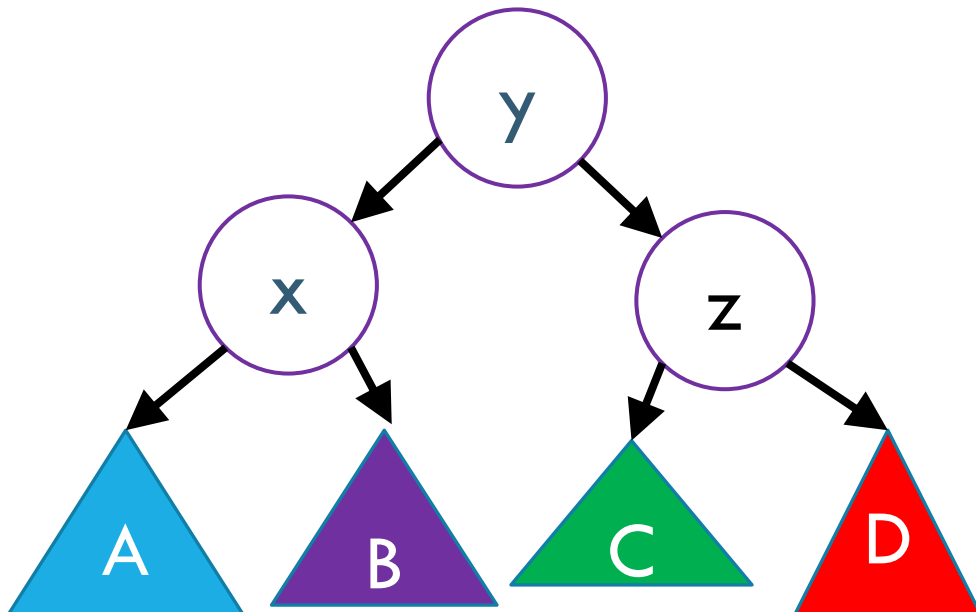
AVL Tree: Practice



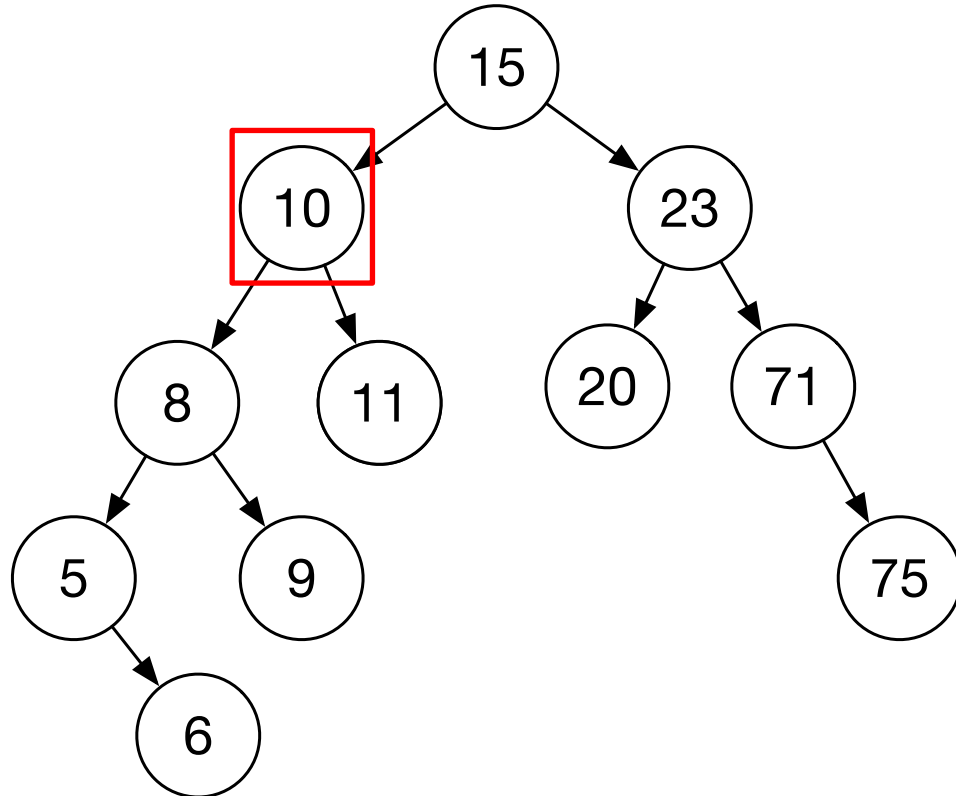
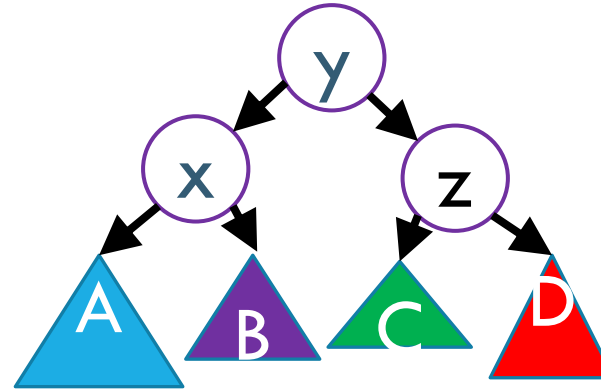
Unbalanced

