CSE 373: Data Structures and Algorithms

Binary Search and BSTs

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Shrirang (Shri) Mare
shri@cs.washington.edu

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Administrivia

Due dates:

- HW2 Part 2 due Friday 11:59pm
- HW1 grades will be released later today

Modeling recursion: Unfolding Method

$$T(n) = \begin{cases} C_1 & \text{when} \quad n = 0, 1 \\ C_2 + T(n-1) & \text{otherwise} \end{cases}$$

$$T(n) = C_2 + T(n-1)$$

$$T(n) = C_2 + C_2 + T(n-2)$$

$$T(n) = C_2 + C_2 + C_2 + T(n-3)$$

$$T(n) = C_2 + C_2 + C_2 + C_2 + \cdots + C_2 + T(2)$$

$$T(n) = C_2 + C_2 + C_2 + C_2 + \cdots + C_2 + C_2 + T(1)$$

$$T(n) = C_2 + C_2 + C_2 + C_2 + \cdots + C_2 + C_2 + C_1$$

$$T(n) = \sum_{i=0}^{n-2} C_2 + C_1 \qquad T(n) = (n-1)C_2 + C_1$$

Modeling binary search recursion

Question: Find the closed form for T(N)

$$T(n) = \begin{cases} C_1 & \text{when} \quad n = 1 \\ C_2 + T\left(\frac{n}{2}\right) & \text{when} \quad n > 1 \end{cases}$$

$$T(n) = C_2 + T\left(\frac{n}{2}\right)$$

$$T(n) = C_2 + \left(C_2 + T\left(\frac{n}{4}\right)\right)$$

$$T(n) = C_2 + C_2 + \left(C_2 + T\left(\frac{n}{8}\right)\right)$$

$$T(n) = \underbrace{C_2 + C_2 + \cdots + C_2}_{\text{t times}} + T\left(\frac{n}{2^t}\right)$$

We want to find a t such that $\frac{n}{2^t} = 1$

Solving for t we get $t = log_2 n$

$$T(n) = C_2 log_2 n + C_1$$

Storing Sorted Items in an Array

get() – O(logn)

put() – O(n)

remove() – O(n)

Can we do better with insertions and removals?

Trees!

A **tree** is a collection of nodes
- Each node has at most 1 parent and 0 or more children

Root node: the single node with no parent, “top” of the tree

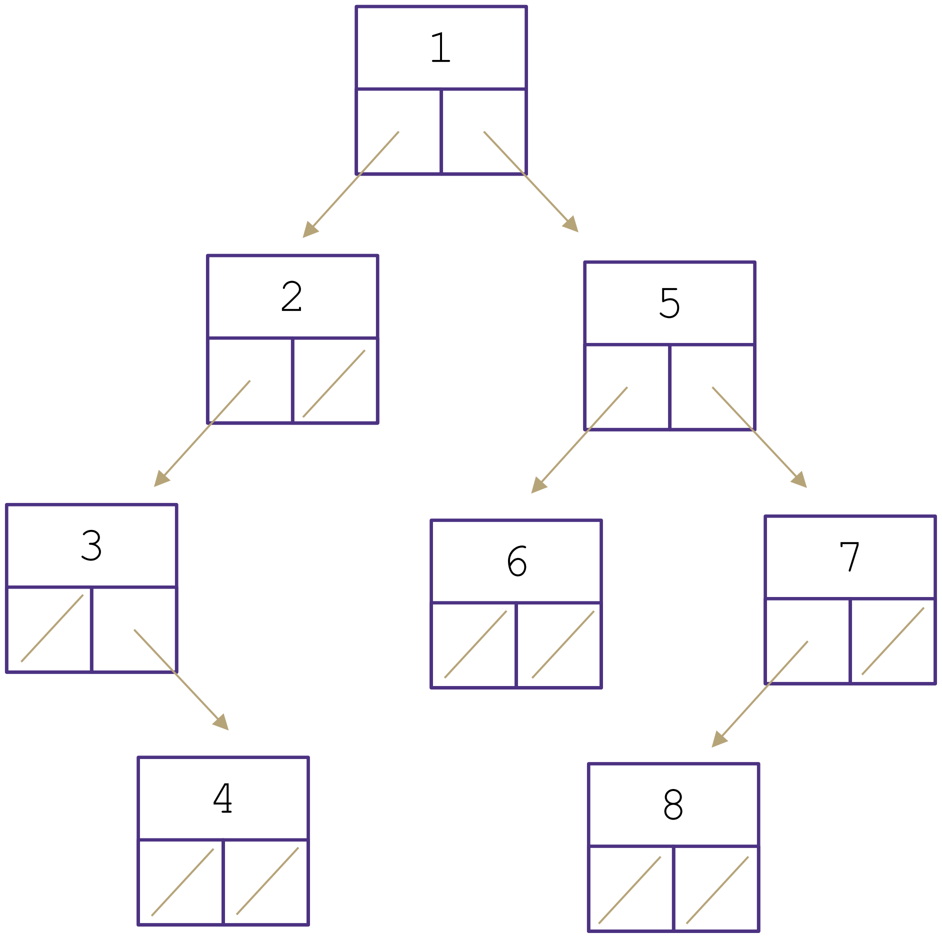
Branch node: a node with one or more children

