More on AVL Trees

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Quick (Anonymous) Feedback

Go to this URL  https://tinyurl.com/373-feedback

1. In general, pace of class:
   - 1 too fast
   - 2 kind of fast
   - 3 just right
   - 4 kind of slow
   - 5 Too slow

2. Please **Keep** doing this

3. Please **Quit** doing this

4. Please **Start** doing this
Outline

So far
- BSTs are efficient for insert and remove operations, but in worst case they take linear time – $O(n)$.
- If we keep BSTs ‘balanced’, we can avoid the worst case.
- We can easily maintain a balanced BSTs using the AVL balance condition

Today
- Maintaining AVL balance condition
- (Maybe) Intro to Hash tables
AVL trees: Balanced BSTs

**AVL Trees** must satisfy the following properties:
- **binary trees**: every node must have between 0 and 2 children
- **binary search tree (BST property)**: for every node, all keys in the left subtree must be smaller and all keys in the right subtree must be larger than the root node
- **Balanced (AVL property)**: for every node, there can be no more than a difference of 1 in the height of the left subtree from the right. \( \text{Math.abs(height(left subtree) – height(right subtree))} \leq 1 \)

AVL stands for Adelson-Velsky and Landis (the inventors of the data structure)

The AVL property:

1. ensures depth is always \( O(\log n) \) – Yes!
2. is easy to maintain – Yes! (using single and double rotations)
Insertion

What happens if when we do an insert(3), we break the AVL condition?
AVL Example: 8, 9, 10
AVL Example: 8,9,10
(Q1) Answer the following questions for the corresponding tree (on the right):

A. Is this a BST? (Y/N):
B. Highlight the AVL unbalanced node:
C. Is this a ‘line’ or ‘kink’ case?
D. To make this AVL balanced, how many rotations do you need? (single/double)
Left Rotation

Rest of the tree

UNBALANCED
Right subtree is 2 longer

BALANCED
Right subtree is 1 longer
It Gets More Complicated

There’s a “kink” in the tree where the insertion happened.

1
   ↓
  3
   ↓
  2

Can’t do a left rotation
Do a “right” rotation around 3 first.

1
   ↓
  2
   ↓
  3

Now do a left rotation.

2
   ↓
  1
   ↓
  3
Four cases to consider

<table>
<thead>
<tr>
<th>Insert location</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left subtree of left child of y</td>
<td>Single right rotation</td>
</tr>
<tr>
<td>Right subtree of left child of y</td>
<td>Double (left-right) rotation</td>
</tr>
<tr>
<td>Left subtree of right child of y</td>
<td>Double (right-left) rotation</td>
</tr>
<tr>
<td>Right subtree of right child of y</td>
<td>Single left rotation</td>
</tr>
</tbody>
</table>
Four cases to consider

The “line” case

The “kink” case
Right Left Rotation

Rest of the tree

UNBALANCED
Right subtree is 2 longer

Left subtree is 1 longer

BALANCED
Right subtree is 1 longer
AVL Example: 8, 9, 10, 12, 11
AVL Example: 8,9,10,12,11
AVL Example: 8, 9, 10, 12, 11
Worksheet (Q10A)
Worksheet (Q10B)
How Long Does Rebalancing Take?

Assume we store in each node the height of its subtree. How do we find an unbalanced node?

How many rotations might we have to do?
How Long Does Rebalancing Take?

Assume we store in each node the height of its subtree.

How do we find an unbalanced node?
- Just go back up the tree from where we inserted.

How many rotations might we have to do?
- Just a single or double rotation on the lowest unbalanced node.
- A rotation will cause the subtree rooted where the rotation happens to have the same height it had before insertion.
Lots of cool Self-Balancing BSTs out there!

Popular self-balancing BSTs include:

- AVL tree
- Splay tree
- 2-3 tree
- AA tree
- Red-black tree
- Scapegoat tree
- Treap

(Not covered in this class, but several are in the textbook and all of them are online!)

(From https://en.wikipedia.org/wiki/Self-balancing_binary_search_tree#Implementations)