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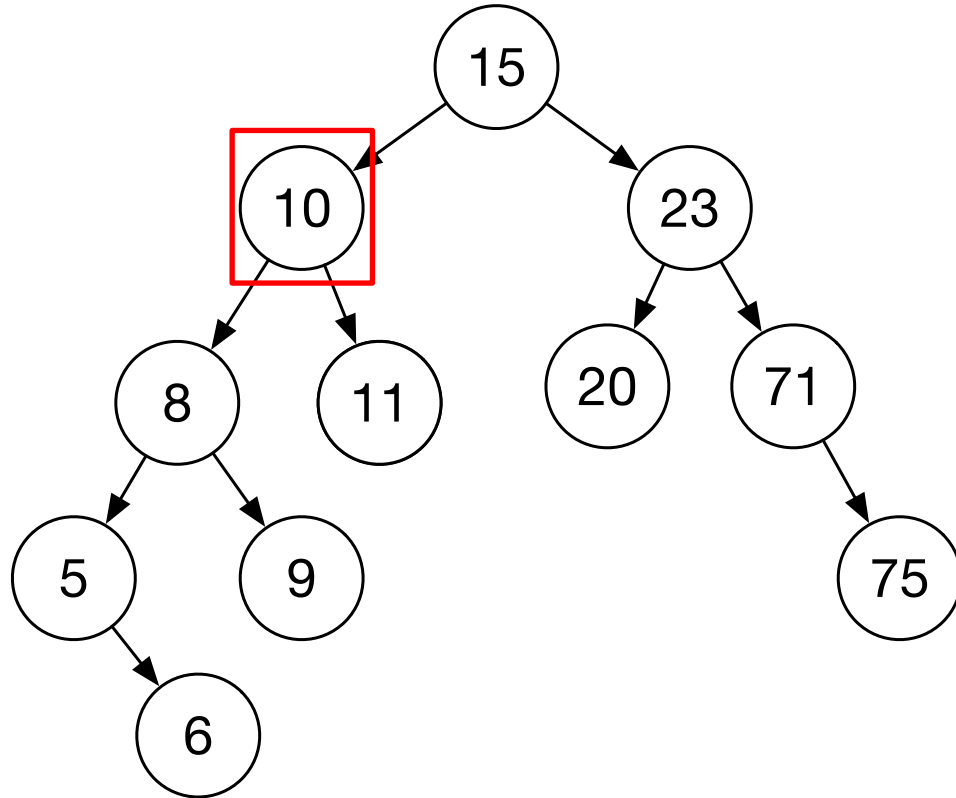
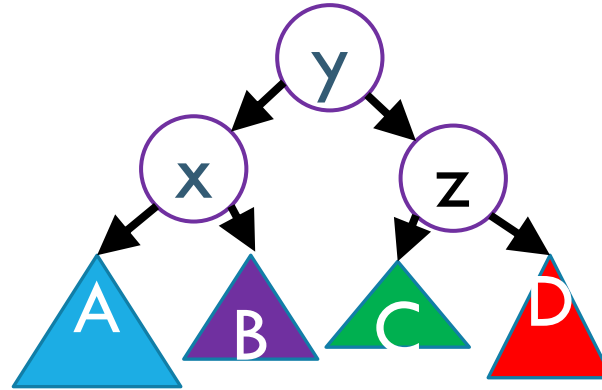