Leaf node: a node with no children

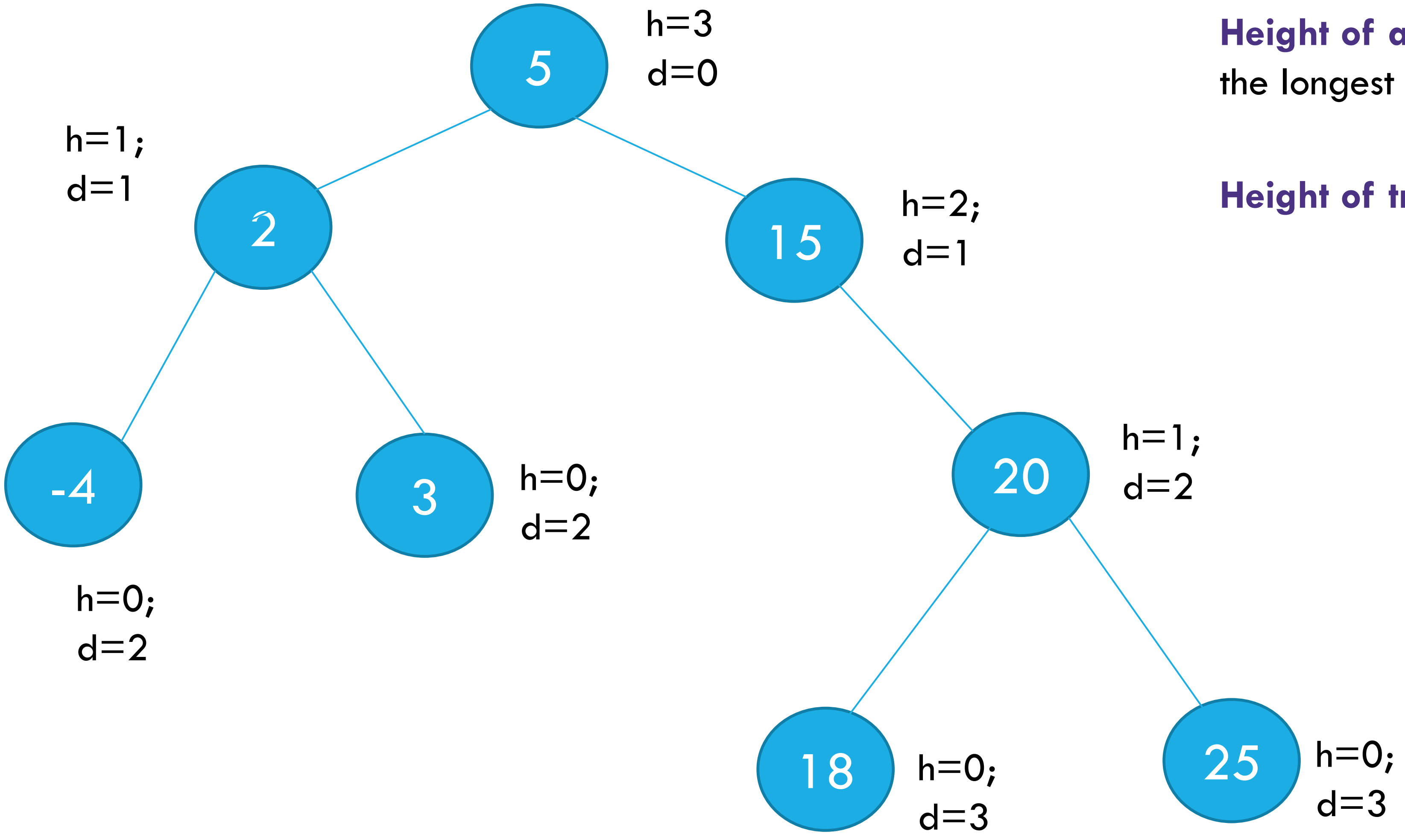
Edge: a pointer from one node to another