AVL Tree: Practice

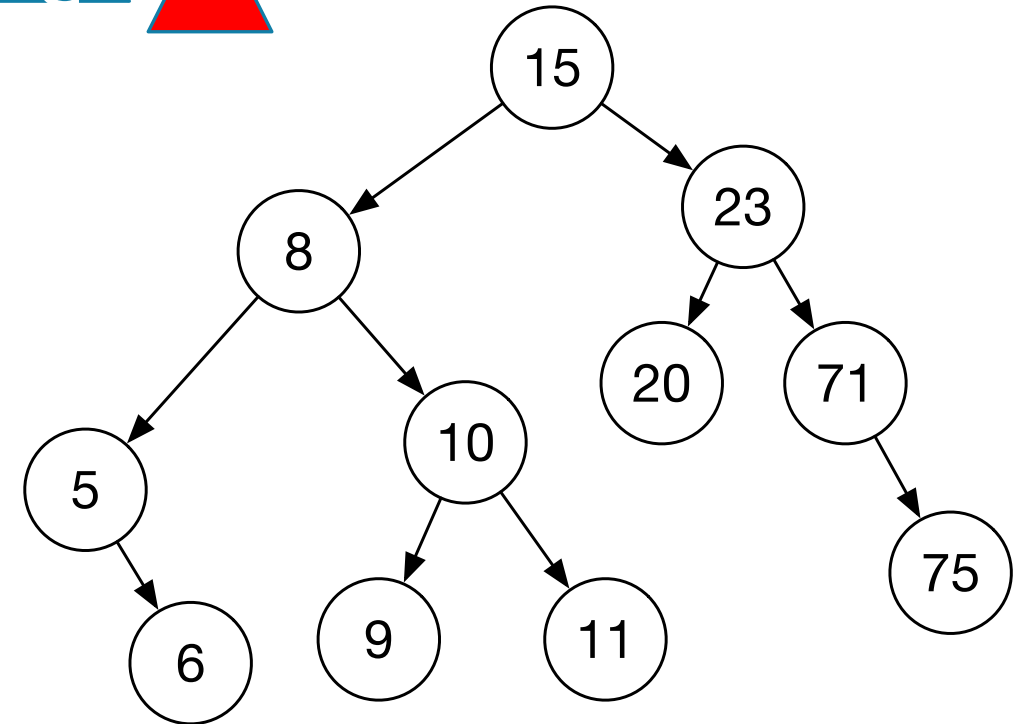


Unbalanced

AVL Tree: Practice

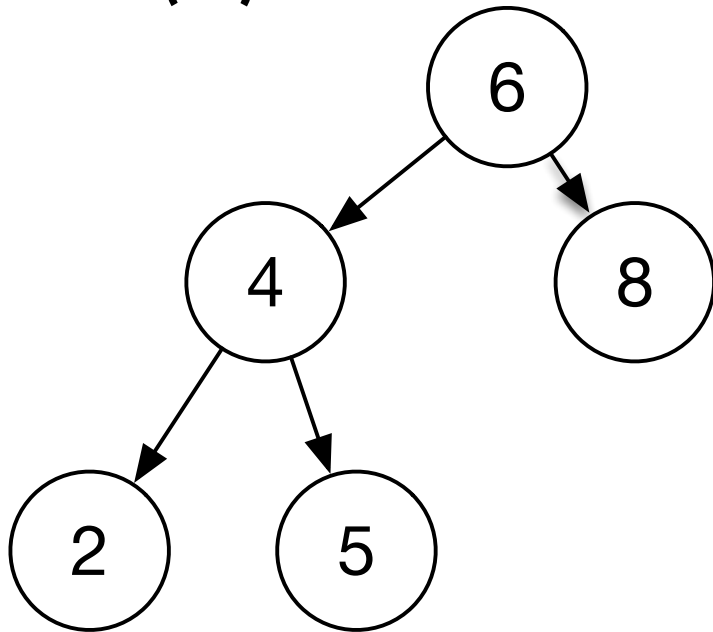


Unbalanced



Worksheet Q1

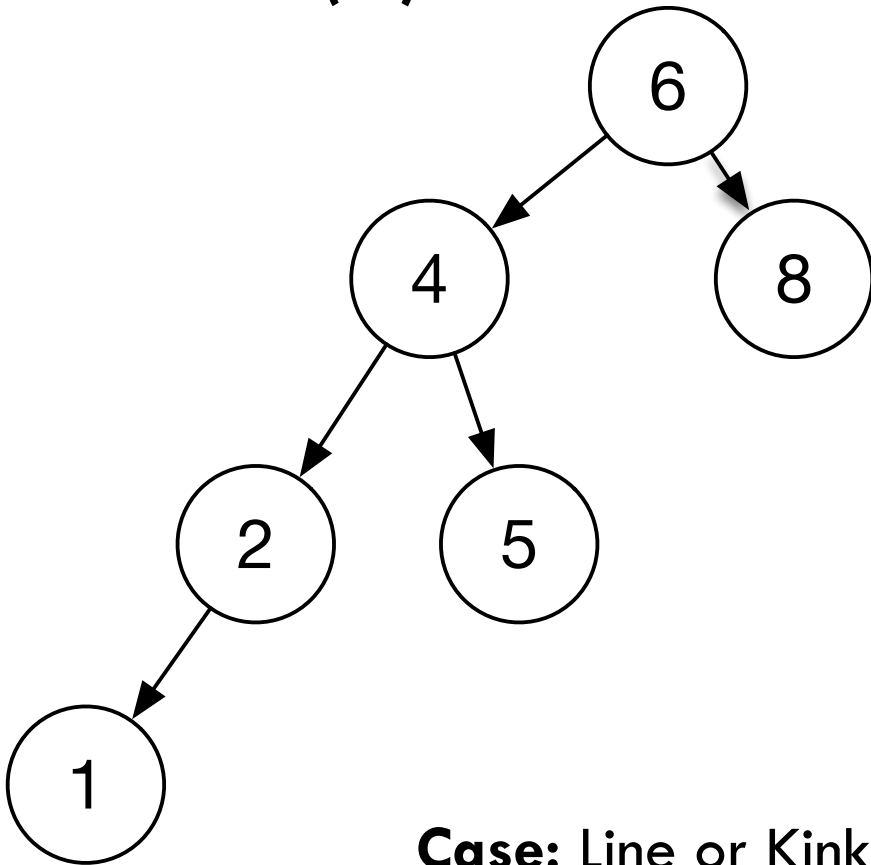
insert(1)



Case: Line or Kink?

Worksheet Q1

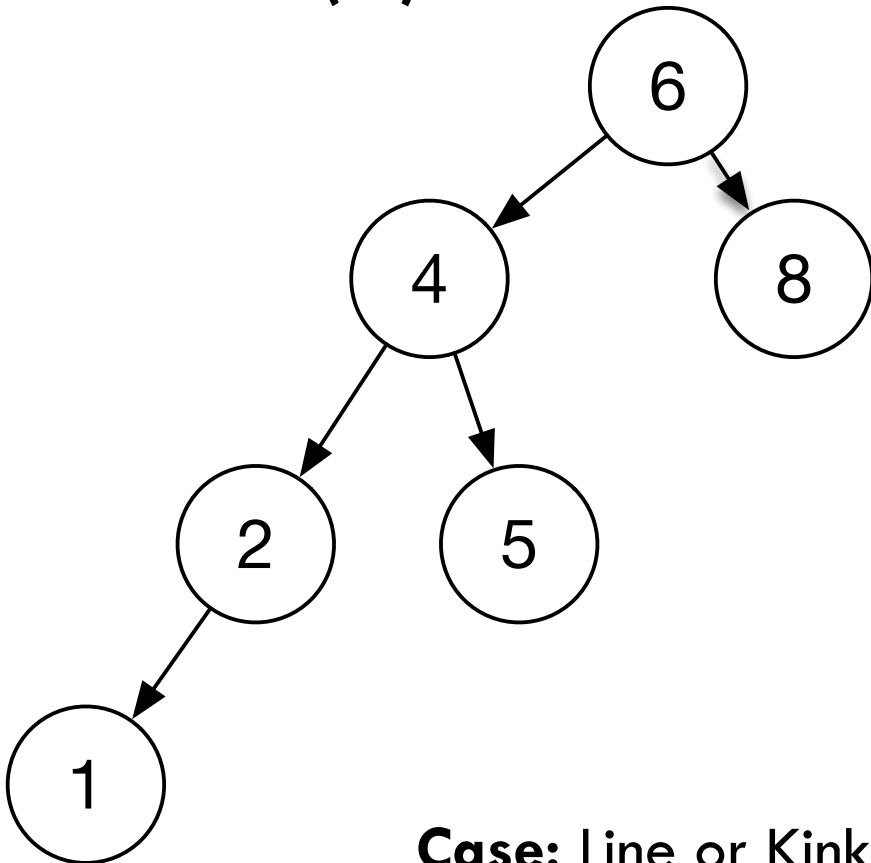
insert(1)