Subtree: a node and all it descendants

Height: the number of edges contained in the longest path from root node to some leaf node



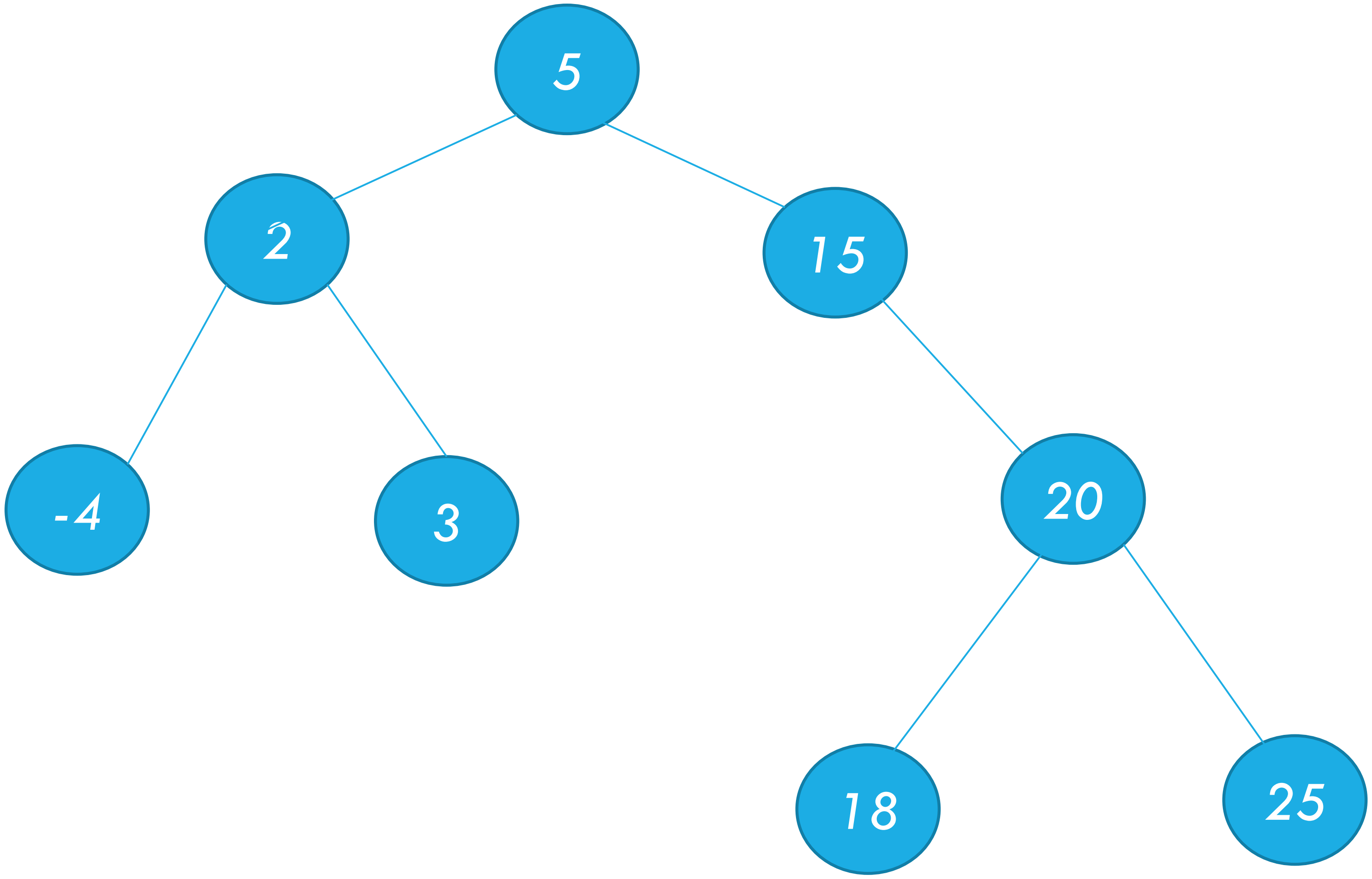
Binary Tree – Height and Depth



Height of a node: the number of edges contained in the longest path from the node to some leaf node

Height of tree = height of root node

Binary Search Tree – O(h) search



Unbalanced Trees

Is this a valid Binary Search Tree?

Yes, but…

We call this a degenerate tree

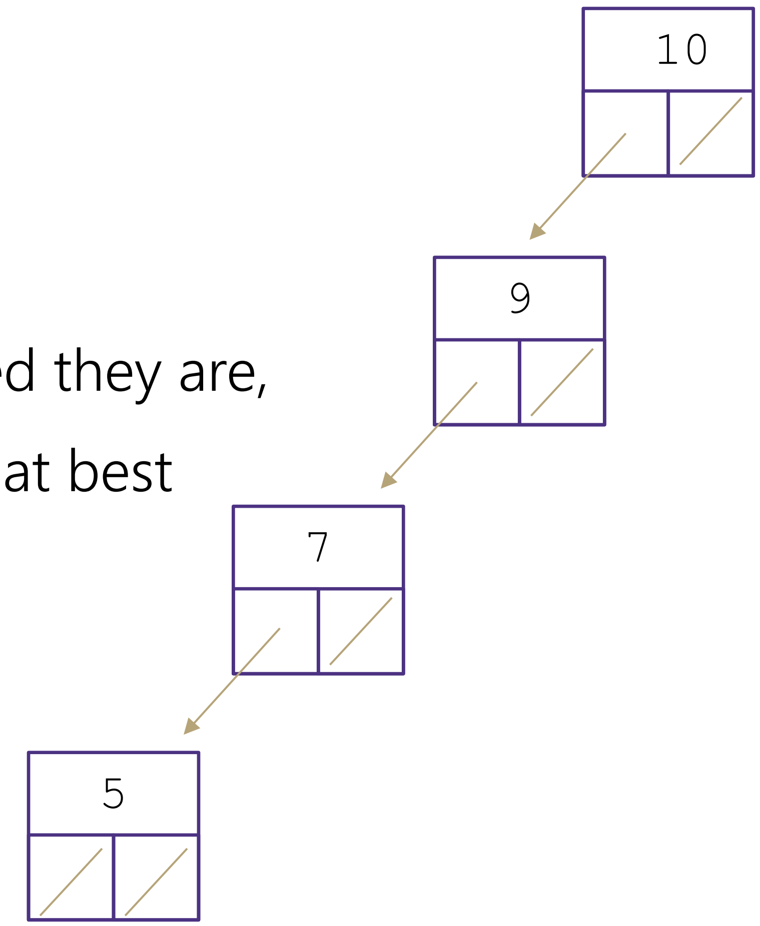
For trees, depending on how balanced they are,

Operations at worst can be O(n) and at best

can be O(logn)

How are degenerate trees formed?

- insert(10)
- insert(9)
- insert(7)
- insert(5)