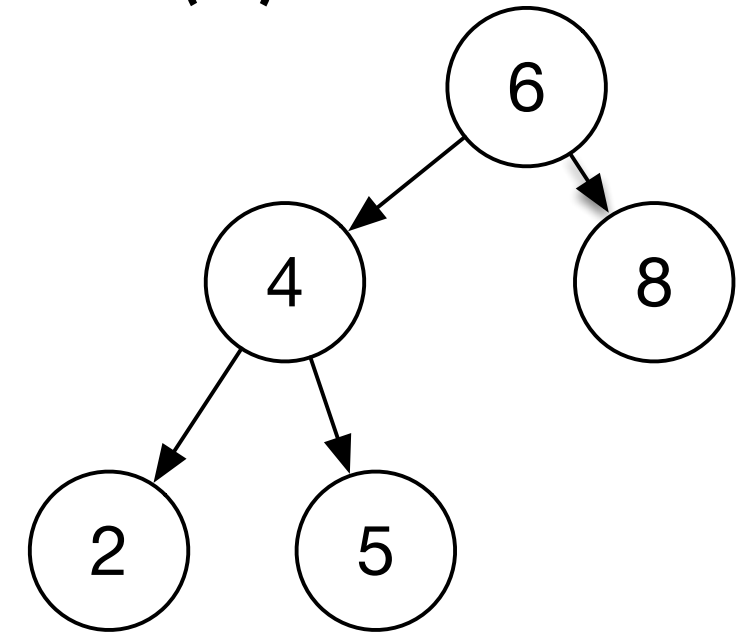
Case: Line or Kink?

Worksheet Q1

insert(1)



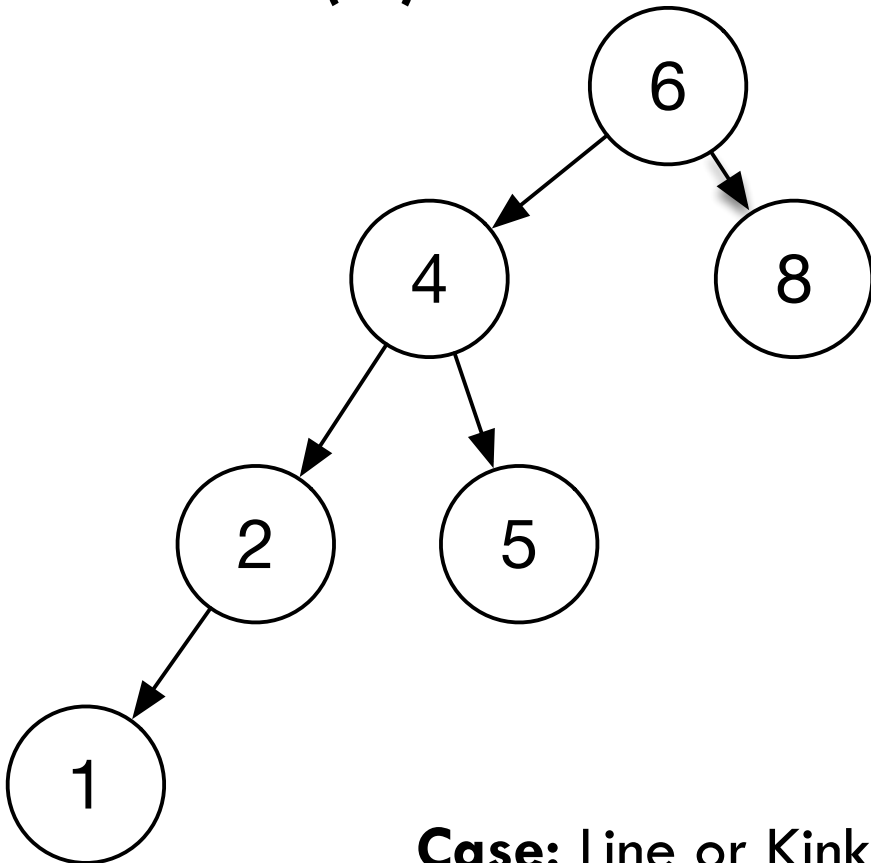
insert(3)



Case: Line or Kink?