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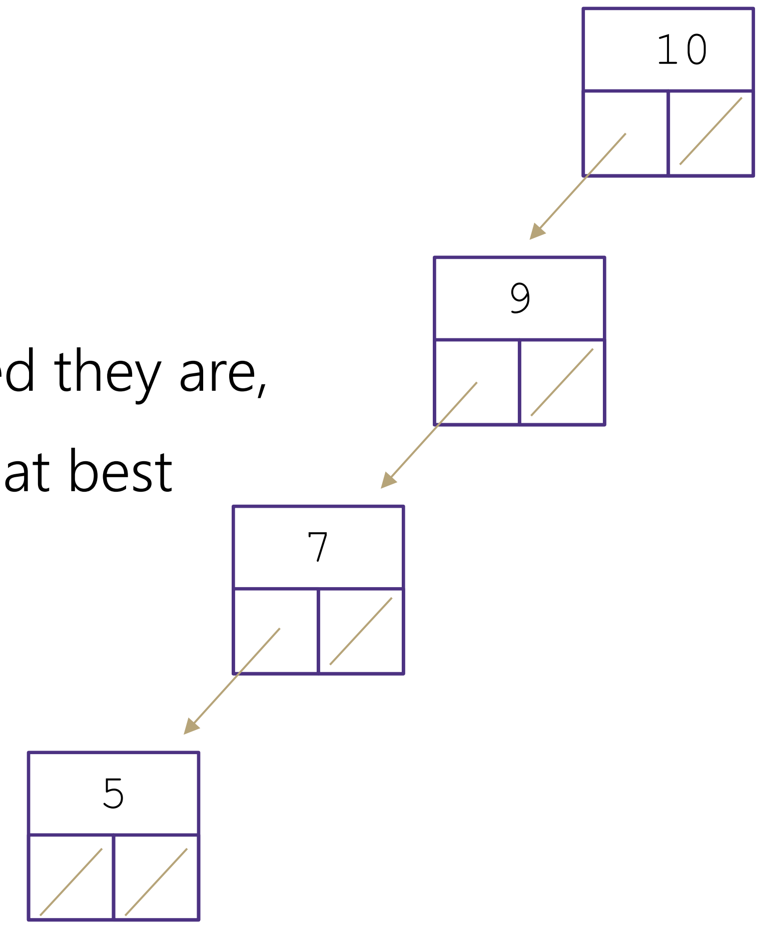
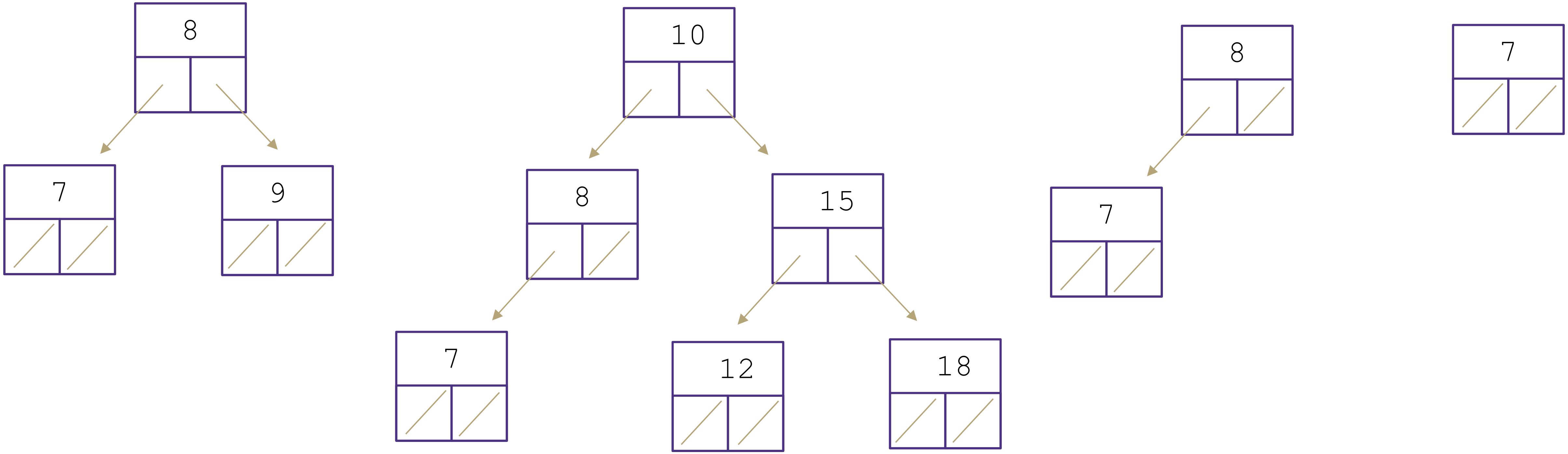