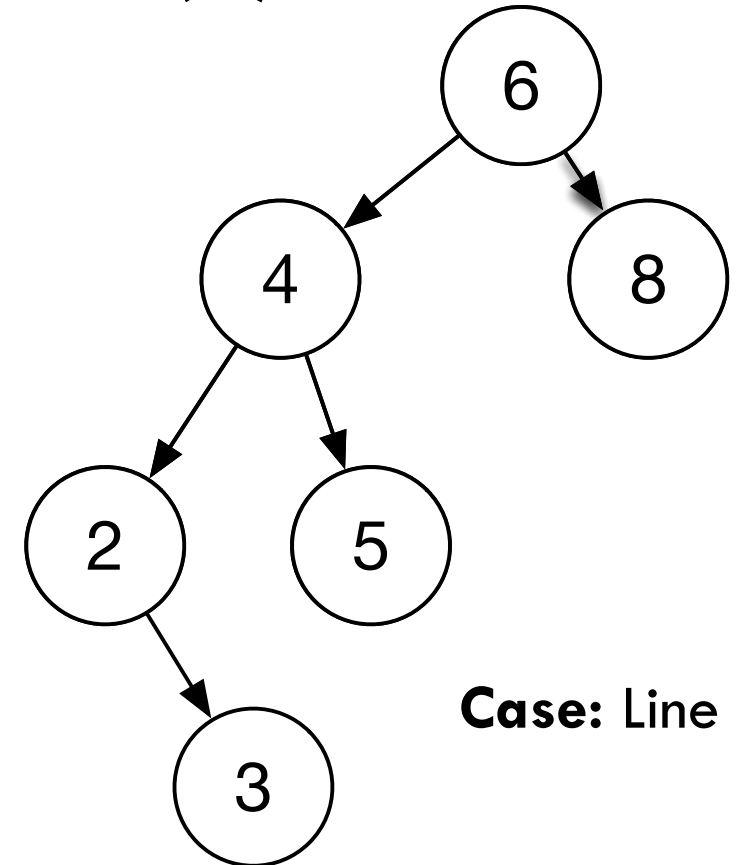
Worksheet Q1

insert(1)



Case: Line or Kink?

insert(3)



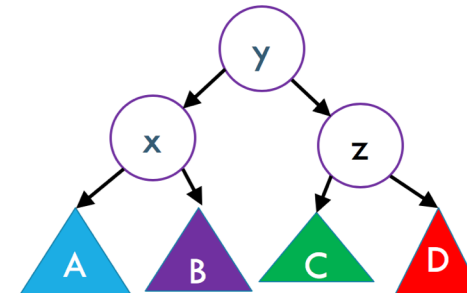
Case: Line or Kink?

AVL Tree insertions

1. Do a BST insert – insert a node as you would in a BST.
2. Check balance condition at each node in the path from the inserted node to the root.
3. If balance condition is not true at a node, identify the case
4. Do the corresponding rotation for the case

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Worksheet Q2

Draw the AVL tree that results from inserting the keys 1, 3, 7, 5, 6, 9 in that order into an initially empty AVL tree. (*Hint*: Drawing intermediate trees as you insert each key can help.)

How long does AVL insert take?

AVL insert time = BST insert time + time it takes to rebalance the tree
= $O(\log n)$ + time it takes to rebalance the tree

How long does rebalancing take?

- Assume we store in each node the height of its subtree.
- How long to find an unbalanced node:
 - Just go back up the tree from where we inserted.
- How many rotations might we have to do?
 - Just a single or double rotation on the lowest unbalanced node.

AVL insert time = $O(\log n) + O(\log n) + O(1) = O(\log n)$

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- How long to find an unbalanced node:
 - Just go back up the tree from where we inserted. $\leftarrow O(\log n)$
- How many rotations might we have to do?
 - Just a single or double rotation on the lowest unbalanced node. $\leftarrow O(1)$

AVL insert time = $O(\log n) + O(\log n) + O(1) = O(\log n)$

AVL wrap up

Pros:

- $O(\log n)$ worst case for find, insert, and delete operations.
- Reliable running times than regular BSTs (because trees are balanced)

Cons:

- Difficult to program & debug [but done once in a library!]
- (Slightly) more space than BSTs to store node heights.

Lots of cool Self-Balancing BSTs out there!

Popular self-balancing BSTs include:

AVL tree

Splay tree

2-3 tree

AA tree

Red-black tree

Scapegoat tree

Treap

(Not covered in this class, but several are in the textbook and all of them are online!)

(From https://en.wikipedia.org/wiki/Self-balancing_binary_search_tree#Implementations)

Announcements

- HW3 out. Due this Friday (10/26) at **Noon** (not at the usual time 11:59pm)
- HW4 out later today (or latest by tomorrow morning). Due next Tuesday (10/30)
- Midterm coming up – Nov 2, 2:30-3:20pm, here in the class
- If you can't take the midterm on Nov 2, let me know ASAP.
- Midterm practice material will be posted on the website tomorrow
- Midterm review next Wednesday



Hash tables

Revisiting Dictionaries

- data = (key, value)
- operations: `put(key, value);` `get(key);` `remove(key)`
- $O(n)$ with Arrays and Linked List
- $O(\log n)$ with BST and AVL trees.
- Can we do better? Can we do this in $O(1)$?