Measuring Balance

Measuring balance:

For each node, compare the heights of its two sub trees

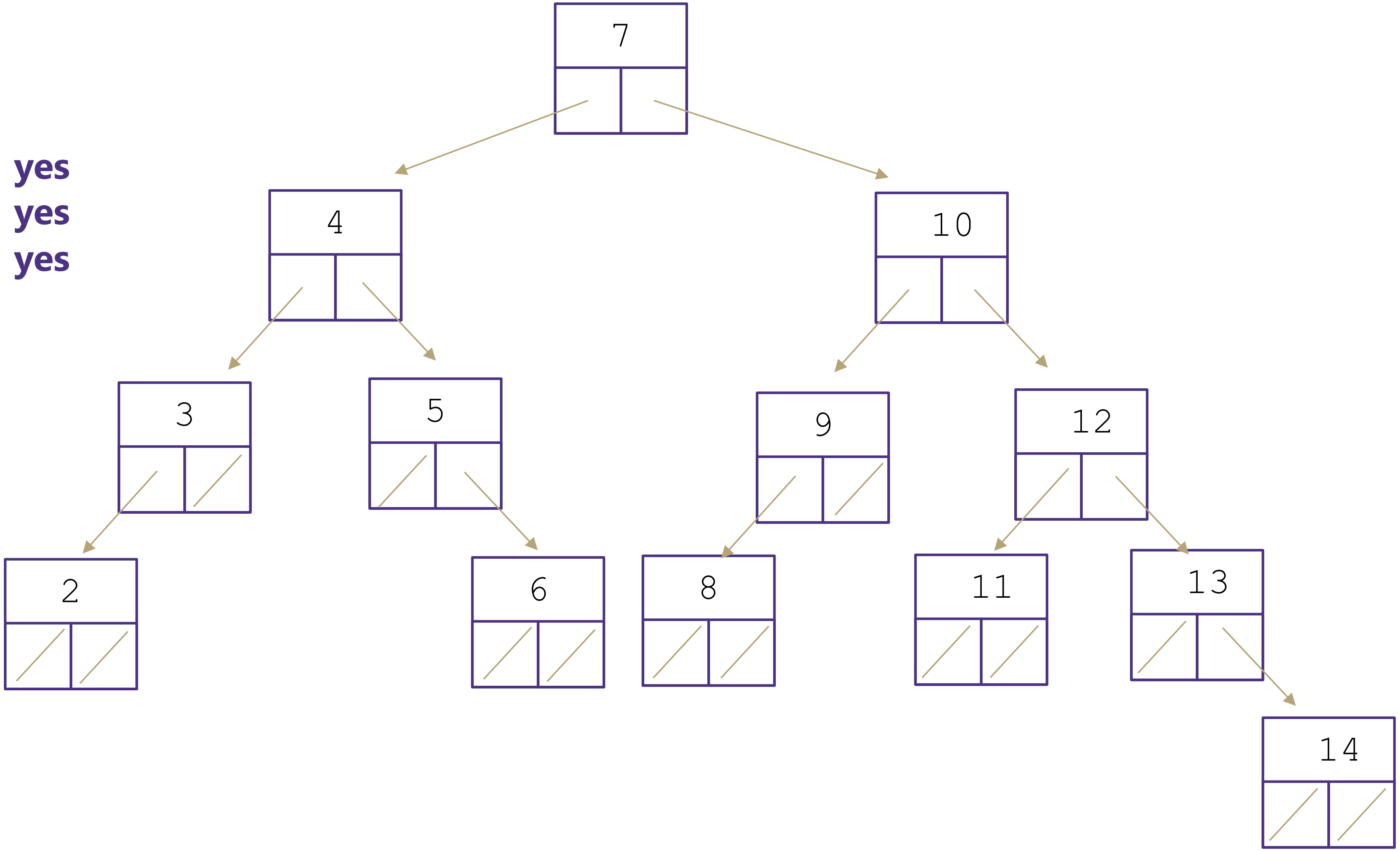
Balanced when the difference in height between sub trees is no greater than 1



Is this a valid AVL tree?