Motivation

Why we are so obsessed with making dictionaries fast?

Dictionaries are extremely most common data structures.

- Databases
- Network router tables
- Compilers and Interpreters
- Faster than $O(\log n)$ search in certain cases
- Data type in most high level programming languages

Question

How would you implement a dictionary such that dictionary operations are $O(1)$?
(Assume all keys are non-zero integers)

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Idea: Create a giant array and use keys as indices

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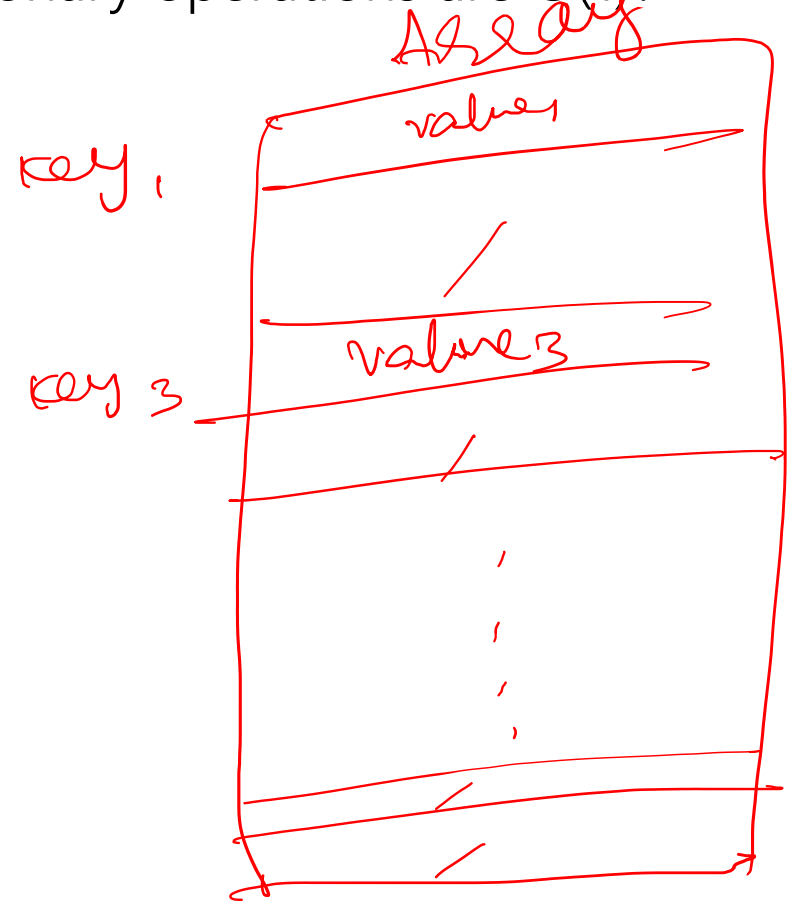
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Idea: Create a giant array and use keys as indices

Problems?

1. ?

2. ?