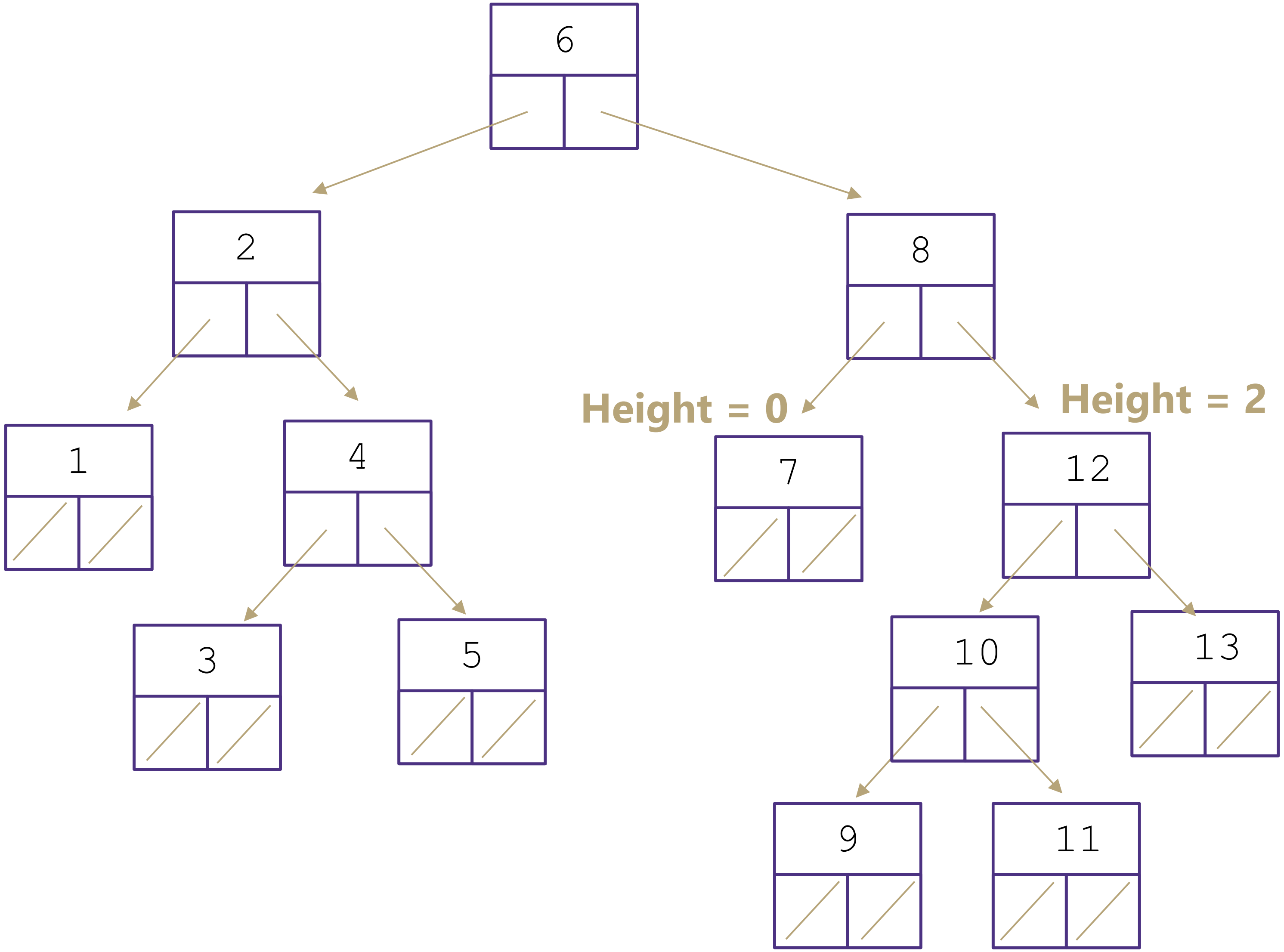
Is it…

- Binary **yes**
- BST **yes**
- Balanced? **yes**



Is this a valid AVL tree?

Is it…
- Binary **yes**
- BST **yes**
- Balanced? **no**



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Is it…
- Binary **yes**
- BST **no**
- Balanced? **yes**