Question

How would you implement a dictionary such that dictionary operations are $O(1)$?
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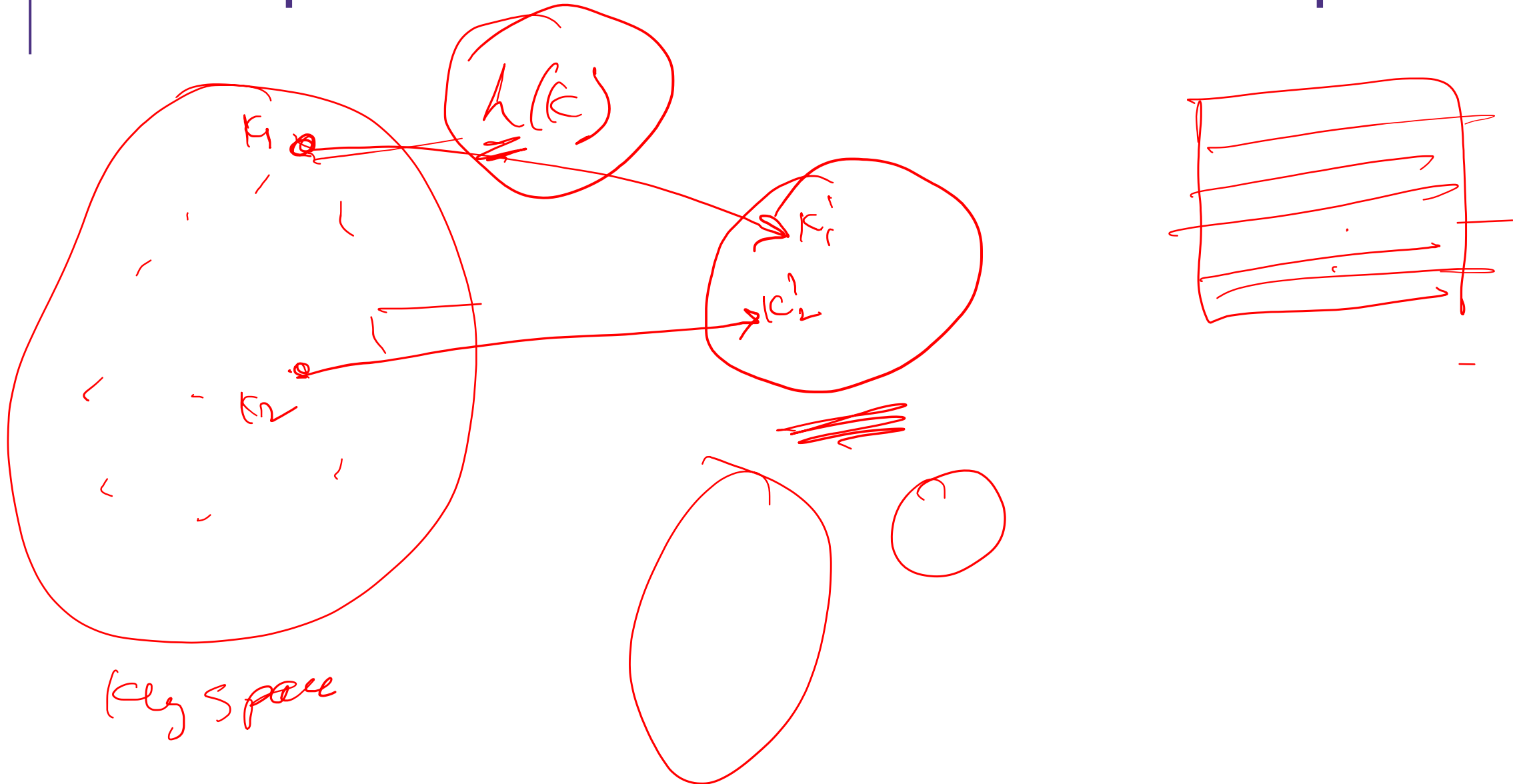
Idea: Create a giant array and use keys as indices

Problems?

1. Can only work with integer keys?
2. Too much wasted space

Idea 2: Can we convert the key space into a smaller set that would take much less memory

Solve problem: Too much wasted space



Review: Integer remainder with %

The % operator computes the remainder from integer division.

14 % 4 is 2

$$\begin{array}{r} 3 \\ 4 \overline{) 14} \\ \underline{12} \\ 2 \end{array}$$

218 % 5 is 3

$$\begin{array}{r} 43 \\ 5 \overline{) 218} \\ \underline{20} \\ 18 \\ \underline{15} \\ 3 \end{array}$$


Applications of % operator:

- Obtain last digit of a number: $230857 \% 10$ is 7
- See whether a number is odd: $7 \% 2$ is 1, $42 \% 2$ is 0
- Limit integers to specific range: $8 \% 12$ is 8, $18 \% 12$ is 6

Implement Direct Access Map



```
public V get(int key) {  
    // input validation  
    return this.array[key].value;  
}  
  
public void put(int key, V value) {  
    this.array[key] = value;  
}  
  
public void remove(int key) {  
    // input validation  
    this.array[key] = null;  
}
```



Implement First Hash Function

```
public V get(int key) {  
    // input validation  
    int newKey = key % this.array.length;  
    return this.array[newkey].value;  
}
```

```
public void put(int key, V value) {  
    this.array[key % this.array.length] = value;  
}
```

```
public void remove(int key) {  
    // input validation  
    int newKey = key % this.array.length;  
    this.array[newKey] = null;  
}
```



newkey



key

First Hash Function: % table size

indices	0	1	2	3	4	5	6	7	8	9
elements	"foo"	"biz"				"bar"			"bop"	

"poo"

```
put(0, "foo"); 0 % 10 = 0
```

```
put(5, "bar"); 5 % 10 = 5
```

```
put(11, "biz"); 11 % 10 = 1
```

```
put(18, "bop"); 18 % 10 = 8
```

```
put(20, "poo"); 20 % 10 = 0
```



Collision!